Grade 10 Tamil Nadu Maths 2015

SECTION - I

Q1. If $f: A \rightarrow B$ is a bijective function and if $n(A) = 5$, then $n(B)$ is equal to:

(a) 10

(b) 4

(c) 5

(d) 25

Solution:

Correct answer: (c)

Given,

f is a bijective function.

That means, f is one-one and onto function.

Therefore, every element of B is an image in f.

Also, f is one-one, i.e. image of every element will be different.

$\Rightarrow n(A) = n(B) = 5$

Q2. The next term of $\frac{1}{20}$ in the sequence $\left(\frac{1}{2}\right), \left(\frac{1}{6}\right), \left(\frac{1}{12}\right), \left(\frac{1}{20}\right), \ldots$ is:

(a) $\frac{1}{24}$

(b) $\frac{1}{22}$

(c) $\frac{1}{30}$

(d) $\frac{1}{18}$

Solution:

Correct answer: (c)

Given,

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \ldots$$

Now, consider the denominator

2, 6, 12, 20,..

$$2 + 4 = 6$$

$$6 + 6 = 12$$

$$12 + 8 = 20$$

$$20 + 10 = 30$$

Hence, the next term is $\frac{1}{30}$

Q3. The common ratio of the GP a^{m-n}, a^m, a^{m+n} is:

(a) a^m

(b) a^{-m}

(c) a^n

(d) a^{-n}

Solution:

Correct answer: (c)

Given,

a^{m-n}, a^m, a^{m+n} are in GP.

Common ratio $= \frac{a^m}{a^{m-n}}$

$= a^{m-(m-n)}$

$= a^{m-m+n}$

$= a^n$

Q4. The remainder when $x^2 - 2x + 7$ is divided by $x + 4$ is:

(a) 28

(b) 29

(c) 30

(d) 31

Solution:

Correct answer: (d)

$$x \quad -6$$
$$x + 4 \enclose{longdiv}{x^2 \quad -2x \quad +7}$$
$$\underline{x^2 \quad +4x}$$
$$-6x \quad +7$$
$$\underline{-6x \quad -24}$$
$$31$$

Therefore, remainder $= 31$

Q5. If $b = a + c$ and the equation $ax^2 + bx + c = 0$ has equal roots, then:

(a) $a = c$

(b) $a = -c$

(c) $a = 2c$

(d) $a = -2c$

Solution:

Correct answer: (a)

Given,

$ax^2 + bx + c = 0$ has equal roots.

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (a + c)^2 - 4ac = 0$ [given $b = a + c$]

$\Rightarrow a^2 + c^2 + 2ac - 4ac = 0$

$\Rightarrow a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - c)^2 = 0$

$\Rightarrow a - c = 0$

$\Rightarrow a = c$

Q6. If $\begin{pmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{pmatrix} = \begin{pmatrix} 1 & y - 2 \\ 8 & 8 \end{pmatrix}$, then the values of x and y respectively are:

(a) $-2, 7$

(b) $-\frac{1}{3}, 7$

(c) $-\frac{1}{3}, -\frac{2}{3}$

(d) $2, -7$

Solution:

Correct answer: (a)

From the given,

$3x + 7 = 1$

$3x = 1 - 7 = -6$

$x = -\frac{6}{3} = -2$

$y - 2 = 5$

$y = 5 + 2 = 7$

Therefore, $x = -2, y = 7$

Q7. The centroid of the triangle with vertices at $(-2, -5), (-2, 12)$ and $(10, -1)$ is:

(a) $(6,6)$

(b) $(4,4)$

(c) $(3,3)$

(d) $(2,2)$

Solution:

Correct answer: (d)

Let the given vertices be:

$(x_1, y_1) = (-2, -5)$

$(x_2, y_2) = (-2, 12)$

$(x_3, y_3) = (10, -1)$

Centroid of the triangle $= \left[\frac{(x_1 + x_2 + x_3)}{3}, \frac{(y_1 + y_2 + y_3)}{3}\right]$

$= \left[\frac{-2 - 2 + 10}{3}, \frac{-5 + 12 - 1}{3}\right]$

$= \left(\frac{6}{3}, \frac{6}{3}\right)$

$= (2,2)$

Q8. The value of k is the straight lines $3x + 6y + 7 = 0$ and $2x + ky = 5$ are perpendicular is:

(a) 1

(b) −1

(c) 2

(d) $\frac{1}{2}$

Solution:

Correct answer: (b)

Given,

$3x + 6y + 7 = 0$

$6y = -3x - 7$

$y = \left(-\frac{3}{6}\right)x - \left(\frac{7}{6}\right)$

Slope $= m_1 = -\frac{3}{6} = -\frac{1}{2}$

and

$2x + ky = 5$

$ky = -2x + 5$

$y = \left(-\frac{2}{k}\right)x + \left(\frac{5}{k}\right)$

Slope $= m_2 = -\frac{2}{k}$

Given that, the two lines are perpendicular.

