# Grade 10 Tamil Nadu Mathematics 2014

## PART – I

Q1. If $\{(7,11),(5,a)\}$ represents a constant function, then the value of ' $a$ ' is:

(a) 7

(b) 11

(c) 5

(d) 9

**Solution:**

Correct answer: (b)

In constant functions the image of every element is equal or the output of every input is constant.

$\therefore a=11$

Q2. If $a, b, c$ are in A.P. then $\frac{a-b}{b-c}$ is equal to:

(a) $\frac{a}{b}$

(b) $\frac{b}{c}$

(c) $\frac{a}{c}$

(d) 1

**Solution:**

Correct answer: (d)

Since $a, b, c$ are in A.P.

$\therefore b-a=c-b$ ($\because$common difference is same for A.P.)

$\Rightarrow a-b=b-c$

$$\Rightarrow \frac{a-b}{b-c}=1$$

Q3. If the $n^{\text{th}}$ term of an A.P. is $t_n=3-5n$, then the sum of the first $n$ terms is:

(a) $\frac{n}{2}[1-5n]$

(b) $n(1-5n)$

(c) $\frac{n}{2}(1+5n)$

(d) $\frac{n}{2}(1+n)$

**Solution:**

Correct answer: (a)

It is given that the nth term of A.P is $t_n = 3 - 5n$, therefore, the first term $a = t_1$ will be obtained by substituting $n = 1$ in $t_n = 3 - 5n$ as follows:

$t_1 = 3 - (5 \times 1) = 3 - 5 = -2$

We know that the sum of an arithmetic progression with first term $a$ and last term $t_n$ is $S_n = \frac{n}{2}(a + t_n)$, therefore, the sum is:

$$S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}(-2 + 3 - 5n) = \frac{n}{2}(1 - 5n)$$

Hence, the sum of the first n terms is $\frac{n}{2}(1 - 5n)$.

Q4. If $x^2 + 5kx + 16 = 0$ has no real roots, then:

(a) $\mathrm{k} > \frac{8}{5}$

(b) $\mathrm{k} > -\frac{8}{5}$

(c) $-\frac{8}{5} < \mathrm{k} < \frac{8}{5}$

(d) $0 < \mathrm{k} < \frac{8}{5}$

**Solution:**

Correct answer: (c)

The discriminant of $x^2 + 5kx + 16 = 0$ is

$D = \sqrt{25k^2 - 64}$

For no real roots $D < 0$

$\Rightarrow \sqrt{25k^2 - 64} < 0$

$\Rightarrow 25k^2 - 64 < 0$

$$\Rightarrow k^2 < \frac{64}{25}$$

$$\Rightarrow |k| < \frac{8}{5}$$

$$\Rightarrow \frac{-8}{5} < \mathrm{k} < \frac{8}{5}$$

Q5. The system of equations $x - 4y = 8, 3x - 12y = 24$

(a) has infinitely many solutions

(b) has no solution

(c) has a unique solution

(d) may or may not have a solution

**Solution:**

Correct answer: (a)

The second equation is equal to the first one multiplied by 3, so basically you have two equations representing two coincident lines (technically one on top of the other) so your system has Infinite solutions corresponding to the Infinite number of common points between the two lines.

Q6. If $A$ is of order $3 \times 4$ and $B$ is of order $4 \times 3$ then the order of $BA$ is:

(a) $3 \times 3$

(b) $4 \times 4$

(c) $4 \times 3$

(d) not defined

**Solution:**

Correct answer: (b)

If matrix A of order $m \times n$ and matrix B is of order $n \times p$,then order of AB is $m \times p$.

Here, matrix A of order $3 \times 4$ and matrix B is of order $4 \times 3$.

$\therefore$Order of BA is $4 \times 4$.

Q7. The $x$ and $y$ intercepts of the line $2x - 3y + 6 = 0$, respectively are:

(a) 2,3

(b) 3,2

(c) $-3,2$

(d) $3,-2$

**Solution:**

Correct answer: (c)

For $x$-intercept, we put $y = 0$

$\Rightarrow 2x + 6 = 0 \Rightarrow x = -3$

Hence, $x$-intercept is $(-3,0)$

For y-intercept, we put $x = 0$

$\Rightarrow -3y + 6 = 0 \Rightarrow y = 2$

Hence, $y$-intercept is $(0,2)$

Q8. Area of the triangle formed by the points $(0,0),(2,0)$ and $(0,2)$ is:

(a) 1 sq. unit

(b) 2 sq. units

(c) 4 sq. units

(d) 8 sq. units

**Solution:**

Correct answer: (b)

Area of triangle having vertices $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ is given by

$$\text{Area } = \frac{1}{2} \times [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

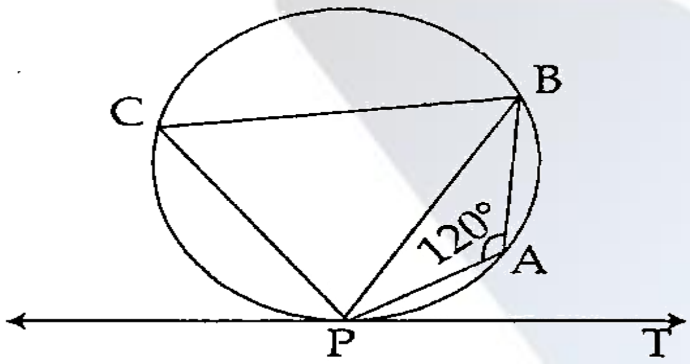
Therefore,

$$= \frac{1}{2} |0(0-2) + 2(2-0) + 0(0-0)|$$

$= \frac{1}{2}|0 + 4 + 0|$

$= \frac{1}{2} \times 4 = 2$ sq. units

Q9. In the figure, if ∠PAB = 120° then ∠BPT =




(a) 120°

(b) 30°

(c) 40°

(d) 60°

**Solution:**

Correct answer: (d)

