Grade 10 Tamil Nadu Mathematics 2017

SECTION – I

Q1. If $f(x) = x^2 + 5$, then $f(-4) =$

(a) 26

(b) 21

(c) 20

(d) -20

Solution:

Correct answer: (b)

Given,

$f(x) = x^2 + 5$

$f(-4) = (-4)^2 + 5$

$= 16 + 5$

$= 21$

Q2. If $k + 2, 4k - 6, 3k - 2$ are the three consecutive terms of an AP, then the value of k is

(a) 2

(b) 3

(c) 4

(d) 5

Solution:

Correct answer: (b)

Given,

$k + 2, 4k - 6, 3k - 2$ are the three consecutive terms of an AP.

$2(4k - 6) = k + 2 + 3k - 2$

$8k - 12 = 4k$

$8k - 4k = 12$

$4k = 12$

$k = \frac{12}{4}$

$k = 3$

Q3. If the product of the first four consecutive terms of a GP is 256 and if the common ratio is 4 and the first term is positive, then its 3rd term is:

(a) 8

(b) $\frac{1}{16}$

(c) $\frac{1}{32}$
(d) 16

Solution:

Correct answer. (a)

Let a be the first term and r be the common ratio of GP.

Given, $r = 4$

Thus, $a, 4a, 16a$, and $64a$ are the first four consecutive terms of a GP.

$a \times 4a \times 16a \times 64a = 256$

$$a^4 = \frac{256}{64 \times 64}$$

$$a^4 = \left(\frac{4}{8}\right)^4$$

$$\Rightarrow a = \frac{4}{8}$$

$$\Rightarrow a = \frac{1}{2}$$

Therefore, the third term $= 16a = 16 \times \left(\frac{1}{2}\right) = 8$

Q4. The remainder when $x^2 - 2x + 7$ is divided by $x + 4$ is:
(a) 28
(b) 29
(c) 30
(d) 31

Solution:

Correct answer: (d)

$$\begin{array}{r|lll} & x & -6 & \\ x+4 & x^2 & -2x & +7 \\ - & x^2 & +4x & \\ \hline & & -6x & +7 \\ & - & & \\ & & -6x & -24 \\ \hline & & & 31 \end{array}$$

Therefore, remainder $= 31$

Q5. The common root of the equations $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ is:
(a) $\frac{c+a}{2b}$
(b) $\frac{c-a}{2b}$

(c) $\frac{c+b}{2a}$
(d) $\frac{a+b}{2c}$

Solution:

Correct answer: (a)

$x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ have a common root.

$\Rightarrow x^2 - bx + c = x^2 + bx - a$

$\Rightarrow c + a = bx + bx$

$\Rightarrow c + a = 2bx$

$\Rightarrow x = \dfrac{c + a}{2b}$

Q6. If $A = \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$ and $A + B = \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix}$, then the matrix B =

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(b) $\begin{pmatrix} 6 & 2 \\ 3 & -1 \end{pmatrix}$
(c) $\begin{pmatrix} -8 & -2 \\ 1 & -7 \end{pmatrix}$
(d) $\begin{pmatrix} 8 & 2 \\ -1 & 7 \end{pmatrix}$

Solution:

Correct answer: (c)

From the given,

$B = A + B - A$

$= \begin{pmatrix} -1 & 0 \\ 2 & -4 \end{pmatrix} - \begin{pmatrix} 7 & 2 \\ 1 & 3 \end{pmatrix}$

$= \begin{pmatrix} -8 & -2 \\ 1 & -7 \end{pmatrix}$

Q7. Slope of the straight line which is perpendicular to the straight line joining the points $(-2,6)$ and $(4,8)$ is equal to:

(a) $\frac{1}{3}$
(b) 3
(c) -3
(d) $-\frac{1}{3}$

Solution:

Correct answer: (c)

Let the given points be:

$(x_1, y_1) = (-2,6)$

$(x_2, y_2) = (4,8)$

Slope of the line joining the given points $= m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \frac{8-6}{4+2}$

$= \frac{2}{6}$

$= \frac{1}{3}$

Slope of the straight line which is perpendicular to the straight line joining the given points $= -\frac{1}{\text{m}} = -\frac{1}{\frac{1}{3}}$

$= -3$

Q8. If the points $(2,5), (4,6)$ and (a, a) are collinear, then the value of ' a ' is equal to:

(a) -8

(b) 4

(c) -4

(d) 8

Solution:

Correct answer: (d)

Given, $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3) = P(2,5), Q(4,6)$ and $R(a, a)$ are collinear.

Thus, the area of the triangle formed by these points is 0 .

$\left(\frac{1}{2}\right)|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3[y_1 - y_2]| = 0$

$\Rightarrow [2(6-a) + 4(a-5) + a(5-6)] = 0$

$\Rightarrow 12 - 2\text{a} + 4\text{a} - 20 + 5\text{a} - 6\text{a} = 0$

$\Rightarrow a - 8 = 0$

$\Rightarrow a = 8$

Q9. The perimeters of two similar triangles are 24 cm and 18 cm respectively. If one side of the first triangle is 8 cm, then the corresponding side of the other triangle is:

(a) 4 cm

(b) 3 cm

(c) 9 cm

(d) 6 cm

Solution:

Correct answer: (d)

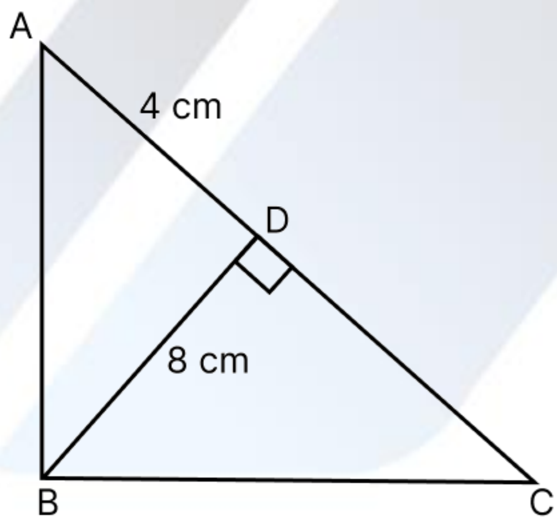
Given,

Perimeters of two similar triangles are 24 cm and 18 cm respectively.

Length of side of the first triangle = 8 cm

Let x be the side of another triangle.
Ratio of perimeters = Ratio of the corresponding sides
$\Rightarrow \frac{24}{18} = \frac{8}{x}$
$\Rightarrow \frac{4}{3} = \frac{8}{x}$
$\Rightarrow x = \frac{24}{4}$
$\Rightarrow x = 6$
Therefore, the corresponding side of the second triangle is 6 cm .