$m_1 \times m_2 = -1$

$\left(-\frac{1}{2}\right) \times \left(-\frac{2}{k}\right) = -1$

$\frac{1}{k} = -1$

$\Rightarrow k = -1$

Q9. If the sides of two similar triangles are in the ratio $2 : 3$, then their areas are in the ratio:

(a) $9 : 4$

(b) $4:9$

(c) $2:3$

(d) $3:2$

Solution:

Correct answer: (b)

Given,

Ratio of sides of two similar triangles $= 2:3$

Ratio of their areas $=$ Square the ratio of sides

$= 2^2:3^2$

$= 4:9$

Q10. If the tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of $40°$ then $\angle POA =$
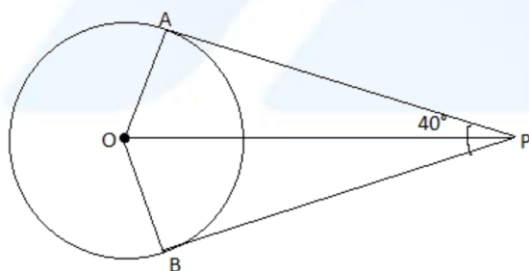
(a) $70°$

(b) $80°$

(c) $50°$

(d) $60°$

Solution:

Correct answer: (a)

Given,

PA and PB are tangents inclined to each other at an angle of $40°$.

We know that,

$\angle AOB + \angle APB = 180°$

$\angle AOB + 40° = 180°$

$\angle AOB = 180° - 40° = 140°$

Also, OP bisects $\angle AOB$.

$\angle POA = \left(\dfrac{1}{2}\right) \times \angle AOB$

$= \left(\dfrac{1}{2}\right) \times 140°$

$= 70°$

Q11. If $x = a\sec\ \theta, y = b\tan\ \theta$, then the value of $\left(\dfrac{x^2}{a^2}\right) - \left(\dfrac{y^2}{b^2}\right) =$

(a) 1

(b) -1

(c) $\tan^2\ \theta$

(d) $\operatorname{cosec}^2\theta$

Solution:

Correct answer: (a)

Given,

$x = a\sec\ \theta, y = b\tan\ \theta$

$\left(\dfrac{x^2}{a^2}\right) - \left(\dfrac{y^2}{b^2}\right)$

$= \left(\dfrac{a^2\sec^2\theta}{a^2}\right) - \left(\dfrac{b^2\tan^2\theta}{b^2}\right)$

$= \sec^2\ \theta - \tan^2\ \theta$

$= 1$

Q12. $(\cos^2\ \theta - 1)(\cot^2\ \theta + 1) + 1 =$

(a) 1

(b) -1

(c) 2

(d) 0

Solution:

Correct answer: (d)

$(\cos^2 \theta - 1)(\cot^2 \theta + 1) + 1$

$= -(1 - \cos^2 \theta)\text{cosec}^2\theta + 1$

$= -\sin^2 \theta \left(\dfrac{1}{\sin^2 \theta}\right) + 1$

$= -1 + 1$

$= 0$

Q13. The ratios of the respective heights and the respective radii of two cylinders are

1: 2 and 2: 1 respectively. Then their respective volumes are in the ratio:

(a) 4: 1

(b) 1: 4

(c) 2: 1

(d) 1: 2

Solution:

Correct answer: (c)

Given,

Ratio of radii of two cylinders $= r_1 : r_2 = 2: 1$

Ratio of heights of two cylinders $= h_1 : h_2 = 1: 2$

Ratio of volume of two cylinders $= V_1 : V_2$

$V_1 : V_2 = \dfrac{\pi r_1{}^2 h_1}{\pi r_2{}^2 h_2}$

$= \left(\dfrac{2}{1}\right)^2 \times \left(\dfrac{2}{1}\right)$

$= 2: 1$

Q14. The variance of 10,10,10,10,10 is:

(a) 10

(b) $\sqrt{10}$

(c) 0

(d) 5

Solution:

Correct answer: (c)

Given,

10,10,10,10,10

Here, all the observations are equal.

Hence, the variance is 0.

Q15. Probability of getting 3 heads and 3 tails in tossing a coin 3 times is:

(a) $\frac{1}{8}$

(b) $\frac{1}{4}$

(c) $\frac{3}{8}$

(d) $\frac{1}{2}$

Solution:

Correct answer: (b)

Given,

Coin tossed 3 times.

Total number of outcomes $= 2^3 = 8$

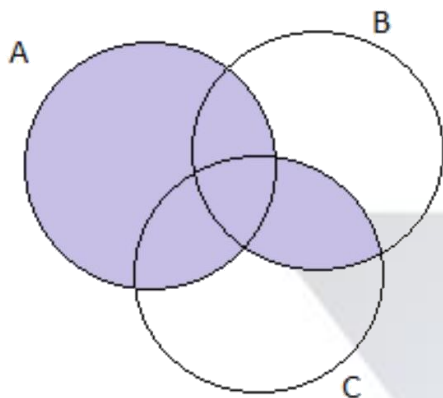Number of favourable outcomes $= 2$

i.e., $\{HHH, TTT\}$

Hence, the required probability $= \frac{2}{8} = \frac{1}{4}$

SECTION - II

Q16. Draw Venn diagram $A \cup (B \cap C)$.