From the figure, we find that ABCP is a cyclic quadrilateral

∵Opposite angles are supplementary

$\therefore \angle BAP + \angle PCB = 180°$

$\Rightarrow \angle PCB = 180° - \angle BAP$

$\Rightarrow \angle PCB = 180° - 120°$

$\Rightarrow \angle PCB = 60°$

Now, using Tangent–Chord theorem

$\angle PCB = \angle BPT$

$\Rightarrow \angle BPT = 60°$

Q10. The perimeter of two similar triangles △ ABC and △ DEF are 36 cm and 24 cm respectively. If DE = 10 cm then AB is:

(a) 12 cm

(b) 20 cm

(c) 15 cm

(d) 18 cm

**Solution:**

Correct answer: (c)

$\because \triangle ABC \cong \triangle DEF$

$\therefore$ Perimeter of $\frac{\Delta ABC}{\Delta DEF} = \frac{AB}{DE}$

$$\Rightarrow \frac{36}{24} = \frac{AB}{10}$$

$$\Rightarrow AB = \frac{36 \times 10}{24} = 15\ cm$$

$\therefore AB = 15\ cm$

Q11. $\sin(90^\circ - \theta)\cos\ \theta + \cos(90^\circ - \theta)\sin\ \theta =$

(a) 1
(b) 0
(c) 2
(d) -1

**Solution:**

Correct answer: (a)

$sin\ (90^\circ - \theta)cos\ \theta + cos\ (90^\circ - \theta)sin\ \theta$

$= cos\ \theta\ cos\ \theta + sin\theta sin\ \theta$

$= cos^2\ \theta + sin^2\ \theta = 1$

Q12. $9\tan^2\ \theta - 9\sec^2\ \theta =$

(a) 1
(b) 0
(c) 9
(d) $-9$

**Solution:**

Correct answer: (d)

$9\tan^2\theta - 9\sec^2\theta = 9(\tan^2\theta - \sec^2\theta) = 9(-1) = -9$

Q13. Curved surface area of solid sphere is 24 $cm^2$. If the sphere is divided into two hemispheres, then the total surface area of one of the hemispheres is:

(a) 12 $cm^2$
(b) 8 $cm^2$
(c) 16 $cm^2$
(d) 18 $cm^2$

**Solution:**

Correct answer: (d)

Curved surface area of solid sphere= $4\pi r^2$

$\Rightarrow 4\pi r^2 = 24$

$\Rightarrow \pi r^2 = 6$

Now, total surface area of hemisphere= $3\pi r^2$

$\therefore$Total surface area of hemisphere= $3(6) = 18cm^2$

Q14. For any collection of n items, $\Sigma(x-\bar{x})=$

(a) $\Sigma x$

(b) $\bar{x}$

(c) $n\bar{x}$

(d) 0

**Solution:**

Correct answer: (d)

The given expression is sum of deviations of items from the actual mean, which is always 0.

Q15. Probability of sure event is:

(a) 1

(b) 0

(c) 100

(d) 0.1

**Solution:**

Correct answer: (a)

The probability of a sure event is always 1.

## PART – II

Q16. If $A=\{4,6,7,8,9\}, B=\{2,4,6\}$ and $C=\{1,2,3,4,5,6\}$ then find $A \cup (B \cap C)$.

**Solution:**

$B=\{2,4,6\}$

$C=\{1,2,3,4,5,6\}$

$B \cap C=\{2,4,6\}$

A $=\{4,6,7,8,9\}$

$A \cup (B \cap C)=\{2,4,6,7,8,9\}$

Q17. Let $x=\{1,2,3,4\}$. Examine whether the relation given below is a function from $x$ to $x$ or not. Explain. $f=\{(2,3),(1,4),(2,1),(3,2),(4,4)\}$.

**Solution:**

Given, $x=\{1,2,3,4\}$, $f=\{(2,3),(1,4),(2,1),(3,2),(4,4)\}$

Thus, $f$ is not a function because 2 has two images 3 and 1.

Q18. Find the $17^{\text{th}}$ term of the A.P.: 4,9,14 … …

**Solution:**

$a=4, d=5$

$t_n = a + (n-1)d$
$t_{17} = 4 + (17-1)5$
$t_{17} = 4 + (16 \times 15)$
$t_{17} = 4 + 80 = 84.$

Q19. Find the GCD of the following: $m^2 - 3m - 18, m^2 + 5m + 6$.

**Solution:**

$m^2 - 3m - 18 = (m+3)(m-6)$
$m^2 + 5m + 6 = (m+2)(m+3)$
GCD of $m^2 - 3m - 18, m^2 + 5m + 6 = (m+3)$

Q20. Form the quadratic equation whose roots are $7+\sqrt{3}$ and $7-\sqrt{3}$.

**Solution:**

Sum of the roots $= \alpha + \beta = 7 + \sqrt{3} + 7 - \sqrt{3} = 14$
Product of the roots $= \alpha\beta = (7+\sqrt{3})(7-\sqrt{3}) = 7^2 - (\sqrt{3})^2$
$= 49 - 3 = 46$
Required equation is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$
$x^2 - 14x + 46 = 0$

Q21. If $A = \begin{pmatrix} 1 & 3 \\ 9 & -6 \end{pmatrix}$ then verify $AI = IA = A$, where $I$ is the unit matrix of order 2.

**Solution:**

$A = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$ $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$AI = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix} \rightarrow (1)$
$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix} \rightarrow (2)$
From (1) and (2)
$AI = IA = A$

Q22. The coordinates of the midpoint of the line segment joining the points $(2a+2,3)$ and $(4,2b+1)$ are $(2a,2b)$. Find the values of $a$ and $b$.

