Tamil Nadu Board SSLC Class 10 Maths 2016

SECTION - I

Q1. If $A = \{p, q, r, s\}, B = \{r, s, tu\}$, then $A \setminus B$ is:

(a) $\{p, q\}$

(b) $\{t, u\}$

(c) $\{r, s\}$

(d) $\{p, q, r, s\}$

Solution:

Correct answer: (a)

Given,

$A = \{p, q, r, s\}, B = \{r, s, tu\}$

$A \setminus B = \{p, q, r, s\} \setminus \{r, s, t, u\}$

$= \{p, q\}$

Q2. The 8 th term of the sequence 1,1,2,3,5,8, ... is:

(a) 25

(b) 24

(c) 23

(d) 21

Solution:

Correct answer: (d)

Given,

1,1,2,3,5,8, ...

This is a Fibonacci series.

$2 = 1 + 1$

$3 = 1 + 2$

$5 = 2 + 3$

$8 = 3 + 5$

7th term = 5 + 8 = 13

8th term = 8 + 13 = 21

Q3. If $1 + 2 + 3 + \cdots + n = k$, then $1^3 + 2^3 + 3^3 + \cdots + n^3$ is equal to:

(a) k^2

(b) k^3

(c) $\frac{k(k+1)}{2}$

(d) $(k + 1)3$

Solution:

Correct answer: (a)

Given,

$1 + 2 + 3 + \cdots + n = k$

Sum of first n natural numbers $= \frac{n(n+1)}{2} = k$ ..... (i)

Sum of the cubes of first n natural numbers

$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n + 1)}{2}\right]^2$

$= k^2 [$ From (i)$]$

Q4. The system of equations $x - 4y = 8, 3x - 12y = 24$

(a) has infinitely many solutions

(b) has no solution

(c) has a unique solution

(d) may or may not have a solution

Solution:

Correct answer: (a)

Given,

$x - 4y = 8$

$3x - 12y = 24$

Comparing with the standard form,

$a_1 = 1, b_1 = -4, c_1 = -8$

$a_2 = 3, b_2 = -12, c_2 = 24$

$\dfrac{a_1}{a_2} = \dfrac{1}{3}$

$\dfrac{b_1}{b_2} = \dfrac{-4}{-12} = \dfrac{1}{3}$

$\dfrac{c_1}{c_2} = \dfrac{-8}{-24} = \dfrac{1}{3}$

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Hence, the given system of equations has infinitely many solutions.

Q5. If $ax^2 + bx + c = 0$ has equal roots, then c is equal to

(a) $\dfrac{b^2}{2a}$

(b) $\dfrac{b^2}{4a}$

(c) $-\dfrac{b^2}{2a}$

(d) $-\dfrac{b^2}{4a}$

Solution:

Correct answer: (b)

Given that, $ax^2 + bx + c = 0$ has equal roots.

Therefore, discriminant $= 0$

$b^2 - 4ac = 0$

$b^2 = 4ac$

$\Rightarrow c = \dfrac{b^2}{4a}$

Q6. If $A = \begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$, then A^2 is:

(a) $\begin{pmatrix} 16 & 4 \\ 36 & 9 \end{pmatrix}$

(b) $\begin{pmatrix} 8 & -4 \\ 12 & -6 \end{pmatrix}$

(c) $\begin{pmatrix} -4 & 2 \\ -6 & 3 \end{pmatrix}$

(d) $\begin{pmatrix} 4 & -2 \\ 6 & -3 \end{pmatrix}$

Solution:

Correct answer: (d)

Given,

$$A = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16 - 12 & -8 + 6 \\ 24 - 18 & -12 + 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix}$$

Q7. The equation of a straight line parallel to y-axis and passing through the point

$(-2,5)$ is:

(a) $x - 2 = 0$

(b) $x + 2 = 0$

(c) $y + 5 = 0$

(d) $y - 5 = 0$

Solution:

Correct answer: (b)

We know that the equation of line parallel to y-axis is $x = k$, where k is a constant.

Given that, the line passes through $(-2,5)$

Hence, the equation of line is $x = -2$ or $x + 2 = 0$

Q8. The equation of a straight-line having slope 3 and y-intercept -4 is:

(a) $3x - y - 4 = 0$

(b) $3x + y - 4 = 0$

(c) $3x - y + 4 = 0$

(d) $3x + y + 4 = 0$

Solution:

Correct answer: (a)

We know that the equation of a line with slope m and y-intercept c is $y = mx + c$

Given,

$m = 3$

$c = -4$

Hence, the required equation of line is:

$y = 3x - 4$

$\Rightarrow 3x - y - 4 = 0$

Q9. In $\triangle$ ABC, DE is parallel to BC, meeting AB and AC at D and E. If AD $= 3$ cm, DB $= 2$ cm and AE $= 2.7$ cm, then AC is equal to:
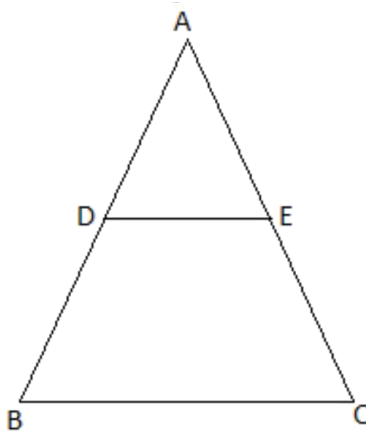
(a) 6.5 cm

(b) 4.5 cm

(c) 3.5 cm

(d) 5.5 cm

Solution:

Correct answer: (b)

Given,

In $\triangle$ ABC, DE is parallel to BC, meeting AB and AC at D and E.

