Grade 10 Tamil Nadu Maths 2018

SECTION - I

Q1. An example for a function which is not a relation, (Domain - R, Co-domain - R) is:

(a) $y = x$

(b) $y = x - 1$

(c) $y = x^2$

(d) not possible

Solution:

Correct answer: (d)

Every function is a relation, but not every relation qualifies as a function.

Therefore, $y = x, y = x - 1$ and $y = x^2$ are functions.

Q2. If a, b, c are in GP, then $\frac{a-b}{b-c}$ is equal to:

(a) $\frac{a}{c}$

(b) $\frac{a}{b}$

(c) $\frac{c}{b}$

(d) $\frac{b}{a}$

Solution:

Correct answer: (c)

Given,

a, b, c are in GP.

$b^2 = ac$

Now,

$\frac{a-b}{b-c}$

Multiply and divide by b,

$$\frac{ab - b^2}{b^2 - bc}$$

$$= \frac{ab - ac}{(b^2 - bc)}$$

$$= \frac{[a(b - c)]}{[b(b - c)]}$$

$$= \frac{a}{b}$$

Q3. The next term of the series $\sqrt{3}, \sqrt{12}, \sqrt{27}, \ldots$ is:

(a) $\sqrt{39}$

(b) $\sqrt{32}$

(c) $\sqrt{54}$

(d) $\sqrt{48}$

Solution:

Correct answer: (d)

Given,

$\sqrt{3}, \sqrt{12}, \sqrt{27}, \ldots.$

$\mathrm{a} = \sqrt{3}$

$d = \sqrt{12} - \sqrt{3}$

$= 2\sqrt{3} - \sqrt{3}$

$= \sqrt{3}$

Next term of the given series is:

$\sqrt{27} + \sqrt{3}$

$= 3\sqrt{3} + \sqrt{3}$

$= 4\sqrt{3}$

$= \sqrt{48}$

Q4. What can be the degree of the remainder at most, when a fourth-degree polynomial is divided by a quadratic polynomial?

(a) 2

(b) 0

(c) 4

(d) 1

Solution:

Correct answer: (d)

When a polynomial of degree four is divided by a quadratic polynomial, the degree of the remainder must be less than that of the quadratic polynomial.

Thus, the maximum possible degree of the remainder is 1.

Q5. The LCM of $x^3 - a^3$ and $(x-a)^2$ is:

(a) $(x-a)^2(x^2+ax+a^2)$

(b) $(x^3-a^3)(x+a)$

(c) $(x+a)^2(x^2+ax+a^2)$

(d) $(x^3-a^3)(x-a)^2$

Solution:

Correct answer: (a)

Given,

$x^3 - a^3$ and $(x-a)^2$

$(x^3-a^3) = (x-a)(x^2+ax+a^2)$

Hence, the LCM is $(x-a)^2(x^2+ax+a^2)$

Q6. If $A = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ is such that $A^2 = I$, then :

(a) $1 - \alpha^2 - \beta\gamma = 0$

(b) $1 + \alpha^2 + \beta\gamma = 0$

(c) $1 + \alpha^2 - \beta\gamma = 0$

(d) $1 - \alpha^2 + \beta\gamma = 0$

Solution:

Correct answer: (a)

Given,

$$A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$A^2 = I$$

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \gamma\alpha - \gamma\alpha & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus, $\alpha^2 + \beta\gamma = 1$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

Q7. Slope of the straight line which is perpendicular to the straight line joining the points $(-2,6)$ and $(4,8)$ is equal to:

(a) -3

(b) $\frac{1}{3}$

(c) $-\frac{1}{3}$

(d) 3

Solution:

Correct answer: (a)

Given:

$(x_1, y_1) = (-2,6)$

$(x_2, y_2) = (4,8)$

Slope $= m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$$= \frac{8-6}{4+2}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

The slope of the line perpendicular to the one passing through the given points is

$= -\frac{1}{\text{m}}$

$$= -\frac{1}{\frac{1}{3}}$$

$= -3$

Q8. The centre of a circle is $(-6,4)$. If one end of the diameter of the circle is at $(-12,8)$, then the other end is at:

(a) $(-3,2)$

(b) $(-18,12)$

(c) $(0,0)$

(d) $(-9,6)$

Solution:

Correct answer: (c)

Given,

Centre $= (-6,4)$

One end of the diameter $= (-12,8)$

Let (x, y) be the other end of the diameter.