**Solution:**

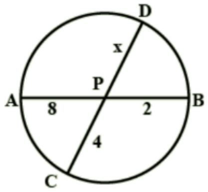
Given points $(x_1, y_1) = (2a+2, 3)$ and $(x_2, y_2) = (4, 2b+1)$
Mid-point $(x,y) = \left(\frac{x_1 x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2a+2+4}{2}, \frac{3+2b+1}{2}\right)$
$\left(\frac{x_1 x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2a+6}{2}, \frac{2b+4}{2}\right) = (2a, 2b)$
Comparing $x$ −coordinates, we get
$\frac{2a+6}{2} = 2a$

$2a + 6 = 4a$
$2a = 6$
$a = 3$
Comparing $y$ −coordinates, we get
$\frac{2b + 4}{2} = 2b$
$2b + 4 = 4b$
$2b = 4$
$b = 2$

Q23. $AB$ and $CD$ are two chords of a circle which intersect each other internally at $P$. If $CP = 4$ cm, AP = 8 cm, PB = 2 cm, then find PD.

**Solution:**

Given, $CP = 4\ cm,\ AP = 8\ cm$ and $PB = 2\ cm$.




Let $PD = x$
We have $AP \times PB = CP \times PD$
$\Rightarrow 8 \times 2 = 4 \times x$
$\Rightarrow 16 = 4x$
$\Rightarrow x = 4$
Thus, $PD = 4\ cm$

Q24. Prove the following identity
$\sqrt{\sec^2\ \theta + \operatorname{cosec}^2\theta} = \tan\ \theta + \cot\ \theta$

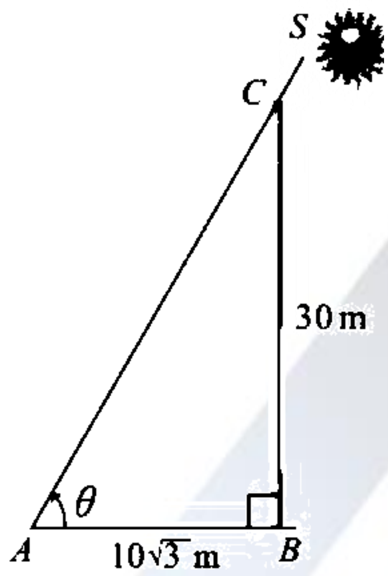
**Solution:**

By using trigonometric identities,
$1 + \tan^2\ A = \sec^2\ A$
$\cot^2\ A + 1 = \operatorname{cosec}^2\ A$
LHS: $\sqrt{(\sec^2\ \theta + \operatorname{cosec}^2\ \theta)}$
$\sec^2\ \theta = 1 + \tan^2\ \theta$
$\operatorname{cosec}^2\ \theta = 1 + \cot^2\ \theta$
So, $\sqrt{(\sec^2\ \theta + \operatorname{cosec}^2\ \theta)} = \sqrt{(1 + \tan^2\ \theta + 1 + \cot^2\ \theta)}$
$= \sqrt{(\tan^2\ \theta + 2 + \cot^2\ \theta)}$
By using algebraic identity,
$(a + b)^2 = a^2 + 2ab + b^2$

Comparing (1) and (2)
$a^2 = \tan^2\ \theta$
$2ab = 2\tan\ \theta \cot\ \theta$
$b^2 = \cot^2\ \theta$
So, $\sqrt{(\tan^2\ \theta + 2 + \cot^2\ \theta)} = \sqrt{(\tan^2\ \theta + 2\tan\ \theta\cot\ \theta + \cot^2\ \theta)}$
We know that $\tan\ \theta \times \cot\ \theta = 1$
Now, $\sqrt{(\tan^2\ \theta + 2 + \cot^2\ \theta)} = \sqrt{(\tan\ \theta + \cot\ \theta)^2}$
$\sqrt{(\tan^2\ \theta + 2 + \cot^2\ \theta)} = \tan\ \theta + \cot\ \theta$
$=$ RHS

Q25. Find the angular elevation (angle of elevation from the ground level) of the sun when the length of the shadow of a 30 m long pole is $10\sqrt{3}$ m.

**Solution:**




Let $S$ be the position of the Sun and BC be the pole.
Let AB denote the length of the shadow of the pole.
Let the angular elevation of the Sun be $\theta$.
Given that $AB = 10\sqrt{3}$ m and $BC = 30$ m
In the right $\triangle CAB, \tan\ \theta = \frac{BC}{AB} = \frac{30}{10\sqrt{3}} = \frac{3}{\sqrt{3}}$
$\Rightarrow \tan\ \theta = \sqrt{3} \therefore \theta = 60^\circ$
Thus, the angular elevation of the Sun from the ground level is $60^\circ$.

Q26. A solid right circular cylinder has radius of 14 cm and height of 8 cm. Find its total surface area.

**Solution:**

Given, radius $r = 14\ cm$, height $h = 8$ cm
Total surface area of cylinder $= 2\pi r(h + r)$ sq. units

$= 2 \times \frac{22}{7} \times 14(8 + 14)$

$= 2 \times \frac{22}{7} \times 14 \times 22$

$= 1936$ sq. cm.

Q27. How many litres of water will a hemispherical tank hold whose diameter is 4.2 m?

**Solution:**

Volume of hemispherical tank

$= \frac{2}{3}\pi r^3$ cubic units

$= \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1$

$= 2 \times 22 \times 0.1 \times 2.1 \times 2.1$

$= 19.404\ \text{m}^3$

Q28. Find the standard deviation of the first 10 natural numbers.

**Solution:**

The standard deviation of the first 10 natural number $= \sqrt{\frac{n^2-1}{12}}$

$= \sqrt{\frac{10^2 - 1}{12}}$

$= \sqrt{\frac{100 - 1}{12}}$

$= \sqrt{\frac{\overset{33}{\cancel{99}}}{\underset{4}{\cancel{12}}}}$

$= \sqrt{\frac{33}{4}}$

$= \sqrt{8.25}$

$= 2.87$

Thus, standard deviation of first 10 natural numbers is 2.87 .