By BPT,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{3}{2} = \frac{2.7}{EC}$$

$$\Rightarrow EC = \frac{2.7 \times 2}{3}$$

$$\Rightarrow EC = 1.8 \text{ cm}$$

$$AC = AE + EC$$

$$= 2.7 + 1.8$$

$$= 4.5 \text{ cm}$$

Q10. AB and CD are two chords of a circle which when produced to meet at a point P such that $AB = 5$ cm, $AP = 8$ cm and $CD = 2$ cm, then PD =

(a) 12 cm

(b) 5 cm

(c) 6 cm

(d) 4 cm

Solution:

Correct answer: (d)
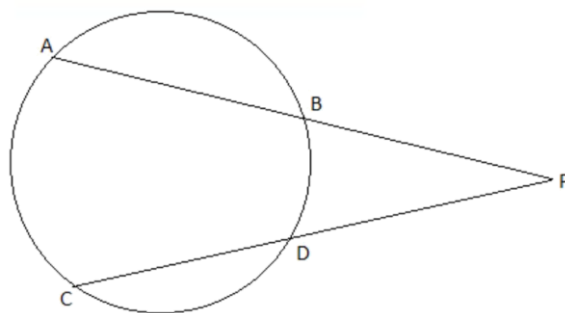
Given,

AB and CD are two chords of a circle which intersect each other externally at P.

$AB = 5$ cm, $AP = 8$ cm and $CD = 2$ cm

Two chords AB and CD meet at P when produced.

Let $PD = x$

$\Rightarrow \text{PA} \times \text{PB} = \text{PC} \times \text{PD}$

$\Rightarrow PA \times (AP - AB) = (PD + CD) \times PD$

$\Rightarrow 8 \times (8 - 5) = (PD + 2) \times PD$

$\Rightarrow 24 = x^2 + 2x \ [PD = x]$

$\Rightarrow x^2 + 2x - 24 = 0$

$\Rightarrow (x + 6)(x - 4) = 0$

$\Rightarrow x = -6, 4$

Length cannot be negative.

Therefore, $x = PD = 4 \ cm$

Q11. $(1 + \tan^2 \theta) \cdot \sin^2 \theta =$

(a) $\sin^2 \theta$

(b) $\cos^2 \theta$

(c) $\tan^2 \theta$

(d) $\cot^2 \theta$
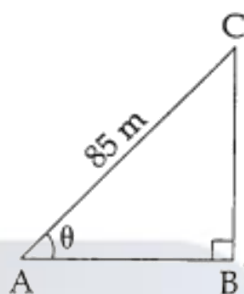
Solution:

Correct answer: (c)

$(1 + \tan^2 \theta) \cdot \sin^2 \theta$

$= \sec^2 \theta \cdot \sin^2 \theta$

$= \left(\dfrac{1}{\cos^2 \theta} \right) \cdot \sin^2 \theta$

$= \tan^2 \theta$

Q12. In the adjoining figure, $\sin \theta = \dfrac{15}{17}$, then $BC =$

(a) 85 m

(b) 65 m

(c) 95 m

(d) 75 m

Solution:

Correct answer: (d)

Given,

$$\sin \theta = \frac{15}{17}$$

$$\Rightarrow \frac{BC}{AC} = \frac{15}{17}$$

$$\Rightarrow \frac{BC}{85} = \frac{15}{17}$$

$$\Rightarrow BC = \frac{15 \times 85}{17} = 75 \text{ m}$$

Q13. The total surface area of a solid hemisphere of diameter 2 cm is equal to:

(a) 12 cm²

(b) 12π cm²

(c) 4π cm²

(d) 3π cm²

Solution:

Correct answer: (d)

Given,

Diameter of hemisphere = 2 cm

Radius $= r = \frac{2}{2} = 1$ cm

Total surface area $= 3\pi r^2$

$= 3\pi(1)^2$

$= 3\pi$ cm^2

Q14. If t is the standard deviation of x, y, z, then the standard deviation of $x + 5, y + 5, z + 5$ is:

(a) $\frac{t}{3}$

(b) $t + 5$

(c) t

(d) xyz

Solution:

Correct answer: (c)

We know that the standard deviation of a set of numbers will not change if every number is increased or decreased by the same amount.

Hence, the standard deviation $of\ x + 5, y + 5, z + 5$ is equal to the standard deviation of x, y, z, i.e., t.

Q15. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected at random, the probability that it is not red is:

(a) $\frac{5}{12}$

(b) $\frac{4}{12}$

(c) $\frac{3}{12}$

(d) $\frac{3}{4}$

Solution:

Correct answer: (d)

Given,

A bag contains 5 black balls, 4 white balls and 3 red balls.

Total number of balls $= 5 + 4 + 3 = 12$

Number of balls other than red $= 5 + 4 = 9$

P(selected ball is not red) $= \frac{9}{12} = \frac{3}{4}$

SECTION - II

Q16. Given $A = \{a, x, y, r, s\}, B = \{1,3,5,7,-10\}$ verify the commutative property of set union.

Solution:

Given,

$A = \{a, x, y, r, s\}$

$B = \{1,3,5,7,-10\}$

$A \cup B = \{a, x, y, r, s\} \cup \{1,3,5,7,-10\}$

$= \{a, x, y, r, s, 1,3,5,7,-10\}$

$B \cup A = \{1,3,5,7,-10\} \cup \{a, x, y, r, s\}$

$= \{1,3,5,7,-10, a, x, y, r, s\}$

$A \cup B = B \cup A$

Hence, verified the commutative property of set union.

Q17. Let $A = \{5,6,7,8\}, B = \{-11,4,7,-10,-7,-9,-13\}$ and $f = \{(x, y) : y = 3 - 2x, x \in A, y \in B\}$

(i) Write down the elements of f.

(ii) What is the range?