Solution:

$A \cup (B \cap C)$

Q17. Let $A = \{1,2,3,4,5\}$, $B = N$ and $f: A \rightarrow B$ be defined by $f(x) = x^2$. Find the range of f. Identify the type of function.

Solution:

Given,

$f(x) = x^2$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

$f(3) = 3^2 = 9$

$f(4) = 4^2 = 16$

$f(5) = 5^2 = 25$

Therefore, the range of $f = \{1,4,9,16,25\}$

Here, the elements in co-domain does not equal to the elements in range

Thus, f is not onto.

Also, the image is not the same for two domain elements in f.

Hence, f is one-one.

Q18. Find the first term and common difference of the AP $\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \dots, \frac{17}{6}$

Solution:

Given AP:

$\frac{1}{2}, \frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \dots, \frac{17}{6}$

First term $= a = \frac{1}{2}$

Common difference $= d = \left(\frac{5}{6}\right) - \left(\frac{1}{2}\right)$

$= \frac{5-3}{6}$

$= \frac{2}{6}$

$= \frac{1}{3}$

Q19. Simplify: $\frac{5x + 20}{7x + 28}$

Solution:

$\frac{5x + 20}{7x + 28}$

$= \frac{[5(x+4)]}{[7(x+4)]}$

$= \frac{5}{7}$

Q20. If α and β are the roots of $3x^2 - 5x + 2 = 0$, then find the value of $\left(\frac{\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha}\right)$.

Solution:

Given,

α and β are the roots of $3x^2 - 5x + 2 = 0$.

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 3, b = -5, c = 2$

Sum of roots $= \alpha + \beta = -\frac{b}{a} = -\frac{-5}{3} = \frac{5}{3}$

Product of roots $= \alpha\beta = \frac{c}{a} = \frac{2}{3}$

$\left(\frac{\alpha}{\beta}\right) + \left(\frac{\beta}{\alpha}\right) = \frac{(\alpha^2 + \beta^2)}{\alpha\beta}$

$= \frac{[(\alpha + \beta)^2 - 2\alpha\beta]}{\alpha\beta}$

$$= \frac{\left[\left(\frac{5}{3}\right)^2 - 2\left(\frac{2}{3}\right)\right]}{\frac{2}{3}}$$

$$= [\left(\frac{25}{9}\right) - \left(\frac{4}{3}\right)] \times \left(\frac{3}{2}\right)$$

$$= \left[\frac{25-12}{9}\right] \times \left(\frac{3}{2}\right)$$

$$= \frac{13}{6}$$

Q21. If $A = \begin{bmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{bmatrix}$, then find A^T and $(A^\mathsf{T})^\mathsf{T}$.

Solution:

Given,

$A = \begin{bmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{bmatrix}$

$A^T = \begin{bmatrix} 8 & 1 \\ 5 & -3 \\ 2 & 4 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 8 & 5 & 2 \\ 1 & -3 & 4 \end{bmatrix}$

Q22. If $A = \begin{bmatrix} 2 & 3 \\ -9 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 7 & -1 \end{bmatrix}$, then find the additive inverse of A.

Solution:

$A = \begin{bmatrix} 2 & 3 \\ -9 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 7 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 2-1 & 3-5 \\ -9-7 & 5+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -2 \\ -16 & 6 \end{bmatrix}$

Additive inverse of $A = -A = \begin{bmatrix} -1 & 2 \\ 16 & -6 \end{bmatrix}$

Q23. The centre of a circle is at $(-6,4)$. If one end of a diameter of the circle is at the origin, then find the other end.

Solution:

Given,

Centre of the circle is $(-6,4)$.

One end of the diameter $= (0,0)$ (i.e. origin)

Let (x, y) be the other end of the diameter.

Centre $=$ Midpoint of diameter

$$(-6,4) = \left[\frac{0+x}{2}, \frac{0+y}{2}\right]$$

$$(-6,4) = \left(\frac{x}{2}, \frac{y}{2}\right)$$

$$\Rightarrow x = -12, y = 8$$

Hence, the other end of the diameter is $(-12,8)$.

Q24. Find the slope and y-intercept of the line whose equation is $4x - 2y + 1 = 0$.

Solution:

Given,

The equation of line is $4x - 2y + 1 = 0$.

$$\Rightarrow 2y = 4x + 1$$

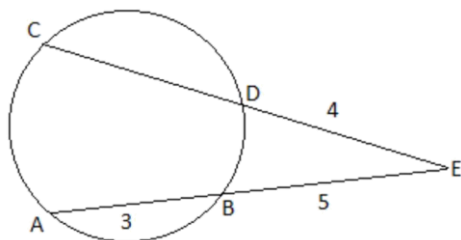$$\Rightarrow y = \left(\frac{4}{2}\right)x + \left(\frac{1}{2}\right)$$

$$\Rightarrow y = 2x + \left(\frac{1}{2}\right)$$

Comparing the slope-intercept form equation $y = mx + c$,

Slope $= m = 2$

y-intercept $= c = \frac{1}{2}$

Q25. In the figure, find the CD.

Solution:

Given,

AB and CD are two chords of a circle, these chords are intersecting at E when produced.