Q29. Three dice are thrown simultaneously. Find the probability of getting the same number on all the three dice.

**Solution:**

$n(s) = 216$

Let A denotes the same number on all the three dice.

$A = \{(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5,), (6,6,6)\}$
$\Rightarrow n(A) = 6$
$\Rightarrow P(A) = \frac{n(A)}{n(S)} = \frac{6}{216} = \frac{1}{36}$

Q30. (a) Construct a $2 \times 3$ matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = |2i - 3j|$.

**Solution:**

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$
$a_{ij} = |2i - 3j|$
$a_{11} = |2(1) - (3)(1)| = |2 - 3| = |-1| = 1$
$a_{12} = |2(1) - (3)(2)| = |2 - 6| = |-4| = 4$
$a_{13} = |2(1) - 3(3)| = |2 - 9| = |-7| = 7$
$a_{21} = |2(2) - 3(1)| = |4 - 3| = 1$
$a_{22} = |2(2) - 3(2)| = |4 - 6| = |-2| = 2$
$a_{23} = |2(2) - 3(3)| = |4 - 9| = |-5| = 5$
Therefore, $A = \begin{pmatrix} 1 & 4 & 7 \\ 1 & 2 & 5 \end{pmatrix}$

(b) Find the equations of the straight lines parallel to the coordinate axes and passing through the point $(3, -4)$.

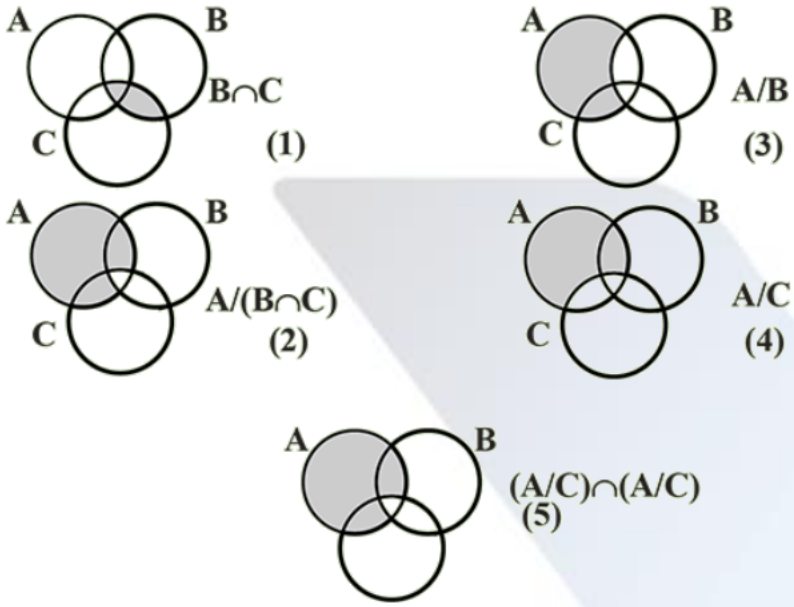
**Solution:**

The equation of line parallel to $x$ is $y = k$
It passes through $(3, -4)$
$\Rightarrow k = -4$
$\therefore$ The equation of line parallel to $x$ axis is $y = -4$.
The equation of line parallel to $y$ axis is $x = k$.
It passes through $(3, -4)$
$\Rightarrow 3 = k$
The equation of line parallel to $y$ axis is $x = 3$.

## PART – III

Q31. Use Venn diagrams to verify De Morgan's law for set difference $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$.

**Solution:**




Q32. A function $f:[1,6) \to R$ is defined as follows:

$$f(x)=\begin{cases}1+x, & 1 \le x < 2\\ 2x-1, & 2 \le x < 4\\ 3x^2-10, & 4 \le x < 6\end{cases} \text{ ( Here, } [1,6)=\{x \in \mathrm{R}: 1 \le x < 6\})$$

Find the values of

(a) $f(5)$

(b) $f(3)$

(c) $f(1)$

(d) $f(2)-f(4)$

(e) $2f(5)-3f(1)$

**Solution:**

Given, $f(x)=\begin{cases}1+x,\ 1 \le x < 2\\ 2x-1,\ 2 \le x < 4\\ 3x^2-10,\ 4 \le x < 6\end{cases}$

a) $f(5)=3(5)^2-10$

$=3(25)-10$

$=75-10=65$

b) $f(3)=2(3)-1=6-1=5$

c) $f(1)=1+1=2$

d) $f(2)=2(2)-1=4-1=3$

$f(4)=3(4)^2-10=3(16)-10=48-10=38$

$f(2)-f(4)=3-38=-35$

e) $2f(5)-3f(1)=2(65)-3(2)$

$=130-6=124$

Q33. Find the total area of 14 squares whose sides are 11 cm, 12 cm, 13 cm, ……., 24 cm.

**Solution:**

Total area of 14 squares

$= 11^2 + 12^2 + 13^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2) - (1^2 + 2^2 + \cdots + 10^2)$

$= \sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$= \frac{24 \times 25 \times 49}{6} - \frac{10 \times 11 \times 21}{6}$

$= 4900 - 385 = 4515$

Q34. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns downstream to the original point in 4 hrs 30 minutes. Find the speed of the stream.

**Solution:**

Speed of boat in still water = 15 km/hr

Let $x$ be speed of the stream in km/hr

Speed of the boat upstream $= 15 - x$

Speed of the boat down stream $= 15 + x$

Time taken for upstream $T_1 = \frac{30}{15-x}$

Time taken for downstream $\mathrm{T}_2 = \frac{30}{15+\mathrm{x}}$

Given, $\mathrm{T}_1 + \mathrm{T}_2 = \frac{9}{2}$

$= \frac{30}{15-x} + \frac{30}{15+x} = \frac{9}{2}$

$= 30\left(\frac{1}{15-x} + \frac{1}{15+x}\right) = \frac{9}{2}$

$= 30\left(\frac{15+x \div 15-x}{(15-x)(15+x)}\right) = \frac{9}{2}$

$= \frac{30 \times 30}{225-x^2} = \frac{9}{2}$

$= \frac{100}{225-x^2} = \frac{1}{2}$

$\Rightarrow 225 - x^2 = 200$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5 \cdots . (\because x \text{ is } +\text{ve})$

$\Rightarrow x = 5$ km/hr

Speed of the stream = 5 km/hr.