Solution:

Given,

$y = 3 - 2x$

When $x = 5$

$y = 3 - 2(5) = 3 - 10 = -7$

When $x = 6$

$y = 3 - 2(6) = 3 - 12 = -9$

When $x = 7$

$y = 3 - 2(7) = 3 - 14 = -11$

When $x = 8$

$y = 3 - 2(8) = 3 - 16 = -13$

(i) $f = \{(5, -7), (6, -9), (7, -11), (8, -13)\}$

(ii) Range of $f = \{-7, -9, -11, -13\}$

Q18. Find the 12 th term of the $AP\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

Solution:

Given AP is: $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, ...$

First term $= a = \sqrt{2}$

Common difference $= d = 3\sqrt{2} - \sqrt{2} = \sqrt{2}(3 - 1) = 2\sqrt{2}$

nth term an $= a + (n - 1)d$

12th term of the $AP = \sqrt{2} + (12 - 1)2\sqrt{2}$

$= \sqrt{2} + 11(2\sqrt{2})$

$= \sqrt{2} + 22\sqrt{2}$

$= 23\sqrt{2}$

Q19. Simplify: $\left[\frac{x^3}{x-2}\right] + \left[\frac{8}{2-x}\right]$

Solution:

$\left[\frac{x^3}{x - 2}\right] + \left[\frac{8}{2 - x}\right]$

$= \left[\frac{x^3}{x - 2}\right] - \left[\frac{8}{x - 2}\right]$

$= \frac{(x^3 - 8)}{x - 2}$

$= \frac{(x^3 - 2^3)}{x - 2}$

$= \left[\frac{(x - 2)(x^2 + 2x + 2^2)}{x - 2}\right]$

$= x^2 + 2x + 4$

Q20. If α and β are the roots of the equation $3x^2 - 6x + 4 = 0$, find the value of $\alpha^2 + \beta^2$.

Solution:

Given,

α and β are the roots of the equation $3x^2 - 6x + 4 = 0$.

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = 3, b = -6, c = 4$

$\alpha + \beta = -\dfrac{b}{a} = -\dfrac{-6}{3} = 2$

$\alpha\beta = \dfrac{c}{a} = \dfrac{4}{3}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2\left(\dfrac{4}{3}\right)$

$= 4 - \left(\dfrac{8}{3}\right)$

$= \dfrac{12 - 8}{3}$

$= \dfrac{4}{3}$

Q21. Find the product of the matrices, if exists. $\begin{pmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{pmatrix}$

Solution:

$A = \begin{bmatrix} 2 & 9 & -3 \\ 4 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$

Order of $A = 2 \times 3$

Order of $B = 3 \times 2$

$\therefore AB$ exists.

$AB = \begin{bmatrix} 2 & 9 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -6 & 7 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 - 54 + 6 & 4 + 63 - 3 \\ 16 + 6 - 0 & 8 - 7 + 0 \end{bmatrix}$

$= \begin{bmatrix} -40 & 64 \\ 22 & 1 \end{bmatrix}$

Q22. Find the values of x, y and z if

$$\begin{bmatrix} x+y \\ y+z \\ z-5 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 0 \end{bmatrix}$$

Solution:

From the given,

$x + y = 7$

$y + z = 9$

$z - 5 = 0$

$z = 5$

Substituting z = 5 in (ii),

$y + 5 = 9$

$y = 9 - 5 = 4$

Substituting y = 4 in (i),

$x + 4 = 7$

$x = 7 - 4 = 3$

Therefore, $x = 3, y = 4$ and $z = 5$.

Q23. If the centroid of a triangle is at (1,3) and two of its vertices are (−7,6) and (8,5), then find the third vertex of the triangle.

Solution:

Given,

Two vertices of the triangle are (−7,6) and (8,5).

Centroid of the triangle is (1,3).

Let (x, y) be the third vertex of the triangle.

Using centroid formula,

$$\text{Centroid} = \left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3} \right]$$

$$(1,3) = \left[\frac{-7+8+x}{3}, \frac{6+5+y}{3} \right]$$

$(1,3) = \left[\dfrac{1+x}{3}, \dfrac{11+y}{3} \right]$

$\Rightarrow 1 + x = 3$

$\Rightarrow x = 2$

And

$11 + y = 9$

$\Rightarrow y = -2$

Hence, the third vertex is (2, -2).

Q24. Find the value of 'a', if the straight lines $5x - 2y - 9 = 0$ and $ay + 2x - 11 = 0$ are perpendicular to each other.

Solution:

Given equations of straight lines are:

$5x - 2y - 9 = 0$

$\Rightarrow 2y = 5x - 9$

$\Rightarrow y = \left(\dfrac{5}{2} \right) x - \left(\dfrac{9}{2} \right)$

Slope $= m_1 = \dfrac{5}{2}$

And

$ay + 2x - 11 = 0$

$ay = -2x + 11$

$y = \left(-\dfrac{2}{a} \right) x + \left(\dfrac{11}{a} \right)$
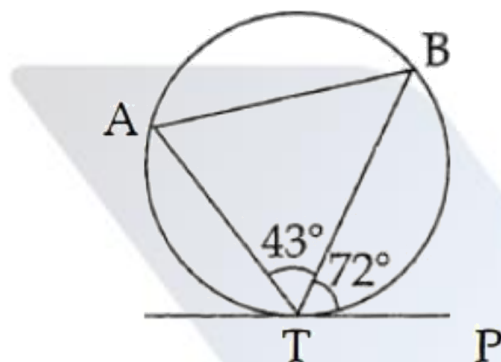
Slope $= m_2 = -\dfrac{2}{a}$

Given that, the straight lines are perpendicular to each other.

$\Rightarrow m_1 \times m_2 = -1$

$\Rightarrow \left(\dfrac{5}{2} \right) \times \left(-\dfrac{2}{a} \right) = -1$

$\Rightarrow a = 5$

Q25. In the figure TP is a tangent to the circle. *A* and *B* are two points on the circle. If $\angle BTP = 72°$ and $\angle ATB = 43°$ find $\angle ABT$.