Q10. $\triangle ABC$ is a right-angled triangle where $\angle B = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm, $AD = 4$ cm, then CD is:
(a) 24 cm
(b) 16 cm
(c) 32 cm
(d) 8 cm
Solution:
Correct answer: (b)
Given,



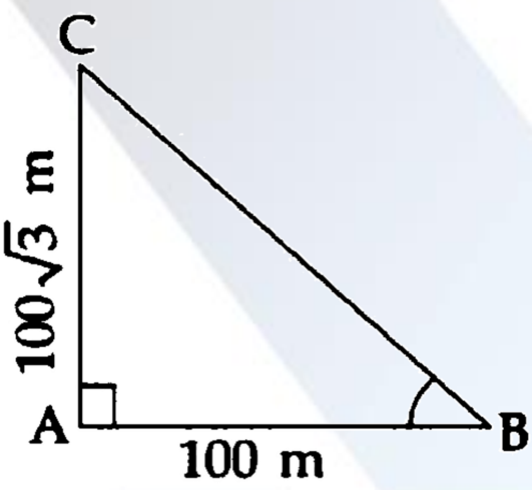
$\triangle$ DBA $\sim \triangle$ DCB
Thus,
$\frac{BD}{CD} = \frac{AD}{BD}$
$BD^2 = AD \times DC$
$(8)^2 = 4 \times DC$

$64 = 4 \times \text{DC}$

$DC = \frac{64}{4}$

$CD = 16 \text{ cm}$

Q11. In the adjoining figure, $\angle ABC =$



(a) 45°
(b) 30°
(c) 60°
(d) 50°

Solution:

Correct answer: (c)

Let $\angle ABC = \theta$

$\tan\ \theta = \frac{\text{AC}}{\text{AB}}$

$\tan\ \theta = \frac{100\sqrt{3}}{100}$

$\tan\ \theta = \sqrt{3}$

$\tan\ \theta = \tan\ 60^\circ$

$\Rightarrow \theta = 60^\circ$

Q12. $9\tan^2\ \theta - 9\sec^2\ \theta =$

(a) 1
(b) 0
(c) 9
(d) -9

Solution:

Correct answer: (d)

$9\tan^2\theta - 9\sec^2\theta = 9(\tan^2\theta - \sec^2\theta)$

$= -9(\sec^2\theta - \tan^2\theta)$

$= -9(1)$

$= -9$

Q13. If the surface area of sphere is $100\pi\ \text{cm}^2$, then its radius is equal to:

(a) 25 cm

(b) 100 cm

(c) 5 cm

(d) 10 cm

Solution:

Correct answer: (c)

Given,

Surface area of sphere $= 100\pi\ \text{cm}^2$

$4\pi r^2 = 100\pi$

$r^2 = \frac{100}{4}$

$r^2 = 25$

$r = 5\ cm$

Q14. Standard deviation of a collection of a data is $2\sqrt{2}$. If each value is multiplied by 3, then the standard deviation of the new data is:

(a) $\sqrt{12}$

(b) $4\sqrt{2}$

(c) $6\sqrt{2}$

(d) $9\sqrt{2}$

Solution:

Correct answer: (c)

Given,

Standard deviation $= 2\sqrt{2}$

Each observation is multiplied by 3, then new standard deviation $= 3 \times (2\sqrt{2}) = 6\sqrt{2}$

Q15. A card is drawn from a pack of 52 cards at random. The probability of getting neither an ace nor a king card is:

(a) $\frac{2}{13}$

(b) $\frac{11}{13}$

(c) $\frac{4}{13}$

(d) $\frac{8}{13}$

Solution:

Correct answer: (b)

Total number of outcomes = 52

Number of ace cards = 4

Number of king cards = 4

Number of cards other than ace and king = $52 - (4 + 4) = 44$

Therefore, P (neither an ace nor aking) = $\frac{44}{52} = \frac{11}{13}$

SECTION – II

Q16. Given, $A = \{1,2,3,4,5\}, B = \{3,4,5,6\}$, and $C = \{5,6,7,8\}$, show that $A \cup (B \cup C) = (A \cup B) \cup C$.

Solution:

Given,

$A = \{1,2,3,4,5\}, B = \{3,4,5,6\}$, and $C = \{5,6,7,8\}$

$B \cup C = \{3,4,5,6] \cup \{5,6,7,8\}$

$= \{3,4,5,6,7,8\}$

$A \cup (B \cup C) = \{1,2,3,4,5\} \cup \{3,4,5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$ ..... (i)

$A \cup B = \{1,2,3,4,5\} \cup \{3,4,5,6\}$

$= \{1,2,3,4,5,6\}$

$(A \cup B) \cup C = \{1,2,3,4,5,6\} \cup \{5,6,7,8\}$

$= \{1,2,3,4,5,6,7,8\}$ ......(ii)

From (i) and (ii),

$A \cup (B \cup C) = (A \cup B) \cup C$

Q17. The following table represents a function from A = {5,6,8,10} to B = {19,15,9,11} where $f(x) = 2x - 1$.

Find the values of a and b.

x	5	6	8	10
$f(x)$	a	11	b	19

Solution:

Given,

$y = f(x) = 2x - 1$
$a = f(5) = 2(5) - 1 = 10 - 1 = 9$
$b = f(8) = 2(8) - 1 = 16 - 1 = 15$
Therefore, $a = 9$ and $b = 15$.

Q18. $-\frac{2}{7}, m, -\frac{7}{2}(m + 2)$ are in GP, find the values of m.

Solution:

Given,

$-\frac{2}{7}, m, -\frac{7}{2}(m + 2)$ are in GP.

If a, b, c are in GP, then $b^2 = ac$.

$\Rightarrow m^2 = \left(-\frac{2}{7}\right) \times \left(-\frac{7}{2}\right)(m + 2)$
$\Rightarrow m^2 = m + 2$
$\Rightarrow m^2 - m - 2 = 0$
$\Rightarrow m^2 - 2m + m - 2 = 0$
$\Rightarrow m(m - 2) + 1(m - 2) = 0$
$\Rightarrow (m - 2)(m + 1) = 0$
$\Rightarrow m = 2, -1$

Q19. Solve by elimination method:

$13x + 11y = 70$
$11x + 13y = 74$

Solution:

Given,

$13x + 11y = 70$ ... (i)
$11x + 13y = 74$ ... (ii)
Adding (i) and (ii).
$24x + 24y = 144$
$24(x + y) = 144$
$x + y = 6$....(iii)
Subtracting (ii) from (i),
$2x - 2y = -4$
$x - y = -2$....(iv)
Adding (iii) and (iv),
$x + y + x - y = 6 - 2$
$2x = 4$
$x = 2$
Substituting $x = 2$ in (iii),
$2 + y = 6$

$y = 6 - 2 = 4$
Hence, the required solution is $x = 2$ and $y = 4$, i.e. $(x, y) = (2,4)$.