Q35. Simplify: $\frac{a^2-16}{a^3-8} \times \frac{2a^2-3a-2}{2a^2+9a+4} \div \frac{3a^2-11a-4}{a^2+2a+4}$.

**Solution:**

We know, $a^2-16=a^2-4^2=(a+4)(a-4)$
and $a^3-8=a^3-2^3=(a-2)(a^2+2a+4)$
Therefore, $\frac{a^2-16}{a^3-18}\times\frac{2a^2-3a-2}{2a^2+9a+4}\div\frac{3a^2-11a-4}{a^2+2a+4}$
$=\dfrac{a^2-16}{a^3-16}\times\dfrac{2a^2-3a-2}{2a^2+9a+4}\times\dfrac{a^2+2a+4}{3a^2+11a-4}$
$=\dfrac{(a+4)(a-4)}{(a-2)(a^2+2a+4)}\times\dfrac{(2a+1)(a-2)}{(2a+1)(a+4)}\times\dfrac{a^2+2a+4}{(3a+1)(a-4)}$
$=\dfrac{1}{3a+1}$

Q36. If $A=\begin{pmatrix}3 & 2\\ -1 & 4\end{pmatrix}, B=\begin{pmatrix}-2 & 5\\ 6 & 7\end{pmatrix}$ and $C=\begin{pmatrix}1 & 1\\ -5 & 3\end{pmatrix}$ verify that $A(B+C)=AB+AC$.

**Solution:**

Now, $B+C=\begin{pmatrix}-2 & 5\\ 6 & 7\end{pmatrix}+\begin{pmatrix}1 & 1\\ -5 & 3\end{pmatrix}=\begin{pmatrix}-1 & 6\\ 1 & 10\end{pmatrix}$

Thus, $\mathrm{A(B+C)}=\begin{pmatrix}3 & 2\\ -1 & 4\end{pmatrix}\begin{pmatrix}-1 & 6\\ 1 & 10\end{pmatrix}=\begin{pmatrix}-1 & 38\\ 5 & 34\end{pmatrix}\ldots\ldots.(1)$

Now, $AB+AC=\begin{pmatrix}3 & 2\\ -1 & 4\end{pmatrix}\begin{pmatrix}-2 & 5\\ 6 & 7\end{pmatrix}+\begin{pmatrix}3 & 2\\ -1 & 4\end{pmatrix}\begin{pmatrix}1 & 1\\ -5 & 3\end{pmatrix}$

$=\begin{pmatrix}-6+12 & 15+14\\ 2+24 & -5+28\end{pmatrix}+\begin{pmatrix}3-10 & 3+6\\ -1-20 & -1+12\end{pmatrix}$

$=\begin{pmatrix}6 & 29\\ 26 & 23\end{pmatrix}+\begin{pmatrix}-7 & 9\\ -21 & 11\end{pmatrix}$

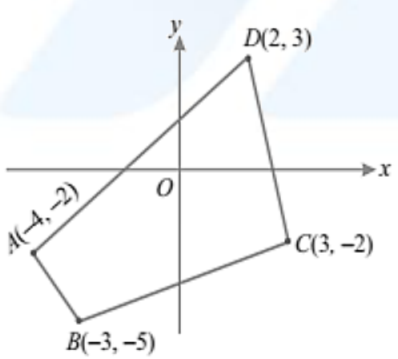
$=\begin{pmatrix}-1 & 38\\ 5 & 34\end{pmatrix}\ldots\ldots.(2)$

From (1) and (2), we have $A(B+C)=AB+AC$.

Q37. Find the area of the quadrilateral formed by the points $(-4,-2),(-3,-5),(3,-2)$ and $(2,3)$.

**Solution:**

Let us plot the points roughly and take the vertices in counter clock-wise direction.




Let the vertices be
A$(-4,-2)$, B$(-3,-5)$, C$(3,-2)$ and D$(2,3)$

Area of the quadrilateral ABCD

$$=\frac{1}{2}\{(x_1y_2+x_2y_3+x_3y_1+x_4y_1)-(x_2y_1+x_3y_2+x_1y_3+x_1y_4)\}$$

$$=\frac{1}{2}[(20+6+9-4)-(6-15-4-12)]$$

$$=\frac{1}{2}(31+25)=28 \text{ sq.units}$$

Q38. Find the equation of the perpendicular bisector of the straight line segment joining the points $(3,4)$ and $(-1,2)$.

**Solution:**

Let the points be:

Point P (3, 4)

Point Q (-1, 2)

The midpoint $M$ of the line segment $PQ$ can be calculated using the midpoint formula:

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Substituting the coordinates of $P$ and $Q$:

$$M=\left(\frac{3+(-1)}{2},\frac{4+2}{2}\right)=\left(\frac{2}{2},\frac{6}{2}\right)=(1,3)$$