Solution:

Given,

$\angle BTP = 72°$ and $\angle ATB = 43°$

$\angle BAT = \angle BTP = 72°$ (alternate angles)

Sum of all angles of a triangle $= 180°$

In triangle ABT,

$\angle ABT + \angle BTA + \angle TAB = 180°$

$\Rightarrow \angle ABT + 43° + 72° = 180°$

$\Rightarrow \angle ABT + 115° = 180°$

$\Rightarrow \angle ABT = 180° - 115°$

$\Rightarrow \angle ABT = 65°$

Q26. If $\triangle ABC$ is right angled at 'C', then find the values of $\cos (A + B)$ and $\sin (A + B)$.

Solution:

Given that, in $\triangle ABC$, C is the right angle.

Thus, $A + B = 90°$

$\cos (A + B) = \cos 90° = 0$

$\sin (A + B) = \sin 90° = 1$

Q27. Curved surface area and circumference at the base of a solid right circular cylinder are 4400 sq. cm and 110 cm respectively. Find its height and diameter.

Solution:

Let r be the radius and h be the height of the right circular cylinder.

Given,

Circumference of the base $= 110$ cm

$2\pi r = 110$

$2 \times \left(\dfrac{22}{7}\right) \times r = 110$

$r = \dfrac{110 \times 7}{22 \times 2}$

$r = 17.5$ cm

Curved surface area $= 4400$ sq. cm (given)

$2\pi rh = 4400$

$2 \times \left(\dfrac{22}{7}\right) \times 17.5 \times h = 4400$

$h = \dfrac{4400}{22 \times 5}$

$h = 40$ cm

Diameter $= 2r = 2 \times 17.5 = 35$ cm

Therefore, height is 40 cm and diameter is 35 cm .

Q28. If the coefficient of variation of a collection of data is 57 and its S.D. is 6.84, then find the mean.

Solution:

Given,

Coefficient of variation $= 57$

Standard deviation $= 6.84$

We know that,

$Coefficient\ of\ variation = \left(\dfrac{standard\ deviation}{mean}\right) \times 100$

$57 = \frac{6.84 \times 100}{\text{mean}}$

$\text{Mean} = \frac{684}{57}$

$= 12$

Q29. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the card is

(i) A multiple of 4

(ii) Not a multiple of 6

Solution:

Total number of cards $= n(S) = 20$

(i) Let A be the event of getting a card with a number as a multiple of 4 .

$A = \{4,8,12,16,20\}$

n(A) = 5

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

(ii) Let B be the event of getting a card with numbers which is not a multiple of 6 .

B = \{1,2,3,4,5,7,8,9,10,11,13,14,15,16,17,19,20\}

n(B) = 17

$P(B) = \frac{n(B)}{n(S)} = \frac{17}{20}$

Q30. (a) The central angle and radius of a sector of a circular disc are 180° and 21 cm respectively. If the edges of the sector are joined together to make a hollow cone, then find the radius of the cone.

Solution:

Given,

Radius of the sector $= R = 21$ cm

Angle of the sector $= \theta = 180°$

The cone is obtained by joining the radius and hence, the radius of the sector is equal to the slant height of the cone.

Slant height = l = 21 cm

Let r be the radius of the cone.

Arc length of the sector = Circumference of the base of the cone

$\left(\dfrac{\theta}{360°}\right) \times 2\pi R = 2\pi r$

$\left(\dfrac{180°}{360°}\right) \times (21) = r$

$\Rightarrow r = \dfrac{21}{2}$

$\Rightarrow r = 10.5$ cm

(b) Prove that: $\dfrac{1+\sec\theta}{\sec\theta} = \dfrac{\sin^2\theta}{1-\cos\theta}$

Solution:

LHS $= \dfrac{1+\sec\theta}{\sec\theta}$

$= \left(\dfrac{1}{\sec\theta}\right) + 1$

$= \cos\theta + 1$

$= [1+\cos\theta] \times \left[\dfrac{1-\cos\theta}{1-\cos\theta}\right]$

$= \dfrac{(1-\cos^2\theta)}{1-\cos\theta}$

$= \dfrac{\sin^2\theta}{1-\cos\theta}$

= RHS

Hence proved.

SECTION - III

Q31. A radio station surveyed 190 students to determine the types of music they liked. The survey revealed that 114 liked rock music, 50 liked folk music and 41 liked classical music, 14 liked rock music and folk music, 15 liked rock music and classical music, 11 liked classical music and folk music, 5 liked all three of music. Find:

(i) How many did not like any of the 3 types?

(ii) How many liked any two types only?

(iii) How many liked folk music but not rock music?

Solution:

Total number of students $= 190$

$R =$ Rock music

$F =$ Folk music

$C =$ Classical music

From the given,

$n(R \cap F) = 14$

$n(R \cap C) = 15$
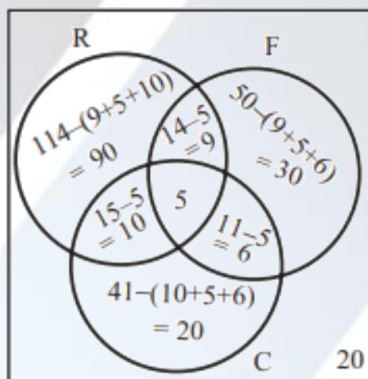
$n(C \cap F) = 11$

$n(R \cap F \cap C) = 5$

$n(R \cap F \cap C') = 14 - 5 = 9$

$n(R \cap C \cap F') = 15 - 5 = 10$

$n(F \cap C \cap R') = 11 - 5 = 6$

The number of students who liked any one of the music $= 90 + 9 + 30 + 10 + 5 + 6 + 20 = 170$