Q20. Simplify: $\frac{(6x^2+9x)}{(3x^2-12x)}$

Solution:

$$\frac{(6x^2 + 9x)}{(3x^2 - 12x)}$$
$$= \frac{[3x(2x + 3)]}{[3x(x - 4)]}$$
$$= \frac{2x + 3}{x - 4}$$

Q21. Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i - j$.

Solution:

$\mathrm{a_{ij}} = 2\mathrm{i} - \mathrm{j}$
$a_{11} = 2(1) - 1 = 2 - 1 = 1$
$a_{12} = 2(1) - 2 = 2 - 2 = 0$
$a_{21} = 2(2) - 1 = 4 - 1 = 3$
$a_{22} = 2(2) - 2 = 4 - 2 = 2$
$\Rightarrow A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$

Q22. Let $A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$. Find the matrix C, if $C = 2A + B$.

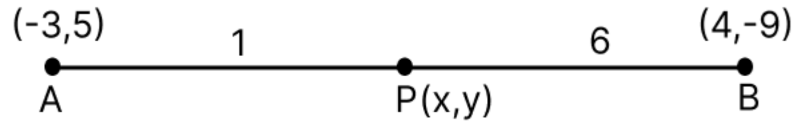
Solution:

$C = 2A + B$
$= 2\begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$
$= \begin{pmatrix} 6 & 4 \\ 10 & 2 \end{pmatrix} + \begin{pmatrix} 8 & -1 \\ 4 & 3 \end{pmatrix}$
$= \begin{pmatrix} 14 & 3 \\ 14 & 5 \end{pmatrix}$

Q23. Find the coordinates of the point which divides the line segment joining $(-3,5)$ and $(4, -9)$ in the ratio $1: 6$ internally.

Solution:

Let $P(x, y)$ divide the line segment joining the points $\mathrm{A}(-3,5)$ and $\mathrm{B}(4, -9)$ internally in the ratio $1: 6$.



Here,

$(x_1, y_1) = (-3,5)$

$(x_2, y_2) = (4, -9)$

$l : m = 1:6$

$$P(x, y) = \left[\frac{(lx_2 + mx_1)}{l+m}, \frac{(ly_2 + my_1)}{l+m}\right]$$

$$= \left[\frac{4-18}{1+6}, \frac{-9+30}{1+6}\right]$$

$$= \left(-\frac{14}{7}, \frac{21}{7}\right)$$

$$= (-2, 3)$$

Q24. "The points $(0, a), a > 0$ lie on x-axis for all a ". Justify the truthness of the statement.

Solution:

For all $a > 0$, the points like $(0, a)$ will exist on the positive side of the y-axis.

Therefore, the given statement is false.

Q25. In $\triangle PQR, AB \| QR$. If AB is 3 cm, PB is 2 cm and PR is 6 cm, then find the length of QR.

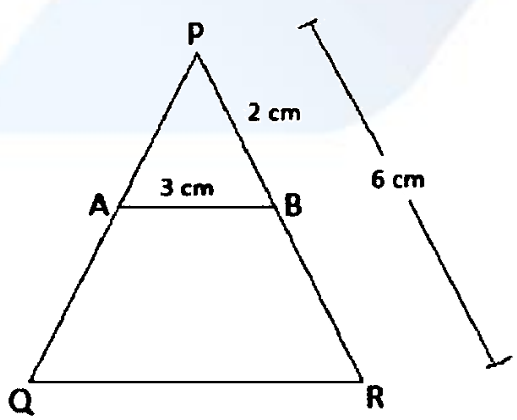
Solution:

Given that, in $\triangle PQR, AB \| QR$.

AB $= 3$ cm

PB $= 2$ cm

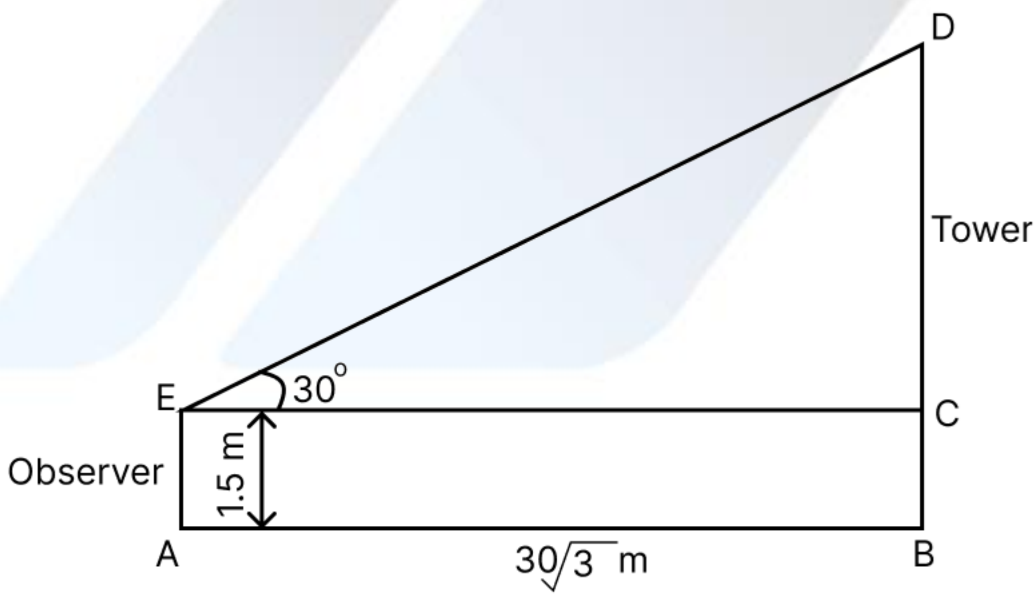
PR $= 6$ cm



In $\triangle$ PAB and $\triangle$ PQR,
$\angle PAB = \angle PQR$ (corresponding angles)
$\angle P = \angle P$ (common)
By AA similarity,
$\triangle PAB \sim \triangle PQR$
By BPT,
$\frac{AB}{QR} = \frac{PB}{PR}$
$QR = \frac{AB \times PR}{PB}$
$= \frac{3 \times 6}{2}$
$= 9$ cm

Q26. The angle of elevation of the top of a tower as seen by an observer is 30°. The observer is at a distance of $30\sqrt{3}$ m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.

Solution:
Let BD be the height of the tower.
AE be the distance between the eye level of the observer and the ground level.