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The slope $m_1$ of the line segment PQ can be calculated using the slope formula:

$$m_1=\frac{y_2-y_1}{x_2-x_1}$$

Substituting the coordinates of $P$ and $Q$:

$$m_1=\frac{2-4}{-1-3}=\frac{-2}{-4}=\frac{1}{2}$$

The slope $m_2$ of the perpendicular bisector is the negative reciprocal of $m_1$ :

$$m_2=-\frac{1}{m_1}=-\frac{1}{\frac{1}{2}}=-2$$

Now that we have the slope of the perpendicular bisector and a point through which it passes (the midpoint M), we can use the point-slope form of the equation of a line:

$$y-y_1=m(x-x_1)$$

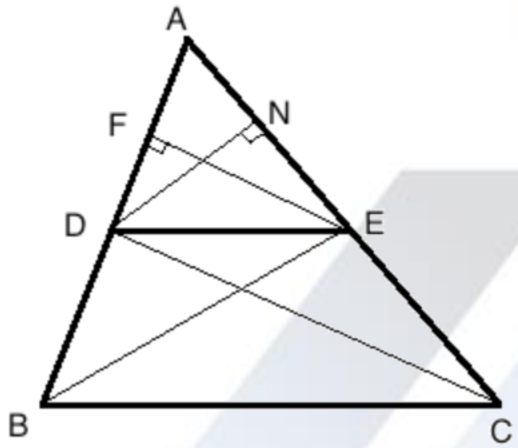
Substituting $m_2=-2$ and the coordinates of $M(1,3)$ :

$y-3=-2(x-1)$
Now we can simplify the equation:
$y-3=-2x+2$
Adding 3 to both sides:
$y=-2x+5$
Rearranging gives:
$2x+y=5$
The equation of the perpendicular bisector of the line segment joining the points $(3,4)$ and $(-1,2)$ is $2x+y=5$

Q39. State and prove Basic Proportionality theorem.

**Solution:**

Statement: If a line is drawn parallel to one side of a triangle, to interest the other two sides at distinct points, the other two sides are divided in the same ratio.

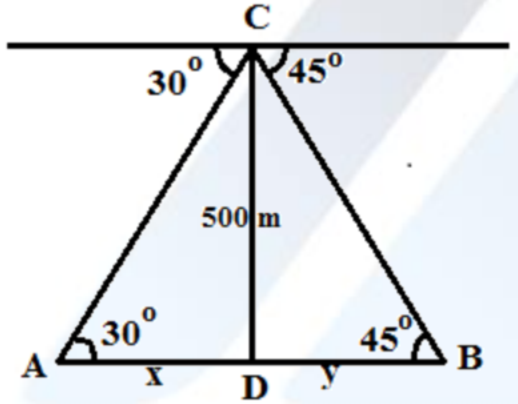



To prove: $\frac{\text{AD}}{\text{BD}}=\frac{\text{AE}}{\text{CE}}$
Consider $\triangle$ ABC.Let DE ∥ BC. Drop FE and DN perpendicular to sides AB and $AC$ respectively.
Now,
Area of $\triangle$ ADE $=\frac{1}{2}\times$ FE $\times$ AD.......(i)
Area of $\triangle$ ADE $=\frac{1}{2}\times$ AE $\times$ DN........(ii)
Also,
Area of $\triangle$ AEB $=\frac{1}{2}\times$ FE $\times$ AB ........(iii)
Area of $\triangle$ ADC $=\frac{1}{2}\times$ AC $\times$ DN.........(iv)
Now, since $\triangle$ BDE and $\triangle$ CED are on the same base DE and between two parallel lines DE and BC , therefore,
Area of $\triangle$ BDE $=$ Area of $\triangle$ CED
Adding area of $\triangle$ ADE on both the sides, we get,

Area of $\triangle$ BDE $+\triangle$ ADE $=$ Area of $\triangle$ CED $+\triangle$ ADE
$\Rightarrow$ Area of $\triangle$ AEB $=$ Area of $\triangle$ ADC ........(v)
Now, (i) $\div$ (iii), we get,
$\frac{\text{ar}\Delta\text{ADE}}{\text{ar}\Delta\text{ADC}} = \frac{\frac{1}{2}\times\text{FE}\times\text{AD}}{\frac{1}{2}\times FE\times AB} = \frac{\text{AD}}{\text{AB}}$ ........(vi)
Now, (ii) $\div$ (iv), we get,
$\frac{\text{ar }\Delta\text{ADE}}{\text{ar }\Delta\text{AEB}} = \frac{\frac{1}{2}\times\text{AE}\times\text{DN}}{\frac{1}{2}\times\text{AC}\times\text{DN}} = \frac{\text{AE}}{\text{AC}}$........(vii)
From (v), (vi) and (vii), we get,
$\Rightarrow \frac{\text{AD}}{\text{AB}} = \frac{\text{AE}}{\text{AC}}$ or $\frac{\text{AB}}{\text{AD}} = \frac{\text{AC}}{\text{AE}}$
Subtracting 1 from both sides, we get,
$\Rightarrow \frac{\text{AB}-\text{AD}}{\text{AD}} = \frac{\text{AC}-\text{AE}}{\text{AE}}$
$\Rightarrow \frac{\text{BD}}{\text{AD}} = \frac{\text{CE}}{\text{AE}}$
Thus, $\frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{CE}}$.

Q40. A person in a helicopter flying at a height of 500 m, observes two objects lying opposite to each other on either bank of a river. The angles of depression of the objects are 30° and 45°. Find the width of the river. ($\sqrt{3} = 1.732$)

**Solution:**




Let AB be the width of the river.
$A$ and $B$ are two points on the opposite water level.
Let C be the helicopter.
Let AD $= x$, BD $= y$
$\triangle ACD$ and $\triangle BCD$,
$\Rightarrow \tan\ 30^\circ = \frac{\text{CD}}{\text{AD}}$ and $\tan\ 45^\circ = \frac{\text{CD}}{\text{DB}}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{500}{x} \text{ and } 1 = \frac{500}{y}$$

$\Rightarrow x = 500\sqrt{3}$ and $y = 500$

The width of river $= x + y$

$= 500\sqrt{3} + 500$

$= 500(\sqrt{3} + 1)$

$= 500(1.732 + 1)$

$= 500(2.732)$

$= 1366$ m

Q41. The perimeter of the ends of a frustum of a cone are 44 cm and $8.4\pi$ cm. If the depth is 14 cm, then find its volume.