(a) The number of students who didn't like any 3 types of music $= 190 - 170 = 20$

(b) Number of students who liked any two types ony $= 9 + 6 + 10 = 25$

(c) Number of students who liked folk music but not rock music $= 30 + 6 = 36$

Q32. A function $f : [-3,7) \to R$ is defined as follows:

$$f(x) = \begin{cases} 4x^2 - 1 & ; -3 \leqslant x < 2 \\ 3x - 2 & ; \quad 2 \leqslant x \leqslant 4 \\ 2x - 3 & ; \quad 4 < x < 7 \end{cases}$$

Find:

(i) $f(-2) - f(4)$

(ii) $\dfrac{[f(3) + f(-1)]}{[2f(6) - f(1)]}$

Solution:

(i) $f(-2) = 4x^2 - 1 = 4(-2)^2 - 1 = 16 - 1 = 15$

$f(4) = 3x - 2 = 3(4) - 2 = 12 - 2 = 10$

$f(-2) - f(4) = 15 - 10 = 5$

(ii) $f(3) = 3x - 2 = 3(3) - 2 = 9 - 2 = 7$

$f(-1) = 4x^2 - 1 = 4(-1)^2 - 1 = 4 - 1 = 3$

$f(6) = 2x - 3 = 2(6) - 3 = 12 - 3 = 9$

$f(1) = 4x^2 - 1 = 4(1)^2 - 1 = 4 - 1 = 3$

$\dfrac{[f(3) + f(-1)]}{[2f(6) - f(1)]} = \left[\dfrac{7 + 3}{2 \times 9 - 3}\right]$

$= \dfrac{10}{18 - 3}$

$= \dfrac{10}{15}$

$= \dfrac{2}{3}$

Q33. Find the sum to n terms of the series $7 + 77 + 777 + \cdots$

Solution:

Given,

$7 + 77 + 777 + \cdots$

Sum of first n terms

$S_n = 7 + 77 + 777 + \cdots.. + (n \text{ terms})$

$S_n = 7(1 + 11 + 111 + \cdots. + n \text{ terms})$

$= \left(\dfrac{7}{9}\right)[9 + 99 + 999 + \cdots + n \text{ terms}]$

$$= \left(\frac{7}{9}\right)[(10-1)+(100-1)+(1000-1)+\cdots.+(10^n-1)]$$

$$= \left(\frac{7}{9}\right)[(10+10^2+10^3+\cdots+10^n)-(1+1+1+\cdots+n \text{ terms })]$$

$$= \left(\frac{7}{9}\right)\left\{\left[\frac{10(10^n-1)}{10-1}\right]-n\right\}$$

$$= \left(\frac{7}{9}\right)\left\{\left[\frac{10(10^n-1)}{9}\right]-n\right\}$$

$$= \left(\frac{70}{81}\right)(10^n-1)-\frac{7n}{9}$$

Q34. If 7 times the 7th term of an Arithmetic Progression is equal to 11 times its 11th term, show that its 18th term is zero. Can you find the first term and the common difference? Justify your answer.

Solution:

Let a be the first term and d be the common difference an AP.

Given,

$7a_7 = 11a_{11}$

$7(a + 6d) = 11(a + 10d)$

$7a + 42\,d = 11a + 110\,d$

$\Rightarrow 11a + 110d - 7a - 42d = 0$

$\Rightarrow 4a + 68\,d = 0$

$\Rightarrow 4(a + 17) = 0$

$\Rightarrow a + 17 = 0$

$\Rightarrow a_{18} = 0$

It is not possible here to find both the first term and common difference. Since only one equation obtained from the given data.

Q35. Factorise: $x^3 - 5x^2 - 2x + 24$

Solution:

Let the given polynomial be $p(x) = x^3 - 5x^2 - 2x + 24$

Let us substitute various values of x until we get $p(x) = 0$.

$p(0) = (0)^3 - 5(0)^2 - 2(0) + 24 = 24 \neq 0$

$p(1) = (1)^3 - 5(1)^2 - 2(1) + 24 = 1 - 5 - 2 + 24 = 18 \neq = p(-2) = (-2)^3 -$

$5(-2)^2 - 2(-2) + 24 = -8 - 10 + 4 + 24 = 0$

Therefore, $x + 2$ is a factor of p(x).

$x^2 - 7x + 12$
$x + 2$

$p(x) = (x + 2)(x^2 - 7x + 12)$

$= (x + 2)[x^2 - 3x - 4x + 12]$

$= (x + 2)[x(x - 3) - 4(x - 3)]$

$= (x + 2)(x - 3)(x - 4)$

Hence, the factors of the given polynomial are $(x + 2), (x - 3)$ and $(x - 4)$.

Q36. Simplify: $\left[\frac{1}{(x^2+3x+2)}\right] + \left[\frac{1}{(x^2+5x+6)}\right] - \left[\frac{2}{(x^2+4x+3)}\right]$

Solution:

$x^2 + 3x + 2 = x^2 + 2x + x + 2 = (x + 1)(x + 2)$
$x^2 + 5x + 6 = x^2 + 2x + 3x + 6 = (x + 2)(x + 3)$
$x^2 + 4x + 3 = x^2 + 3x + x + 3 = (x + 1)(x + 3)$

Now,

$\left[\frac{1}{(x^2 + 3x + 2)}\right] + \left[\frac{1}{(x^2 + 5x + 6)}\right] - \left[\frac{2}{(x^2 + 4x + 3)}\right]$

$= \left[\frac{1}{(x + 1)(x + 2)}\right] + \left[\frac{1}{(x + 2)(x + 3)}\right] - \left[\frac{2}{(x + 1)(x + 3)}\right]$

$= \frac{[(x + 3) + (x + 1) - 2(x + 2)]}{[(x + 1)(x + 2)(x + 3)]}$

$= \frac{[2x + 4 - 2x - 4]}{[(x + 1)(x + 2)(x + 3)]}$

$= 0$

Q37. If $A = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}; B = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$ verify that $A(B + C) = AB + AC$.