$AB = CE = 30\sqrt{3}$ m
$AE = BC = 1.5$ m

In right triangle DEC,

$\tan 30^\circ = \frac{CD}{EC}$

$\frac{1}{\sqrt{3}} = \frac{CD}{30\sqrt{3}}$

$\Rightarrow CD = \frac{30\sqrt{3}}{\sqrt{3}}$

$\Rightarrow CD = 30$ m

Therefore, height of the tower = BD

= BC + CD

= 1.5 + 30

= 31.5 m

Q27. The total surface area of a solid right circular cylinder is 1540 cm^2. If the height is four times the radius of the base, then find the height of the cylinder.

Solution:

Let r be the radius of the base and h be the height of the cylinder.

Given,

$h = 4r$

Total surface area = 1540 cm^2

$2\pi r(r + h) = 1540$

$2 \times \left(\frac{22}{7}\right) \times r(r + 4r) = 1540$

$r(5r) = \frac{1540 \times 7}{22 \times 2}$

$5r^2 = 35 \times 7$

$r^2 = 7 \times 7$

$r = 7$ m

Height of the cylinder = $h = 4r = 4(7) = 28$ cm

Q28. The smallest value of a collection of data is 12 and the range is 59. Find the largest value of the collection of data.

Solution:

Given,

Smallest number = 12

Range = 59

We know that,

Range = Largest number − Smallest number

59 = Largest number − 12

$\Rightarrow$ Largest number $= 59 + 12 = 71$

Q29. In tossing a fair coin twice, find the probability of getting:

(i) Two heads

(ii) Exactly one tail

Solution:

Sample space $= \mathrm{S} = \{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$

$\mathrm{n(S)} = 4$

(i) Let A be the event of getting two heads.

$\mathrm{A} = \{\mathrm{HH}\}$

Number of outcomes favourable to $\mathrm{A} = \mathrm{n(A)} = 1$

$$\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(S)}}$$

$$= \frac{1}{4}$$

(ii) Let B be the event of getting exactly one tail.

$\mathrm{B} = \{\mathrm{HT}, \mathrm{TH}\}$

Number of outcomes favorable to $B = n(B) = 2$

$$\mathrm{P(B)} = \frac{\mathrm{n(B)}}{\mathrm{n(S)}}$$

$$= \frac{2}{4}$$

$$= \frac{1}{2}$$

Q30. (a) If the volume of a solid sphere is $\frac{72411}{7}$ cu. cm, then find its radius. (take $\pi = \frac{22}{7}$)

Solution:

Let r be the radius of a solid sphere.

Given,

Volume of sphere $= \frac{72411}{7}$ cu. cm

$$\left(\frac{4}{3}\right) \pi r^3 = \frac{50688}{7}$$

$$\left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times r^3 = \frac{50688}{7}$$

$$r^3 = \frac{50688 \times 3}{4 \times 22}$$

$$r^3 = 1728$$

$$r^3 = 43 \times 33$$

$$\Rightarrow \mathrm{r} = 4 \times 3$$

$$\Rightarrow r = 12$$

Hence, the radius of the sphere $= 12$ cm

(b) If $x = a\sec\ \theta + b\tan\ \theta$ and $y = a\tan\ \theta + b\sec\ \theta$, then prove that: $x^2 - y^2 = a^2 - b^2$

Solution:

Given,

$x = a\sec\theta + b\tan\ \theta$

$y = a\tan\ \theta + b\sec\ \theta$

LHS $= x^2 - y^2$

$= (a\sec\ \theta + b\tan\ \theta)^2 - (a\tan\ \theta + b\sec\ \theta)^2$

$= a^2\sec^2\ \theta + b^2\tan^2\ \theta + 2ab\sec\ \theta\tan\ \theta - a^2\tan^2\ \theta - b^2\sec^2\ \theta - 2ab\tan\ \theta\sec\ \theta$

$= a^2(\sec^2\ \theta - \tan^2\ \theta) - b^2(\sec^2\ \theta - \tan^2\ \theta)$

$= a^2(1) - b^2(1)$

$= a^2 - b^2$

= RHS

Hence proved.

SECTION – III

Q31. Let $A = \{a, b, c, d, e, f, g, x, y, z\}, B = \{1,2, c, d, e\}$ and $C = \{d, e, f, g, 2, y\}$. Verify $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

Solution:

Given,

$A = \{a, b, c, d, e, f, g, x, y, z\}, B = \{1,2, c, d, e\}$ *and* $C = \{d, e, f, g, 2, y\}$

$B \cup C = \{1,2, c, d, e\} \cup \{d, e, f, g, 2, y\}$

$= \{1,2, c, d, e, f, g, y\}$

$A \setminus (B \cup C) = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{1,2, c, d, e, f, g, y\}$

$= \{a, b, x, z\}$ ... *(i)*

$A \setminus B = \{a, b, c, d, e, f, g, x, y, z\} \setminus \{1,2, c, d, e\}$

$= (a, b, f, g, x, y, z)$

$A \setminus C = \{a, b, c, d, e, f, g, x, y, z\} \setminus (d, e, f, g, 2, y\}$

$= \{a, b, c, x, z\}$

$(A \setminus B) \cap (A \setminus C) = \{a, b, f, g, x, y, z\} \cap \{a, b, c, x, z\}$

$= \{a, b, x, z\}$ ... *(ii)*

From (i) and (ii),

$A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

Hence verified.

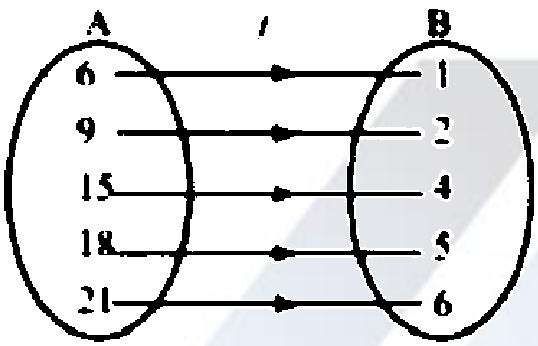
Q32. Let $A = \{6,9,15,18,21\}, B = \{1,2,4,5,6\}$ and $f: A \to B$ be defined by $f(x) = \frac{x-3}{3}$.