Therefore, AE × BE = CE × DE

$(AB + BE) \times BE = CE \times DE$

$(3 + 5) \times 5 = CE \times 4$

$\Rightarrow CE = \dfrac{8 \times 5}{4}$
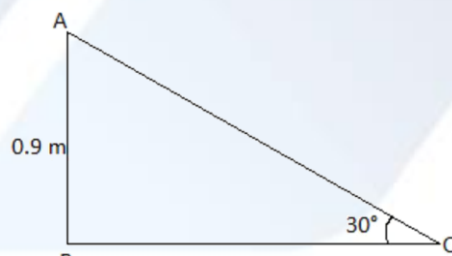
$\Rightarrow CE = 10$

$CD + DE = 10$

$CD = 10 - 4$

$CD = 6$ units

Q26. A ramp for unloading a moving truck has an angle of elevation of 30°. If the top of the ramp is 0.9 m above the ground level, then find the length of the ramp.

Solution:

Let AB be the top of the ramp from ground level.

AC be the length of the ramp.

In right-triangle ABC, $\sin 30° = \dfrac{AB}{AC}$

$\dfrac{1}{2} = \dfrac{0.9}{AC}$

$\Rightarrow AC = 0.9 \times 2$

$\Rightarrow AC = 1.8$ m

Hence, the length of the ramp = 1.8 m.

Q27. If the circumference of the base of a solid right circular cylinder is 154 cm and its height is 16 cm, find its curved surface area.

Solution:

Given,

Height of the cylinder $= h = 16$ cm

Circumference of the base of the cylinder $= 2\pi r = 154$ cm

Curved surface area $= 2\pi rh$

$= 154 \times 16$

$= 2464$ cm^2

Q28. Find the range and the coefficient of range of 43,24,38,56,22,39,45.

Solution:

Given,

43, 24, 38, 56, 22, 39, 45

Smallest value $= S = 22$

Largest value $= L = 56$

Range $= L - S = 56 - 22 = 34$

Coefficient of range $= \dfrac{L - S}{L + S}$

$= \dfrac{56 - 22}{56 + 22}$

$= \dfrac{34}{78}$

$= 0.436$ (approx)

Q29. A bag contains 6 white balls numbered from 1 to 6 and 4 red balls numbered from 7 to 10. A ball is drawn at random. Find the probability of getting:

(i) an even numbered ball

(ii) a white ball

Solution:

6 white balls numbered from 1 to 6 and 4 red balls numbered from 7 to 10 .

Total number of outcomes $= n(S) = 6 + 4 = 10$

(i) Let A be the event of getting an even numbered ball.

$A = \{2,4,6,8,10\}$

$n(A) = 5$

$P(A) = \dfrac{n(A)}{n(S)} = \dfrac{5}{10} = \dfrac{1}{2}$

(ii) Let B be the event of getting a white ball.

Number of white balls $= n(B) = 6$

$P(B) = \dfrac{n(B)}{n(S)} = \dfrac{6}{10} = \dfrac{3}{5}$

Q30. (a) Prove the following identity:

$$\sqrt{(\sec^2 \theta + \operatorname{cosec}^2\theta)} = \tan \theta + \cot \theta$$

Solution:

$LHS = \sqrt{\sec^2 \theta + \operatorname{cosec}^2\theta}$

$= \sqrt{\dfrac{1}{\cos^2 \theta} + \dfrac{1}{\sin^2 \theta}}$

$= \sqrt{\dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}}$

$= \dfrac{1}{\sin \theta \cos \theta} \dots (i)$

$RHS = \tan \theta + \cot \theta$

$= \dfrac{\sin \theta}{\cos \theta} + \dfrac{\cos \theta}{\sin \theta}$

$= \dfrac{\sin 2 + \cos \theta}{\cos \theta \sin \theta}$

$= \dfrac{1}{\sin \theta \cos \theta} \dots (ii)$

From (i) and (ii),

LHS = RHS

Hence proved.

(b) The thickness of a hemispherical bowl is 0.25 cm. The inner radius of the bowl is 5 cm. Find the outer curved surface area of the bowl. (take $\pi = 22/7$)

Solution:

Given,

Inner radius $= r = 5$ cm

Thickness of the bowl $= 0.25$ cm

Outer radius $= R = r +$ thickness $= 5 + 0.25 = 5.25$ cm
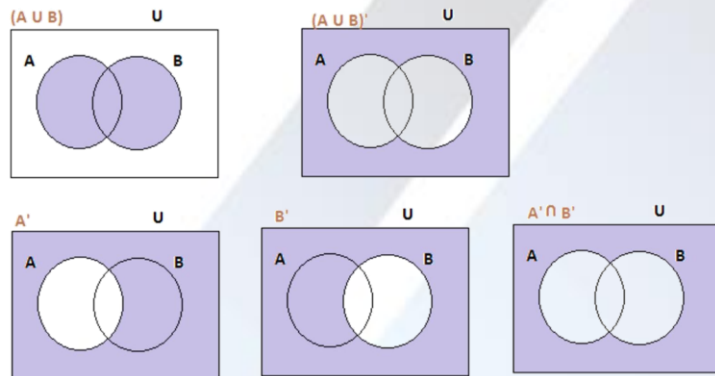
Outer curved surface area $= 2\pi R^2$

$$= 2 \times \left(\frac{22}{7}\right) \times 5.25 \times 5.25$$

$$= 173.25 \text{ cm}^2$$

SECTION - III

Q31. Use Venn diagrams to verify De Morgan's law of complementation $(A \cup B)' = A' \cap B'$.