**Solution:**

Height $h = 14$ cm

Given, $2\pi R = 44\ cm$ and $2\pi r = 8.4\pi$

$$\Rightarrow 2 \times \frac{22}{7} \times R = 44 \text{ and } 2 \times \frac{22}{7} \times r = 8.4$$

$\Rightarrow R = 7$ cm and $r = 4.2$ cm

Volume of the frustum $= \frac{1}{3}\pi\, h(R^2 + r^2 + Rr)$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 = (7^2 + (4.2)^2 + 7(4.2))$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 = (49 + 17.64 + 29.4)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14 \times 96.04 = 1408.58 \text{ cm}^3$$

$= 1408.6 \text{ cm}^3$

Q42. Using clay, a student made a right circular cone of height 48 cm and base radius 12 cm. Another student reshapes it in the form of a sphere. Find the radius of the sphere.

**Solution:**

Consider the dimensions of cone

Given, height $h = 48$ cm, radius $r = 12$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$ cubic cm

$$= \frac{1}{3}\pi(12)^2(48) \text{ cubic cm}$$

$= \pi(12)^2(16)$ cubic cm

Let $r$ be the radius of sphere.

Volume of sphere and cone will be equal.

Volume of sphere $= \pi(12)^2(16)$

$\Rightarrow \frac{4}{3}\pi r^3 = \pi(12)^2(16)$

$\Rightarrow \frac{4}{3} \times \pi \times r^3 = \pi \times 12 \times 12 \times 16$

$\Rightarrow r^3 = 3 \times 3 \times 12 \times 16$

$\Rightarrow r^3 = 3 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\Rightarrow r = 3 \times 2 \times 2 = 12\ cm$

Thus, radius of sphere = 12 cm.

Q43. Calculate the standard deviation of the following data:

| $x$ | 3 | 8 | 13 | 18 | 23 |
|---|---|---|---|---|---|
| $f$ | 7 | 10 | 15 | 10 | 8 |

**Solution:**

Assumed mean $A = 13, d = x - A = x - 13$

| $x$ | $f$ | $d = x - 13$ | $d^2$ | $fd$ | $fd^2$ |
|---|---|---|---|---|---|
| 3 | 7 | $-10$ | 100 | $-70$ | 700 |
| 8 | 10 | $-5$ | 25 | $-50$ | 250 |
| 13 | 15 | 0 | 0 | 0 | 0 |
| 18 | 10 | 5 | 25 | 50 | 250 |
| 23 | 8 | 10 | 100 | 80 | 800 |
| | $\sum f = 50$ | | | $\sum fd = 10$ | $\sum fd^2 = 2000$ |

Standard deviation $= \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$

$= \sqrt{\frac{2000}{50} - \left(\frac{10}{50}\right)^2}$

$= \sqrt{40 - \frac{1}{25}} = \sqrt{\frac{1000 - 1}{25}}$

$= \sqrt{\frac{999}{25}} = 6.321$

$\therefore \sigma = 6.321$

Q44. Two unbiased dice are rolled once. Find the probability of getting:

(a) a sum 8

(b) a doublet

(c) a sum greater than 8

**Solution:**

$$S=\begin{Bmatrix}(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6)\\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6)\\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6)\\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6)\\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6)\\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)\end{Bmatrix}$$

$n(S)=36$

i) Let A denotes the sum being 8.

$A\{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$n(A)=5$

$$P(A)=\frac{n(A)}{n(S)}=\frac{5}{36}$$

ii) Let B denotes the doublet

$B=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$

$n(B)=6$

$$P(B)=\frac{n(B)}{n(S)}=\frac{6}{36}=\frac{1}{6}$$

iii) Let C denotes the sum greater than 8.

$C=\{(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)\}$

$n(C)=10$

$$P(C)=\frac{n(C)}{n(S)}=\frac{10}{36}=\frac{5}{18}$$

Q45. (a) If $S_1, S_2$ and $S_3$ are the sum of first $n$, $2n$ and $3n$ terms of a geometric series respectively, then prove that $S_1(S_3-S_2)=(S_2-S_1)^2$.

**Solution:**

From given we can write:

$$S_1=\frac{a(r^n-1)}{r-1}$$

$$S_2=\frac{a(r^{2n}-1)}{r-1}$$

$$S_3=\frac{a(r^{3n}-1)}{r-1}$$

$$S_1(S_3-S_2)=\frac{a(r^n-1)}{r-1}\left[\frac{a(r^{3n}-1)}{r-1}-\frac{a(r^{2n}-1)}{r-1}\right]$$

$$=\frac{a^2(r^n-1)}{(r-1)^2}[r^{3n}-1-r^{2n}+1]$$

$$= \frac{a^2(r^n - 1)}{(r-1)^2}[r^{3n} - r^{2n}]$$
$$= \frac{a^2(r^n - 1)}{(r-1)^2}[r^{3n} - 1 - r^{2n} + 1]$$
$$= \frac{a^2(r^n - 1)}{(r-1)^2}[r^{3n} - r^{2n}]$$
$$= \frac{a^2}{(r-1)^2} r^{2n}(r^n - 1)^2 \ldots (1)$$
$$(S_2 - S_1)^2 = \left[\frac{a(r^{2n} - 1)}{r-1} - \frac{a(r^n - 1)}{r-1}\right]^2$$
$$= \frac{a^2}{(r-1)^2}[r^{2n} - 1 - r^n + 1]^2$$
$$= \frac{a^2}{(r-1)^2} r^{2n}(r^n - 1)^2 \ldots (2)$$

From (1) and (2), we have:

$S_1(S_3 - S_2) = (S_2 - S_1)^2$

(b) If $\alpha$ and $\beta$ are the roots of the equation $3x^2 - 4x + 1 = 0$ form a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