Represent f by:

(i) an arrow diagram

(ii) a set of ordered pairs

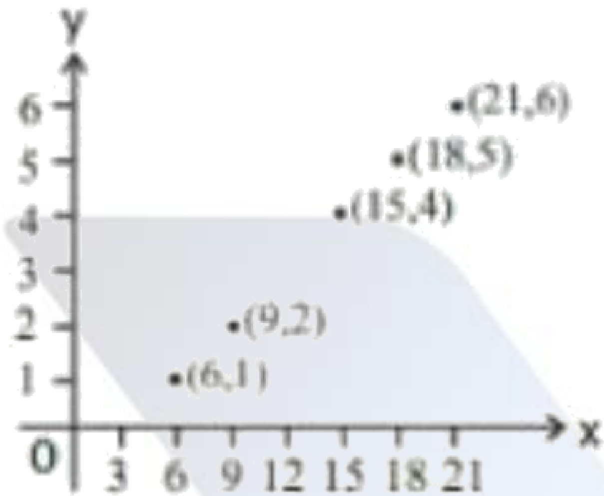
(iii) a table
(iv) a graph
Solution:
Given,
$$f(x)=\frac{x-3}{3}$$
$$f(6)=\frac{6-3}{3}=\frac{3}{3}=1$$
$$f(9)=\frac{9-3}{3}=\frac{6}{3}=2$$
$$\mathrm{f}(15)=\frac{15-3}{3}=\frac{12}{3}=4$$
$$\mathrm{f}(18)=\frac{18-3}{3}=\frac{15}{3}=5$$
$$f(21)=\frac{21-3}{3}=\frac{18}{3}=6$$
(i) An arrow diagram:



(ii) The set of ordered pairs
f = {(6,1), (9,2), (15,4), (18,5), (21,6)}
(iii) Table

x	6	9	15	18	21
$f(x)$	1	2	4	5	6

(iv) Graph:



Q33. Find the sum of the first $2n$ terms of the series $1^2 - 2^2 + 3^2 - 4^2 + \cdots$.

Solution:

Given,

$1^2 - 2^2 + 3^2 - 4^2 + \cdots$

$= 1 - 4 + 9 - 16 + 25 - \cdots$.. to $2n$ terms

$= (1-4) + (9-16) + (25-36) + \cdots$. to n terms (after grouping)

$= -3 + (-7) + (-11) + \cdots$.. n terms

This is an AP with $a = -3, d = -4$

Hence, the required sum $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(-3) + (\mathrm{n}-1)(-4)]$

$= \left(\frac{n}{2}\right)[-6 - 4n + 4]$

$= \left(\frac{n}{2}\right)[-4n - 2]$

$= \left(-\frac{2n}{2}\right)(2n+1)$

$= -n(2n+1)$

Q34. Find the sum of the first n terms of the series $7 + 77 + 777 + \cdots$

Solution:

Given,

$7 + 77 + 777 + \cdots$

Sum of first n terms

$S_n = 7 + 77 + 777 + \cdots .. + (n \text{ terms})$

$S_n = 7(1 + 11 + 111 + \cdots . + \text{n terms})$

$= \left(\frac{7}{9}\right)[9 + 99 + 999 + \cdots + n \text{ terms}]$

$$= \left(\frac{7}{9}\right)[(10-1)+(100-1)+(1000-1)+\cdots.+(10^n-1)]$$
$$= \left(\frac{7}{9}\right)[(10+10^2+10^3+\cdots+10^n)-(1+1+1+\cdots+n\text{ terms })]$$
$$= \left(\frac{7}{9}\right)\left\{\left[\frac{10(10^n-1)}{10-1}\right]-n\right\}$$
$$= \left(\frac{7}{9}\right)\left\{\left[\frac{10(10^n-1)}{9}\right]-n\right\}$$
$$= \left(\frac{70}{81}\right)(10^n-1)-\frac{7n}{9}$$

Q35. The speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns downstream to the original point in 4 hrs. 30 minutes. Find the speed of the stream.

Solution:

Given,

Speed in still water = 15 km/hr

Total time taken $= 4\text{hrs}30\text{ min} = 4\frac{1}{2}\text{hr} = \frac{9}{2}\text{hrs}$

Let x km/hr be the speed of the stream.

Speed of the boat during upstream $= 15 - x$

Speed of the boat during downstream $= 15 + x$

Time taken for upstream $= T_1 = \frac{30}{15-x}$

Time taken for downstream $= T_2 = \frac{30}{15+x}$

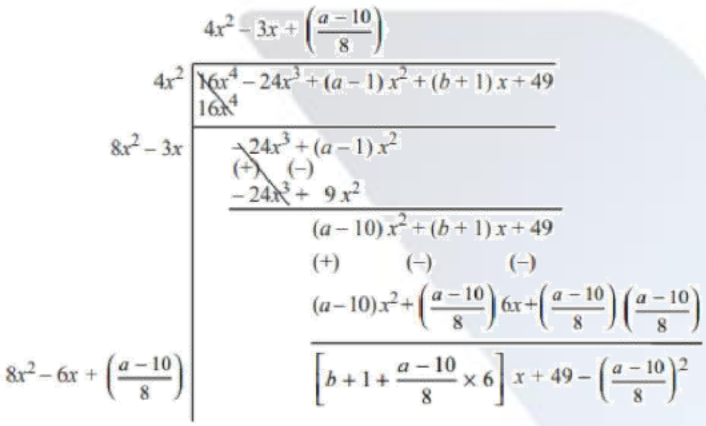
According to the given,

$$T_1 + T_2 = \frac{9}{2}$$
$$\left[\frac{30}{15-x}\right]+\left[\frac{30}{15+x}\right]=\frac{9}{2}$$
$$30\left[\frac{15+x+15-x}{(15-x)(15+x)}\right]=\frac{9}{2}$$
$$\frac{30}{(225-x^2)}=\left(\frac{9}{2}\times 30\right)$$
$$\frac{30}{(225-x^2)}=\frac{3}{20}$$
$$\Rightarrow 600 = 675 - 3x^2$$
$$\Rightarrow 3x^2 = 675 - 600$$
$$\Rightarrow x^2 = \frac{75}{3}$$
$$\Rightarrow x^2 = 25$$
$$\Rightarrow x = 5$$

Therefore, speed of the stream $= 5$ km/hr

Q36. Find the value of a and b if $16x^4 - 24x^3 + (a-1)x^2 + (b+1)x + 49$ is a perfect square.

Solution:



Given that, the polynomial is a perfect square.

Therefore, remainder $= 0$

$b + 1 + \left[\frac{a-10}{8}\right] \times 6 = 0$ ... (i)

$$49 - \left[\frac{a-10}{8}\right]^2 = 0$$

$$\Rightarrow \left[\frac{a-10}{8}\right]^2 = 49$$

$$\Rightarrow \frac{a-10}{8} = 7$$

$\Rightarrow a - 10 = 56$

$\Rightarrow a = 56 + 10 = 66$

Substituting $a = 66$ *in* (i).

$$\text{b} + 1 + \left[\frac{66-10}{8}\right] \times 6 = 0$$

$$b + 1 + \left(\frac{56}{8}\right) \times 6 = 0$$

$b + 1 + 42 = 0$

$b = -43$

Therefore, $a = 66$ and $b = -43$.