Solution:

Therefore, $(A \cup B)' = A' \cap B'$

Q32. Let $A = \{4,6,8,10\}$ and $\{3,4,5,6,7\}$ and $f : A \rightarrow B$ be defined by $f(x) = \left(\frac{1}{2}\right)x + 1$, then represent f by:

(i) an arrow diagram

(ii) a set of ordered pairs and

(iii) a table

Solution:

Given,

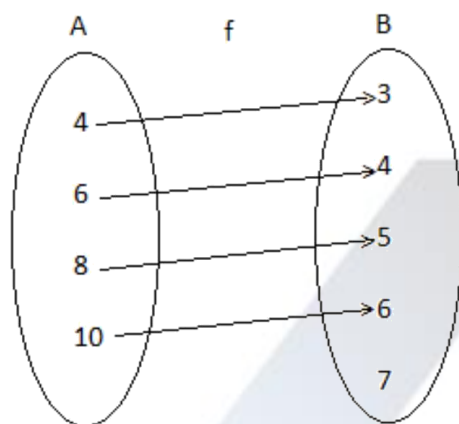$$f(x) = \left(\frac{1}{2}\right)x + 1$$

$$f(4) = \left(\frac{1}{2}\right) \times 4 + 1 = 2 + 1 = 3$$

$$f(6) = \left(\frac{1}{2}\right) \times 6 + 1 = 3 + 1 = 4$$

$$f(8) = \left(\frac{1}{2}\right) \times 8 + 1 = 4 + 1 = 5$$

$$f(10) = \left(\frac{1}{2}\right) \times 10 + 1 = 5 + 1 = 6$$

(i) An arrow diagram

(ii) The set of ordered pairs are $f = \{(4,3), (6,4), (8,5), (10,6)\}$

(iii) Table

x	4	6	8	10
$f(x)$	3	4	5	6

Q33. Find the sum of the first n terms of the series $6 + 66 + 666 + \cdots..$

Solution:

Given,

$6 + 66 + 666 + \cdots..$

Sum of first n terms

$S_n = 6 + 66 + 666 + \cdots\ldots.+(\text{n terms})$

$S_n = 6(1 + 11 + 111 + \cdots.+\text{n terms})$

$= \left(\frac{6}{9}\right)[9 + 99 + 999 + \cdots + n \text{ terms}]$

$= \left(\frac{6}{9}\right)[(10 - 1) + (100 - 1) + (1000 - 1) + \cdots.+(10^n - 1)]$

$= \left(\frac{6}{9}\right)[(10 + 10^2 + 10^3 + \cdots + 10^n) - (1 + 1 + 1 + \cdots + n \text{ terms})]$

$= \left(\frac{6}{9}\right)\left\{\left[\frac{10(10^n - 1)}{10 - 1}\right] - n\right\}$

$= \left(\frac{6}{9}\right)\left\{\left[\frac{10(10^n - 1)}{9}\right] - n\right\}$

$= \left(\frac{60}{81}\right)(10^n - 1) - \frac{6n}{9}$

$= \left(\frac{20}{27}\right)(10^n - 1) - \frac{2n}{3}$

Q34. The sum of four consecutive terms of an AP is 20 and the sum of their squares is 120. Find those numbers.

Solution:

Let $a - 3d, a - d, a + d, a + 3d$ be the four consecutive terms of an AP.

According to the given,

$a - 3d + a - d + a + d + a + 3d = 20$

$4a = 20$

$a = \frac{20}{4} = 5$

Also,

$(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

$a^2 + 9d^2 - 6ad + a^2 + d2 - 2ad + a^2 + d^2 + 2ad + a^2 + 9d^2 + 6ad = 120$

$4a^2 + 20d^2 = 120$

$4(5)^2 + 20d^2 = 120$

$20\,d^2 = 120 - 100$

$$d^2 = \frac{20}{20}$$

$$d^2 = 1$$

$$d = \pm 1$$

If a = 5 and d = −1, then AP is 8,6,4,2

If a = 5 and d = 1, then AP is 2,4,6,8

Q35. Factorize: $x^3 - 7x + 6$

Solution:

$$x^3 - 7x + 6 = (x^3 - x^2) + (x^2 - 7x + 6)$$

$$= (x^3 - x^2) + (x^2 - 7x + 6)$$

$$= (x^3 - x^2) + (x^2 - 6x - x + 6)$$

$$= x^2(x - 1) + (x - 1)(x - 6)$$

$$= (x - 1)(x^2 + x - 6)$$

$$= (x - 1)(x^2 + 3x - 2x - 6)$$

$$= (x - 1)(x - 2)(x + 3)$$

Q36. Find the square root of $4 + 25x^2 - 12x - 24x^3 + 16x^4$

Solution:

Given,

$$4 + 25x^2 - 12x - 24x^3 + 16x^4$$

$$= 16x^4 - 24x^3 + 25x^2 - 12x + 4$$

Therefore, the square root of the polynomial $= |4x^2 - 3x + 2|$

Q37. If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$, then verify that $(AB)^T = B^T A^T$.