**Solution:**

Since $\alpha, \beta$ are are the roots of the equation $3x^2 - 4x + 1 - 0$. we have $\alpha + \beta = \frac{4}{3}, \alpha\beta = \frac{1}{3}$

Now, for the required equation, the sum of the roots $= \left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) = \frac{\alpha^3 + \beta^3}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(\frac{4}{3}\right)^3 - 3 \times \frac{1}{3} \times \frac{4}{3}}{\frac{1}{3}} = \frac{28}{9}$$

Also, product of the roots $= \left(\frac{\alpha^2}{\beta}\right)\left(\frac{\beta^2}{\alpha}\right) = \alpha\beta = \frac{1}{3}$

$\therefore$ The required equation is $x^2 - \frac{28}{9}x + \frac{1}{3} = 0$ or $9x^2 - 28x + 3 = 0$

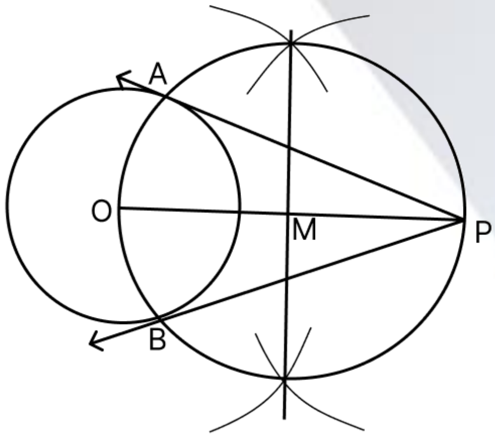
## PART – IV

Q46. (a) Draw a circle of radius 3 cm. From an external point 7 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

**Steps of construction:**

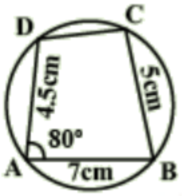
1. With $O$ as centre, draw a circle of radius 3 cm.
2. Draw a line $OP = 7$ cm.
3. Draw a perpendicular bisector of $OP$, which cuts $OP$ at $M$.
4. With $M$ as centre and $MO$ as radius, draw a circle which cuts the previous circles $A$ and $B$.

5. Join $AP$ and $BP$. $AP$ and $BP$ are the required tangents. Length of the tangents
$PA = PB = 6.3$ cm
Verification: In the right-angle triangle OAP
$PA^2 = OP^2 - OA^2 = 7^2 - 3^2 = 49 - 9 = 40$
$PA = \sqrt{40} = 6.3$ cm
Length of the tangents = 6.3 cm.



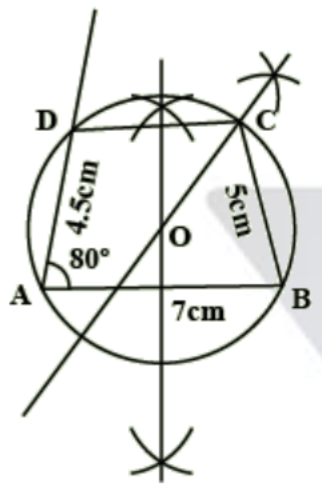

(b) Construct a cyclic quadrilateral ABCD with AB = 7 cm, ∠ A = 80°, AD = 4.5 cm and $BC = 5$ cm.

**Rough Diagram**




Construction:

1. Draw a rough diagram.
2. Draw a line segment AB = 7 cm.
3. Through A draw $AX$ such that $\angle XAB = 80°$.
4. Mark a point $D$ on $AX$ such that $AD = 4.5$ cm.
5. Complete the △ ABD.
6. Draw the ⊥ r of AB and BD, they meet at O.
7. Draw a circle O and radius as OA.
8. Mark a point $C$ on the circle such that BC = 5 cm.
9. ABCD is the required cyclic quadrilateral.




Q47. (a) Draw the graph of $y = 2x^2$ and hence solve $2x^2 + x - 6 = 0$.

$y = 2x^2$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $x^2$ | 9 | 4 | 1 | 0 | 1 | 4 | 9 |
| $y = 2x^2$ | 18 | 8 | 2 | 0 | 2 | 8 | 18 |

The points are $(-3,18), (-2,8), (-1,2), (0,0), (1,2), (2,8), (3,18)$

$y = 2x^2, 2x^2 + x - 6 = 0$

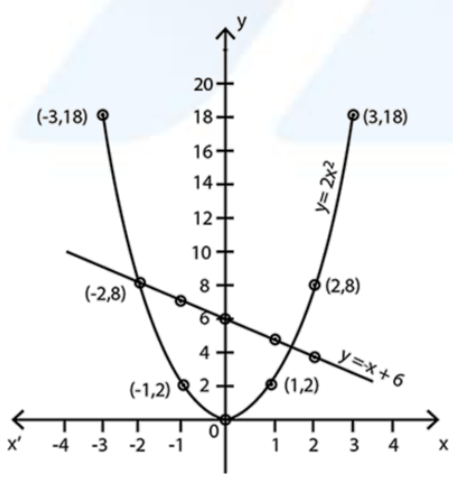
$y + x - 6 = 0$

$y = -x + 6$

| $x$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|
| $-x$ | 1 | 0 | $-1$ | $-2$ |
| 6 | 6 | 6 | 6 | 6 |
| | 7 | 6 | 5 | 4 |

The points are $(-1,7), (0,6), (1,5), (2,4)$

The solution set is $\{-2, 1.5\}$



(b) The cost of the milk per litre is ₹ 15. Draw the graph for the relation between the quantity and cost. Hence find:

(i) the proportionality constant

(ii) the cost of 3 litres of milk

Let $x$ be the quantity of milk (in litre)

$y$ denotes the cost of milk (in ₹)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $y$ | 15 | 30 | 45 | 60 | 75 | 90 | 105 |

The proportionality constant $k = 15$

The cost of 3 litres of milk $= 45$