Q37. If $A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$.

Solution:

Given,

$$A = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix} \Rightarrow A^T = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$
$$B = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} \Rightarrow B^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$
$$AB = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 10-2 & -5+2 \\ 14 & -3 \\ & -7+3 \end{pmatrix}$$
$$= \begin{pmatrix} 8 & -3 \\ 11 & -4 \end{pmatrix}$$
$$\therefore (AB)^T = \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix}$$
$$B^T A^T = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 10-2 & 14-3 \\ -5+2 & -7+3 \end{pmatrix}$$
$$= \begin{pmatrix} 8 & 11 \\ -3 & -4 \end{pmatrix}$$
Hence, $(AB)^T = B^T A^T$

Q38. Find the area of the quadrilateral formed by the points $(-4,-2), (-3,-5), (3,-2)$ and $(2,3)$.

Solution:

Given, vertices of the quadrilateral are $(-4,-2), (-3,-5), (3,-2)$ and $(2,3)$.

Area of the quadrilateral

$$= \frac{1}{2}\left|\begin{array}{ccccc} -4 & -3 & 3 & 2 & -4 \\ -2 & -5 & 2 & 3 & -2 \end{array}\right|$$

$$= \left(\frac{1}{2}\right)[(20+6+9-4)-(6-15-4-12)]$$

$$= \left(\frac{1}{2}\right)[31+25]$$

$$= \left(\frac{1}{2}\right)\times 56$$

$= 28$ sq.units

Q39. State and prove Pythagoras theorem.

Solution:

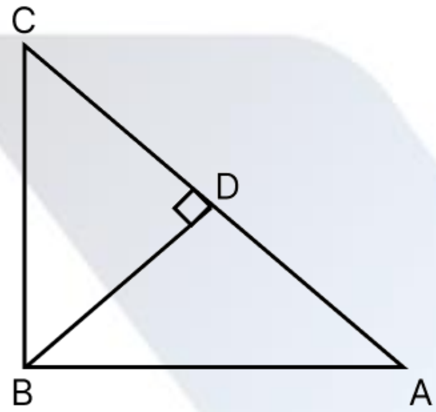
Pythagoras theorem - statement:

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Given:

In a right triangle ABC, $\angle B = 90°$

To prove:

$AC^2 = AB^2 + BC^2$
Construction:
Draw a perpendicular BD onto the side AC.

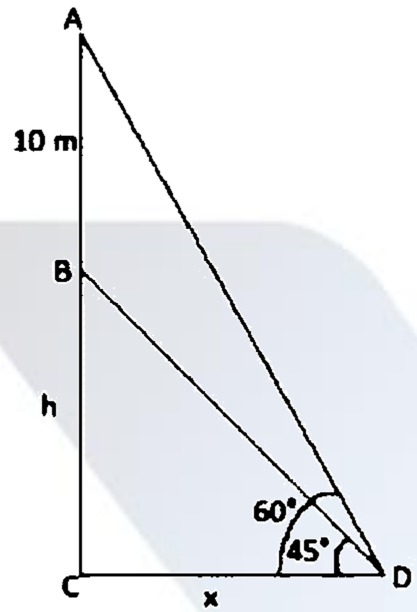


We know that,
$\triangle ADB \sim \triangle ABC$
Therefore, $\frac{AD}{AB} = \frac{AB}{AC}$ (by similarity)
$AB^2 = AD \times AC$ ..... (i)
Also, $\triangle BDC \sim \triangle ABC$
Therefore, $\frac{CD}{BC} = \frac{BC}{AC}$ (by similarity)
$BC^2 = CD \times AC$....(ii)
Adding (i) and (ii),
$AB^2 + BC^2 = AD \times AC + CD \times AC$
$AB^2 + BC^2 = AC(AD + CD)$
Since, $AD + CD = AC$
Therefore, $AC^2 = AB^2 + BC^2$
Hence, the Pythagorean theorem is proved.

Q40. A flag post stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flag post are 60° and 45° respectively. If the height of the flag post is 10 m, find the height of the building. ($\sqrt{3} = 1.732$)

Solution:

Let AB be the flag post and BC be the building.



In right triangle BCD ,

$\tan\ 45^\circ = \frac{\mathrm{BC}}{\mathrm{CD}}$

$1 = \frac{h}{x}$

$\Rightarrow h = x$....(i)

In right triangle ACD ,

$\tan\ 60^\circ = \frac{\mathrm{AC}}{\mathrm{CD}}$

$\sqrt{3} = \frac{10 + h}{x}$

$\sqrt{3}x = 10 + h$

$\sqrt{3}h = 10 + h$[From (i)]

$(\sqrt{3} - 1)h = 10$

$h = \frac{10}{\sqrt{3} - 1}$

$h = \left[\frac{10}{\sqrt{3} - 1}\right]\left[\frac{\sqrt{3} + 1}{\sqrt{3} + 1}\right]$

$h = \frac{10(\sqrt{3} + 1)}{3 - 1}$

$h = \frac{10(1.732 + 1)}{2}$

$= 5(2.732)$

$= 13.66\ \mathrm{m}$

Therefore, the height of the building is 13.66 m .

Q41. The perimeter of the ends of a frustum of a cone are 44 cm and 8.4π cm. If the depth is 14 cm, then find its volume.

Solution:

Given,

Depth of the frustum of a cone $= h = 14$ cm

Perimeter of circular base with radius R = 44 cm

$2\pi R = 44$

$\left(\frac{22}{7}\right) R = 22$

$R = 7$ cm

Perimeter of circular end with radius $r = 8.4\pi$ cm

$2\pi r = 8.4\pi$

$r = \frac{8.4}{2}$

$r = 4.2\ cm$

Volume of the frustum $= \left(\frac{1}{3}\right)\pi h[R^2 + r^2 + Rr]$

$= \left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times 14 \times [7^2 + 4.2^2 + (7)(4.2)]$

$= \left(\frac{44}{3}\right) \times [49 + 17.64 + 29.4]$

$= \left(\frac{44}{3}\right) \times 96.04$

$= 1408.58\ cm^3$

Q42. The length, breadth and height of a solid metallic cuboid are 44 cm, 21 cm and 12 cm respectively. It is melted and a solid cone is made out of it. If the height of the cone is 24 cm, then find the diameter of its base.

Solution:

Given, dimensions of the cuboid are:

Length $= l = 44$ cm

Breadth $= b = 21$ cm

Height $= h = 12$ cm

Let r be the radius of the cone.