Solution:

$B + C = \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$

$$= \begin{pmatrix} (-2)+1 & 5+1 \\ 6+(-5) & 7+3 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$A(B+C) = \begin{pmatrix} (-2)+1 & 5+1 \\ 6+(-5) & 7+3 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ 1 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 3(-1)+2\times 1 & 3\times 6+2\times 10 \\ (-1)(-1)+4\times 1 & (-1)\times 6+4\times 10 \end{pmatrix} = \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 5 \\ 6 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 3(-2)+2\times 6 & 3\times 5+2\times 7 \\ (-1)(-2)+4\times 6 & (-1)\times 5+4\times 7 \end{pmatrix} = \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix}$$

$$AC = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -5 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

$$AB + AC = \begin{pmatrix} 6 & 29 \\ 26 & 23 \end{pmatrix} + \begin{pmatrix} -7 & 9 \\ -21 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 6+(-7) & 29+9 \\ 26+(-21) & 23+11 \end{pmatrix} = \begin{pmatrix} -1 & 38 \\ 5 & 34 \end{pmatrix}$$

Therefore, $A(B + C) = AB + AC$

Q38. Find the area of the quadrilateral whose vertices are $(-4,5), (0,7), (5, -5)$ and $(-4, -2)$.

Solution:

Given, vertices of the quadrilateral are $(-4,5), (0,7), (5, -5)$ and $(-4, -2)$.

Area of the quadrilateral

$$= \frac{1}{2} \begin{vmatrix} -4 & 0 & 5 & -4 & -4 \\ 5 & 7 & -5 & -2 & 5 \end{vmatrix}$$

$$= \frac{1}{2}|(-28-0-10-20)-(0+35+20+8)|$$

$$= \left(\frac{1}{2}\right) | -58 - 63] |$$

$$= \left(\frac{1}{2}\right) \times 121$$

$$= 60.5 \text{ sq.units}$$

Q39. State and prove Thales theorem.
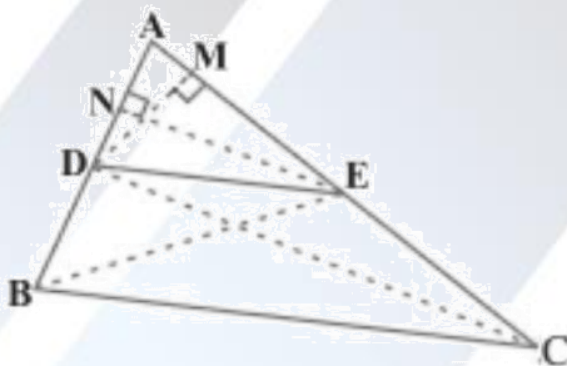
Solution:

Statement:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof:

In triangle ABC, a line parallel to side BC intersects other two sides namely AB and AC at D and E respectively. Join BE and CD.

Also, draw DM ⊥ AC and EN ⊥ AB.

area of $\triangle ADE = \frac{1}{2}(AD \times EN)$

$$ar(\triangle ADE) = \frac{1}{2}(AD \times EN)$$

Similarly,

$$ar(\triangle BDE) = \frac{1}{2}(DB \times EN)$$

$$ar(\triangle ADE) = \frac{1}{2}(AE \times DM)$$

$$\text{ar}(\triangle DEC) = \frac{1}{2}(EC \times DM)$$

Now,

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\left[\frac{1}{2}AD \times EN\right]}{\left[\frac{1}{2}(DB \times EN)\right]} = \frac{AD}{DB} \ldots\text{(i)}$$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\left[\frac{1}{2}(AE \times DM)\right]}{\left[\frac{1}{2}(EC \times DM)\right]} = \frac{AE}{EC} \ldots\text{(i)}$$

Triangle BDE and DEC are on the same base DE and between the same parallels.

Therefore, $\text{ar}(\triangle BDE) = \text{ar}(\triangle DEC)$

From (i), (ii) and (iii),
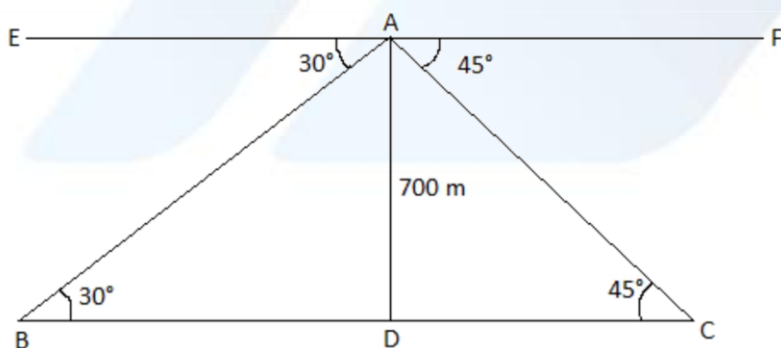
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

BC and DE.

Q40. A person in a helicopter flying at a height of 700 m, observes two objects lying opposite to each other on either banks of a river. The angles of depression of the objects are 30° and 45°; find the width of the river. ($\sqrt{3} = 1.732$)

Solution:

Let A be the position of flight.

B and C be the positions of two objects.

In right triangle ADC ,

$$\tan 45° = \frac{AD}{CD}$$

$$1 = \frac{700}{CD}$$

$\Rightarrow CD = 700$ m

In right triangle ADB ,

$$\tan 30° = \frac{AD}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{700}{BD}$$

$\Rightarrow BD = 700\sqrt{3}$ m

Width of the river $= BD + CD = 700\sqrt{3} + 700$

$= 700(\sqrt{3} + 1)$

$= 700(1.732 + 1)$

$= 700 \times 2.732$

$= 1912.40$ m

Q41. The radii of two circular ends of a frustum shaped bucket are 15 cm and 8 cm . If its depth is 63 cm, find the capacity of the bucket in litres. (Take $\pi = 22/7$)

Solution:

Given,

The radii of two circular ends of a frustum shaped bucket are 15 cm and 8 cm .