Solution:

Given,

$A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 10 - 2 & -5 + 2 \\ 14 - 3 & -7 + 3 \end{pmatrix}$

$= \begin{pmatrix} 8 & -3 \\ 11 & -4 \end{pmatrix}$

$(AB)^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix}$

$B^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}, A^T = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$

$B^T A^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 10 - 2 & 14 - 3 \\ -5 + 2 & -7 + 3 \end{pmatrix}$

$= \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix}$

Therefore, $(AB)^T = B^T A^T$

Q38. Prove that $(0,5), (-2,-2), (5,0)$ and $(7,7)$ are the vertices of a rhombus.

Solution:

Let the given vertices be $A(0,5), B(-2,-2), C(5,0)$ and $D(7,7)$.

$AB = \sqrt{(-2 - 0)^2 + (-2 - 5)^2}$

$= \sqrt{2^2 + 7^2}$

$= \sqrt{4 + 49}$

$= \sqrt{53}$

$CD = \sqrt{(7 - 5)^2 + (7 - 0)^2}$

$= \sqrt{2^2 + 7^2}$

$= \sqrt{4 + 49}$

$= \sqrt{53}$

DA $= \sqrt{(0 - 7)^2 + (5 - 7)^2}$

$= \sqrt{7^2 + (-2)^2}$

$= \sqrt{49 + 4}$

$= \sqrt{53}$

AB = BC = CD = DA

$AC = \sqrt{(5 - 0)^2 + (0 - 5)^2}$

$AC = \sqrt{5^2 + (-5)^2}$

$AC = \sqrt{25 + 25}$

$AC = \sqrt{50}$

$AC = 5\sqrt{2}$

$BD = \sqrt{(7 - (-2))^2 + (7 - (-2))^2}$
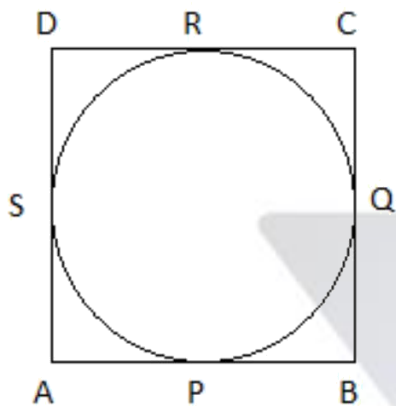
$BD = \sqrt{(9^2 + 9^2)}$

$BD = 9\sqrt{2}$

Diagonal AC ≠ Diagonal BD

Hence, the given vertices form a rhombus.

Q39. If all sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.

Solution:

Given: Sides *AB*, *BC*, *CD* and *DA* of a parallelogram *ABCD* touch a circle at *P*, *Q*, *R* and *S* respectively.

To prove: Parallelogram $ABCD$ is a rhombus.

Proof:

We know that the tangents drawn from an external point to a circle are equal.

$AP = AS$....(i) (tangents from point A)

$BP = BQ$....(ii) (tangents from point B)

$CR = CQ$....(iii) (tangents from point C)

$DR = DS$....(iv) (tangents from point D)

Adding (i), (ii), (iii), and (iv),

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$AB + CD = AD + BC$

$AB + AB = AD + AD$ [opposite sides of a parallelogram are equal]

$2AB = 2AD$ or $AB = AD$

Also, $AB = CD$ and $AD = BC$

$AB = BC = CD = DA$

Hence, the parallelogram $ABCD$ is a rhombus.

Q40. If $\tan \theta + \sin \theta = m, \tan \theta - \sin \theta = n$ and $m \neq n$, then show that $m^2 - n^2 = 4\sqrt{mn}$.

Solution:

Given,

$\tan \theta + \sin \theta = m$

$\tan\theta - \sin\theta = n$

$m^2 - n^2 = (\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2$

$= \tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta$ ......(i)

$mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta$

$= \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right) - \sin^2\theta$

$= \sin^2\theta\left[\dfrac{(1-\cos^2\theta)}{\cos^2\theta}\right]$

$= \sin^2\theta\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)$

$= \sin^2\theta\tan^2\theta$

$\Rightarrow \sqrt{mn} = \tan\theta\sin\theta$

From (i) and (ii),

$m^2 - n^2 = 4\sqrt{mn}.$

Q41. A sector containing an angle of 120° is cut off a circle of radius 21 cm and folded into a cone by joining the radii. Find the curved surface area of the cone. ($\pi = 22/7$)
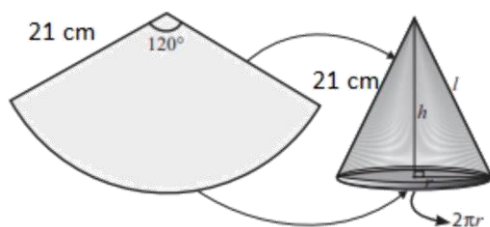
Solution:

Given,

Radius of the sector $= R = 21$ cm

Angle of the sector $= \theta = 120°$

Let r be the radius and l be the slant height of the cone.