Height of the cone $= H = 24$ cm (given)

Volume of cone = Volume of cuboid

$\left(\frac{1}{3}\right)\pi r^2 H = lbh$

$\left(\frac{1}{3}\right) \times \left(\frac{22}{7}\right) \times r^2 \times 24 = 44 \times 21 \times 21$

$r^2 = \frac{44 \times 21 \times 12 \times 7 \times 3}{22 \times 24}$

$r^2 = 7 \times 3 \times 7 \times 3$

$r^2 = (7 \times 3)^2$

$r = 21\ cm$

Diameter of the base of cone $= 2r = 2(21) = 42$ cm

Q43. Find the coefficient of variation of the following data.

18,20,15,12,25

Solution:

Given,

18,20,15,12,25

Mean $= \frac{18+20+15+12+25}{5}$

$= \frac{90}{5}$

$= 18$

x	$d = x - 18$	d^2
18	0	0
20	2	4
15	-3	9
12	-6	36
25	7	49
	$\Sigma d = 0$	$\sum d^2 = 98$

Standard deviation $(\sigma) = \sqrt{\left(\frac{\Sigma d^2}{n}\right)}$

$= \sqrt{\left(\frac{98}{5}\right)}$

$= \sqrt{19.6}$

$= 4.428$

$Coefficient\ of\ variation\ = \left(\frac{standard\ deviation}{mean}\right) \times 100$

$= \left(\frac{4.428}{18}\right) \times 100$

$= \frac{442.8}{18}$

$= 24.6$

Q44. If a die is rolled twice, find the probability of getting an even number in the first time or a total of 8.

Solution:

Total number of outcomes $= n(S) = 6^2 = 36$

Let A be the event of getting an even number for the first time.

$A =$
$\{(2,1),(2,2),(2,3)(2,4),(2,5),(2,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(A) = 18$

$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$

Let B be the event of getting a sum of numbers on dice 8 .

$B = \{(2,6),(6,2),(3,5),(5,3),(4,4)\}$

$n(B) = 5$

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B = \{(2,6),(6,2),(4,4)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \left(\frac{18}{36}\right) + \left(\frac{5}{36}\right) - \left(\frac{3}{36}\right)$

$= \frac{18 + 5 - 3}{36}$

$= \frac{20}{36}$

$= \frac{5}{9}$

Hence, the required probability is $\frac{5}{9}$.

Q45. (a) Find the GCD of the following polynomials $3x^4 + 6x^3 - 12x^2 - 24x$ and $4x^4 + 14x^3 + 8x^2 - 8x$.

Solution:

Let $f(x) = 3x^4 + 6x^3 - 12x^2 - 24x$

$= 3x(x^3 + 2x2 - 4x - 8)$

$g(x) = 4x^4 + 14x^3 + 8x^2 - 8x$

$= 2x(2x^3 + 7x^2 + 4x - 4)$

Now, let us find the GCD of the below polynomials.

$x^3 + 2x^2 - 4x - 8$ and $2x^3 + 7x^2 + 4x - 4$

$$x^3+2x^2-4x-8 \,\overline{)\, 2x^3+7x^2+4x-4 \,}\; 2$$
$$2x^3+4x^2-8x-16$$
$$3x^2+12x+12$$
$$\Rightarrow x^2+4x+4$$
Remainder $\neq 0$

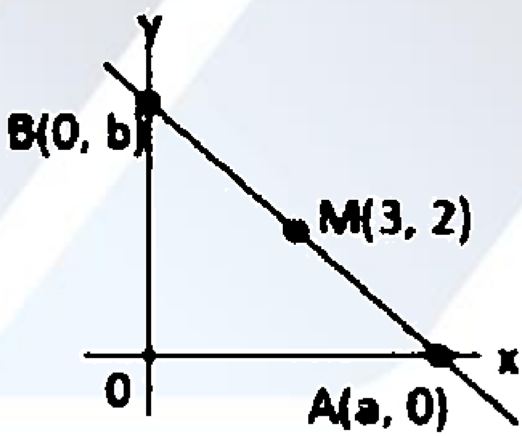
$$x^2+4x+4 \,\overline{)\, x^3+2x^2-4x-8 \,}\; x-2$$
$$x^3+4x^2+4x$$
$$-2x^2-8x-8$$
$$-2x^2-8x-8$$
0 = Remainder

Thus, common factor of x^3+2x^2-4x-8 and $2x^3+7x^2+4x-4$ is x^2+4x+4.
Also, the common factor of $3x$ and $2x$ is x.
Therefore, the required GCD the given polynomials $= x(x^2+4x+4)$.

(b) A straight line cuts the coordinate axes at A and B. If the midpoint of AB is (3,2), then find the equation of AB.

Solution:

Let $A(a,0)$ and $B(0,b)$ the points on the x -axis and y -axis respectively. Given that, (3,2) is the midpoint of AB.



M(3,2) = Midpoint of AB

$$(3,2)=\left[\frac{a+0}{2},\frac{0+b}{2}\right]$$

$$(3,2)=\left(\frac{a}{2},\frac{b}{2}\right)$$

$\Rightarrow a=6$ and $b=4$

The equation of line passing through A and B is $\left(\frac{x}{a}\right)+\left(\frac{y}{b}\right)=1$

$\Rightarrow\left(\frac{x}{6}\right)+\left(\frac{y}{4}\right)=1$

$\Rightarrow\frac{2x+3y}{12}=1$

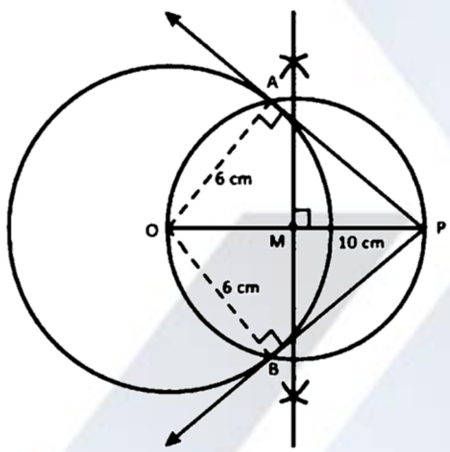
$\Rightarrow 2x+3y=12$

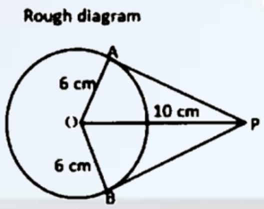
Hence, the required equation is $2x+3y-12=0$.

SECTION – IV

Q46. (a) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 6 cm. Also, measure the lengths of the tangents.