$R = 15$ cm and $r = 8$ cm

Depth $= h = 63$ cm

Volume of frustum shaped bucket $== (1/3)\pi h[R^2 + r^2 + Rr]$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 63 \times [15^2 + 8^2 + (15)(8)]$

$= (66) \times [225 + 64 + 120]$

$= 66 \times 409$

$= 26994$ cm^3 $= 26.994$ lit

Hence, the capacity of the bucket is 26.994 lit.

Q42. A circus tent is to be erected in the form of a cone surmounted on a cylinder. The total height of the tent is 49 m. Diameter of the base is 42 m and the height of the cylinder is 21 m. Find the cost of canvas needed to make the tent, if the cost of canvas is ₹ 12.50/m². (Take $\pi = 22/7$)

Solution:

Given,

Diameter of cylinder = 42 m

Radius = r = 21 m

Height of cylinder = h = 21 m

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \left(\frac{22}{7}\right) \times 21 \times 21$$

$$= 44 \times 62$$

$$= 2772 \text{ m}^2$$

Radius of the cone = R = 21 m (given)

Height = H = 28 m

Slant height = $l = \sqrt{(R^2 + H^2)}$

$$= \sqrt{(21^2 + 28^2)}$$

$$= \sqrt{(441 + 784)}$$

$$= \sqrt{1225}$$

$$= 35 \text{ m}$$

Curved surface area of cone = πRl

$$= \left(\frac{22}{7}\right) \times 21 \times 35$$

$$= 2310 \text{ m}^2$$

Total surface area of circus tent = 2772 + 2310 = 5082 m²

Cost of canvas 1 m² = ₹ 12.50

Total cost of canvas = 5082 × ₹ 12.50 = ₹ 63525

Q43. Calculate the standard deviation of the following data.

x	3	8	13	18	23
f	7	10	15	10	8

Solution:

x	f	$d = x - 13$	d^2	fd^2
3	7	-10	100	700
8	10	-5	25	250
13	15	0	0	0
18	10	5	25	250
23	8	10	100	800
	$\sum d = 50$			$\sum d^2 = 2000$

$$\sigma = \sqrt{\left(\frac{\sum fd^2}{\sum f}\right)}$$

$$= \sqrt{\left(\frac{2000}{50}\right)}$$

$$= \sqrt{40}$$

$$= 6.325$$

Therefore, the standard deviation is 6.325.

Q44. A box contains 4 white balls, 6 red balls, 7 black balls and 3 blue balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is :

(i) neither white nor black

(ii) red or white, and

(iii) either white or red or black or blue

Solution:

Given,

4 white balls, 6 red balls, 7 black balls and 3 blue balls

$n(S) = 4 + 6 + 7 + 3 = 20$

(i) Number of balls other than white and black = 6 (red) +3 (blue) = 9

P(neither white nor black) $= \frac{9}{20}$

(ii) Probability of getting red ball $= P(R) = \frac{6}{20}$

Probability of getting white ball $= P(W) = \frac{4}{20}$

P(red or white ball) $= P(R) + P(W)$

$$= \left(\frac{6}{20}\right) + \left(\frac{4}{20}\right)$$

$$= \frac{10}{20}$$

$$= \frac{1}{2}$$

(iii) Probability of getting black ball $= \frac{7}{20}$

Probability of getting blue ball $= \frac{3}{20}$

P(either white or red or black or blue) $= \left(\frac{6}{20}\right) + \left(\frac{4}{20}\right) + \left(\frac{7}{20}\right) + \left(\frac{3}{20}\right)$

$$= \frac{20}{20}$$

$$= 1$$

Q45. (a) If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, then prove that $c^2 = a^2(1 + m^2)$

Solution:

Given that the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots.

Therefore, discriminant $= 0$

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$

$\Rightarrow 4m^2c^2 - 4[c^2 - a^2 + m^2c^2 - m^2a^2] = 0$

$\Rightarrow 4m^2c^2 - 4c^2 + 4a^2 - 4m^2c^2 + 4m^2a^2 = 0$

$$\Rightarrow -4c^2 + 4a^2(1 + m^2) = 0$$

$$\Rightarrow 4c^2 = 4a^2(1 + m^2)$$

$$\Rightarrow c^2 = a^2(1 + m^2)$$

Hence proved.

(b) Find the equation of the straight-line segment whose endpoints are the points of intersection of the straight lines $2x - 3y + 4 = 0, x - 2y + 3 = 0$ and the midpoint of the line joining the points $(3, -2)$ and $(-5, 8)$.

Solution:

Given equations of lines are:

$2x - 3y + 4 = 0$ ....(i)

$x - 2y + 3 = 0$ ....(ii)

$(ii) \times 2 - (i)$,

$2x - 4y + 6 - (2x - 3y + 4) = 0$

$-y + 2 = 0$

$y = 2$

Substituting $y = 2$ in (ii)