Curved surface area of the cone = Area of the sector

$$= \left(\frac{\theta}{360°}\right) \times \pi R^2$$

$$= \left(\frac{120°}{360°}\right) \times \left(\frac{22}{7}\right) \times 21 \times 21$$

$$= 22 \times 21$$

$$= 462 \text{ cm}^2$$

Q42. An iron right circular cone of diameter 8 cm and height 12 cm is melted and recast into spherical lead shots each of radius 4 mm. How many lead shots can be made?

Solution:

Given,

Diameter of cone = 8 cm

Radius of cone = R = 4 cm

Height of the cone = h = 12 cm

Radius of spherical shot = 4 mm = 0.4 cm

$$Number\ of\ spherical\ lead\ shots\ = \frac{Volume\ of\ cone}{Volume\ of\ one\ spherical\ shot}$$

$$= \frac{\left[\left(\frac{1}{3}\right)\pi R^2\ h\right]}{\left[\left(\frac{4}{3}\right)\pi r^3\right]}$$

$$= \frac{4 \times 4 \times 12}{4 \times 0.4 \times 0.4 \times 0.4}$$

$$= 750$$

Hence, the required number of lead shots are 750.

Q43. The following table shows the marks obtained by 48 students in a quiz competition in mathematics. Calculate the standard deviation.

Data x	6	7	8	9	10	11	12
Frequency f	3	6	9	13	8	5	4

Solution:

x	F	fx	$d = x - 9$	d^2	fd^2
6	3	18	-3	9	27
7	6	42	-2	4	24
8	9	72	-1	1	9
9	13	117	0	0	0
10	8	80	1	1	8
11	5	55	2	4	20
12	4	48	3	9	36
	$\Sigma f = 48$	$\Sigma fx = 432$	$\Sigma d = 0$		$\Sigma fd^2 = 124$

Arithmetic mean $= \frac{\Sigma fx}{\Sigma f} = \frac{432}{48} = 9$

Standard deviation $= \sqrt{\left(\frac{\Sigma fd^2}{\Sigma f}\right)}$

$= \sqrt{\left(\frac{124}{48}\right)}$

$= \sqrt{2.58}$

$= 1.61$ (approx)

Q44. The probability that a new car will get an award for its design is 0.25, the probability that it will get an award for efficient use of fuel is 0.35 and the probability that it will get both the awards is 0.15. Find the probability that:

(i) it will get at least one of the two awards

(ii) it will get only one of the awards

Solution:

Let $P(A)$ be the probability of winning the award for design and $P(B)$ be the probability of getting an award for efficient use of fuel.

$P(A) = 0.25$

$P(B) = 0.35$

$P(A \cap B) = 0.15$

(i) Probability that it will get at least one of the two awards

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.25 + 0.35 - 0.15$

$= 0.45$

(ii) Probability of getting only one award $= P(A) + P(B) - 2P(A \cap B)$

$= 0.25 + 0.35 - 2(0.15)$

$= 0.3$

Q45. (a) Find the LCM (Least Common Multiple) of the following.

$x^3 + y^3, x^3 - y^3, x^4 + x^2y^2 + y^4$

Solution:

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$x^4 + x^2y^2 + y^4 = (x^2)^2 + 2x^2y^2 + (y^2)^2 - x^2y^2$

$= (x^2 + y^2)^2 - (xy)^2$

$= (x^2 + y^2 + xy)(x^2 + y^2 - xy)$

LCM $= (x + y)(x - y)(x^2 + y^2 + xy)(x^2 + y^2 - xy)$

$\qquad\qquad = (x^2 - y^2)(x^2 + y^2 + xy)(x^2 + y^2 - xy)$

(b) Find the equation of the line whose gradient is $\frac{3}{2}$ and which passes through P,

where P divides the line segment joining $A(-2,6)$ and $B(3,-4)$ in the ratio $2:3$

internally.

Solution:

Given, $P(x, y)$ divides the line segment joining A$(-2,6)$ and B$(3,-4)$ in the ratio $2:$

3 internally.

Here,

$(x_1, y_1) = (-2,6)$

$(x_2, y_2) = (3, -4)$

$l: m = 2: 3$

Using section formula,

$$P(x,y) = \left[\frac{(lx_2 + mx_1)}{l+m}, \frac{(ly_2 + my_1)}{l+m}\right]$$

$$= \left[\frac{6-6}{2+3}, \frac{-8+18}{2+3}\right]$$

$$= \left(\frac{0}{5}, \frac{10}{5}\right)$$

$$= (0,2)$$

Equation of the line passing through $P(0,2)$ with slope $\frac{3}{2}$ is:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \left(\frac{3}{2}\right)(x - 0)$$

$$y - 2 = \left(\frac{3}{2}\right)x$$

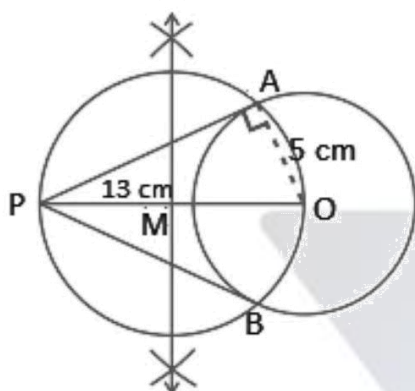$$2y - 4 = 3x$$

$$\Rightarrow 3x - 2y + 4 = 0$$

SECTION - IV

Q46. (a) Draw a circle of diameter 10 cm. From a point P, 13 cm away from its centre, draw the two tangents PA and PB to the circle and measure their lengths.