Solution:

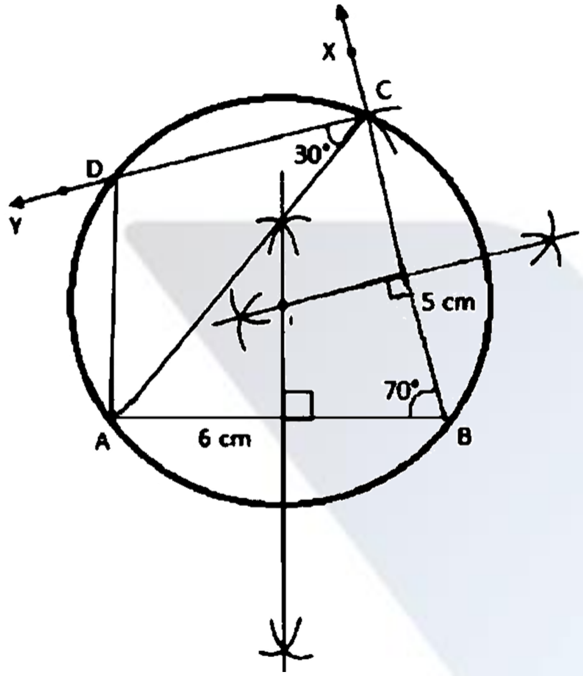




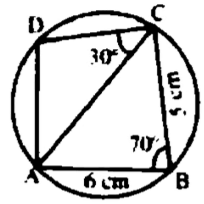
Measure of length of the tangents = PA = PB = 8 cm

(b) Construct a cyclic quadrilateral $ABCD$, given $AB=6$ cm, $\angle ABC=70^{\circ}, BC=$ 5 cm and $\angle ACD=30^{\circ}$.

Solution:



Rough diagram



Q47. (a) Solve graphically $2x^2 + x - 6 = 0$

Solution:

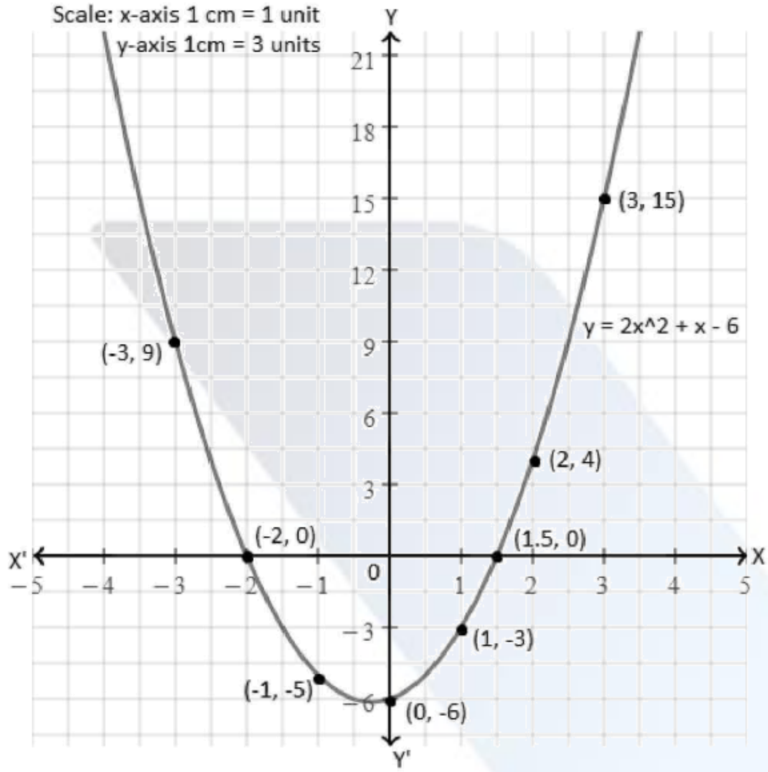
Given equation is:

$2x^2 + x - 6 = 0$

Let $y = 2x^2 + x - 6$

Plot the ordered pair of points $(-3,9), (-2,0), (-1,-5), (0,-6), (1,-3), (2,4)$ and $(3,15)$ on the graph.

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
2×2	18	8	2	0	2	8	18
x	-3	-2	-1	0	1	2	3
-6	-6	-6	-6	-6	-6	-6	-6
y	9	0	-5	-6	-3	4	15



The curve in the graph, cuts the x-axis at the points $(-2,0)$ and $(1.5,0)$.
Hence, the solution set is $x = -2$ and $x = 1.5$.

(b) Draw the graph of $xy = 20, x, y > 0$. Use the graph to find y when $x = 5$, and to find x when $y = 10$.

Solution:

Given, $xy = 20$

x	1	2	4	5
y	20	10	5	4

From the above table, it is clear that as x increases y decreases. This type of variation is called indirect variation. $y \propto \frac{1}{x}$

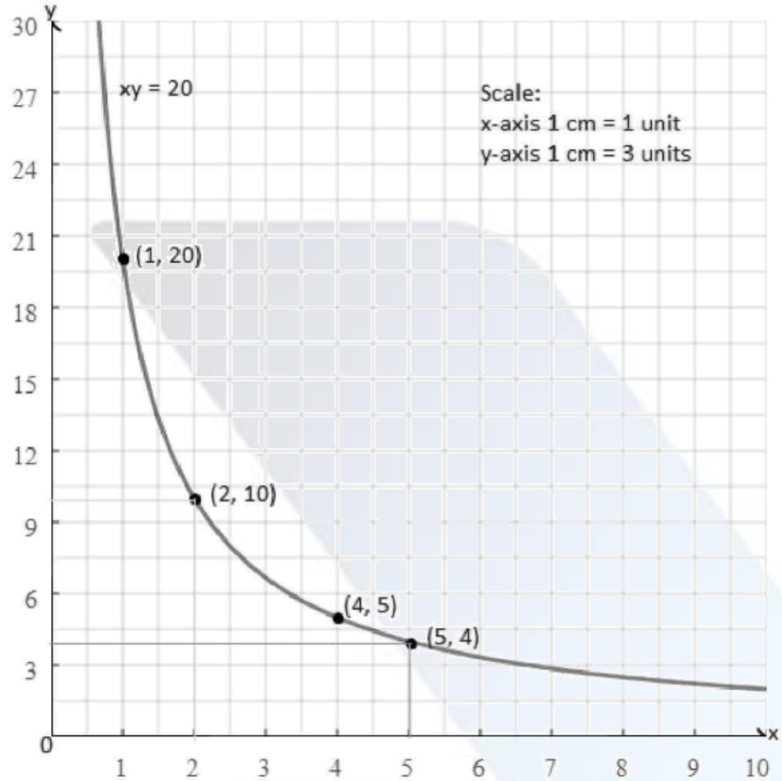
$xy = k$, where $k =$ proportionality constant

Now,

$1 \times 20 = 2 \times 10 = 4 \times 5 = 20 = k$

Thus, $k = 20$

Plot the points $(1,20), (2,10), (4,5)$ and $(5,4)$ in the graph.



When $x = 5, y = 4$

When $y = 10, x = 2$