$x - 2(2) + 3 = 0$

$x - 4 + 3 = 0$

$x = 1$

Therefore, the point of intersection of lines $= A(1, 2)$

Midpoint of the line joining the points $(3, -2)$ and $(-5, 8) = B = \left[\frac{3-5}{2}, \frac{-2+8}{2}\right]$

$$= \left(-\frac{2}{2}, \frac{6}{2}\right)$$

$$= (-1, 3)$$

Equation of the line segment joining A and B is

$$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$$

$$\frac{y - 2}{3 - 2} = \frac{x - 1}{-1 - 1}$$
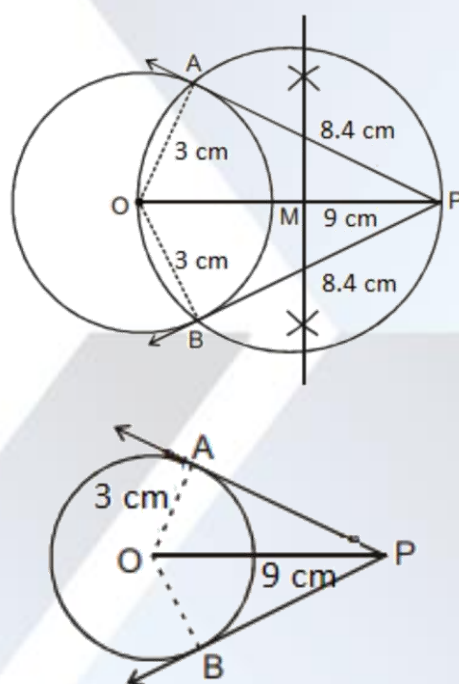
$$\frac{y - 2}{1} = \frac{x - 1}{-2}$$

$$(-2)(y - 2) = x - 1$$

$-2y + 4 = x - 1$

$\Rightarrow x + 2y - 1 - 4 = 0$

$\Rightarrow x + 2y - 5 = 0$

SECTION - IV

Q46. (a) Draw the two tangents from a point which is 9 cm away from the centre-of a circle of radius 3 cm. Also, measure the lengths of the tangents.
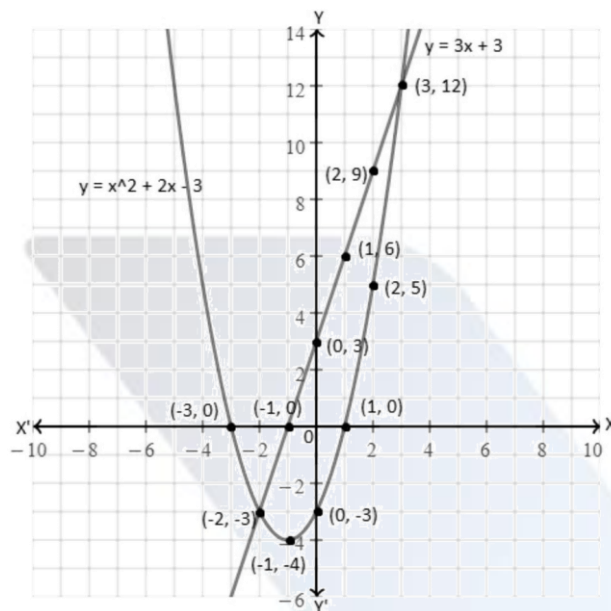
Solution:

Length of the tangents = 8.4 cm

(b) Construct a cyclic quadrilateral $PQRS$ given $PQ = 5\ cm, QR = 4\ cm, \angle QPR = 35°$ and $\angle PRS = 70°$.

Solution:

Q47. (a)Draw the graph of $y = x^2 + 2x - 3$ and hence find the roots of $x^2 - x - 6 = 0$.

Solution:

Given,

$y = x^2 + 2x - 3$

x	-3	-2	-1	0	1	2	3
y	0	-3	-4	-3	0	5	12

Now, subtracting $x^2 - x - 6 = 0$ from $y = x^2 + 2x - 3$,

$y - 0 = x^2 + 2x - 3 - (x^2 - x - 6)$

$y = 3x + 3$

x	-2	-1	0	1	2	3
y	-3	0	3	6	9	12

The graph of given equations intersect each other at $(-2, -3)$ and $(3, 12)$.

Hence, the solution set is $(-2, -3)$ and $(3, 12)$.

(b)

Number of workers	3	4	6	8	9	16
Number of days	96	72	48	36	32	18

Draw the graph for the data given in the table. Hence find the number of days taken by 12 workers to complete the work.

Solution:
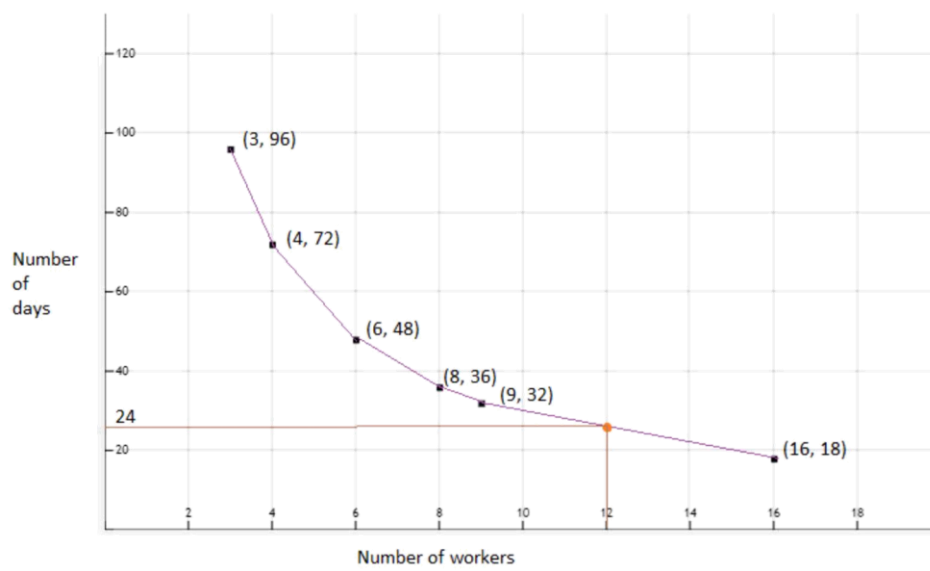
Hence, the number of days taken by 12 workers to complete the work is 24.