Solution:

Given,

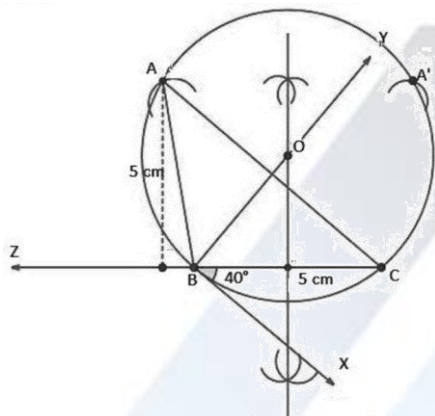Diameter of circle $= 10$ cm

Radius $= \frac{10}{2} = 5$ cm

PA and PB are required tangents.

Length of the tangents = 12 cm

(b) Construct a $\triangle ABC$ in which the base $BC = 5$ cm, $\angle BAC = 40°$ and the median from A to BC is 6 cm.

Also, measure the length of the altitude from A.

Solution:

Q47. (a) Draw the graph of $y = x^2 + 2x - 3$ and hence find the roots $x^2 - x - 6 = 0$.

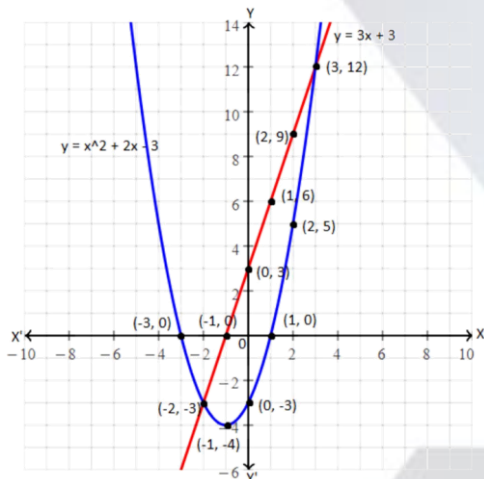**Solution:**

Given,

$y = x^2 + 2x - 3$

x	-3	-2	-1	0	1	2	3
y	0	-3	-4	-3	0	5	12

Now, subtracting $x^2 - x - 6 = 0$ from $y = x^2 + 2x - 3$,

$y - 0 = x^2 + 2x - 3 - (x^2 - x - 6)$

$y = 3x + 3$

x	-2	-1	0	1	2	3
y	-3	0	3	6	9	12

The graph of given equations intersects each other at $(-2, -3)$ and $(3, 12)$.

Hence, the solution set is $(-2, -3)$ and $(3, 12)$.

(b) A bank gives 10% SI (simple interest) on deposits for senior citizens. Draw the graph for the relation between the sum deposited and the interest earned for one year. Hence, find:

(i) the interest on the deposit of ₹ 650

(ii) The amount to be deposited to earn an interest of ₹ 45
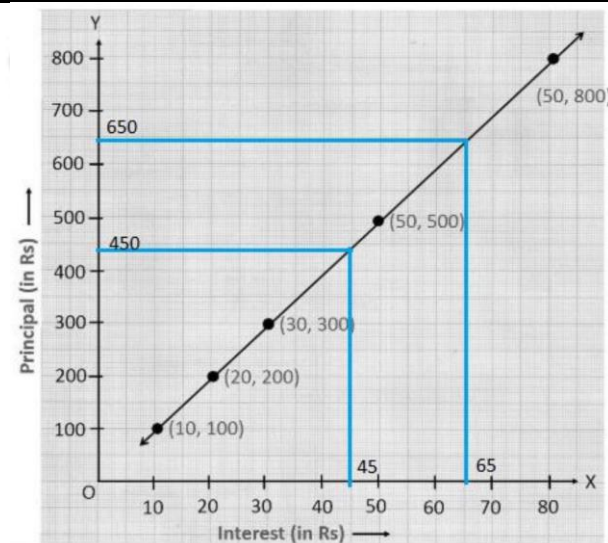
Solution:

Simple interest $= \dfrac{PTR}{100}$

Given, R = 10%

$SI = \dfrac{P \times T \times 10}{100} = \dfrac{PT}{100}$

SI for 1 year $= \dfrac{P}{100}$

Let us take different values of P as shown below.

P	100	200	300	400	500	600	700	800
SI	10	20	30	40	50	60	70	80

(i) Simple interest earned on a deposit of ₹ 650 = ₹ 65

(ii) The amount to be deposited to earn an interest of ₹ 45 = ₹ 450