We know that centre = midpoint of diameter.

$(-6,4) = \left[\frac{-12+x}{2}, \frac{8+y}{2}\right]$

$-12 + x = -12; 8 + y = 8$

$x = 12 - 12; y = 8 - 8$

$x = 0, y = 0$

Hence, the required point is (0,0).

Q9. The area of two similar triangles are 16 cm^2 and 36 cm^2 respectively. If the altitude of the first triangle is 3 cm, then the corresponding altitude of the other triangle is:

(a) 4 cm

(b) 6.5 cm

(c) 4.5 cm

(d) 6 cm

Solution:

Correct answer: (c)

Let x be the altitude of the second triangle.

We know that,

$$\frac{Area\ of first\ triangle}{Area\ of\ second\ triangle} = \frac{Square\ the\ altitude\ of first\ triangle}{Square\ the\ altitude\ of second\ triangle}$$

$$\frac{16}{36} = \frac{3^2}{x^2}$$

$$\Rightarrow x^2 = \frac{9 \times 36}{16}$$

$$\Rightarrow x^2 = \left(\frac{9}{2}\right)^2$$

$$\Rightarrow x = \frac{9}{2}$$

$$\Rightarrow x = 4.5\ cm$$

Q10. If a vertical stick of 12 m long casts a shadow of 8 m long on the ground and at the same time, a tower casts a shadow of 40 m long on the ground, then the height of the tower is:

(a) 75 m

(b) 40 m

(c) 60 m

(d) 50 m

Solution:

Correct answer: (c)

Given,

Height of the stick = 12 m

Length of the shadow of stick = 8 m

Let x be the height of the tower.

Length of the shadow of tower = 40 m

Thus,

$$\frac{12}{8} = \frac{x}{40}$$

$$\Rightarrow x = \frac{12 \times 40}{8}$$

$$\Rightarrow x = 60 \text{ m}$$

Q11. $(1 + \cot^2 \theta)(1 - \cos \theta)(1 + \cos \theta) =$

(a) $\sec^2 \theta - \tan^2 \theta$

(b) $\tan^2 \theta - \sec^2 \theta$

(c) $\cos^2 \theta - \sin^2 \theta$

(d) $\sin^2 \theta - \cos^2 \theta$

Solution:

Correct answer: (a)

$(1 + \cot^2 \theta)(1 - \cos \theta)(1 + \cos \theta)$

$= \text{cosec}^2\theta(1 - \cos^2 \theta)$

$= \text{cosec}^2\theta \sin^2 \theta$

$= \left(\frac{1}{\sin^2\theta}\right) \sin^2 \theta$

= 1

$= \sec^2 \theta - \tan^2 \theta$

Q12. If A is an acute angle of a $\triangle ABC$, right angles at B, then the value of $\sin A + \cos A$ is:

(a) less than one

(b) equal to one

(c) equal to two

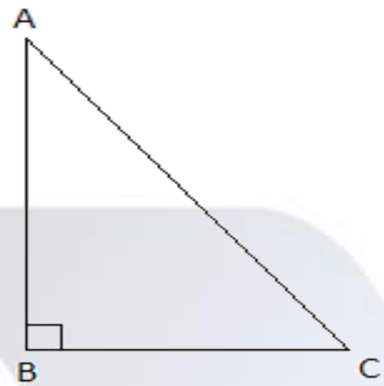
(d) greater than one

Solution:

Correct answer: (d)

Given,

A is an acute angle in $\triangle ABC$, where $\angle$B is a right angle.



$\sin A + \cos A = \left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)$

$= \frac{BC + AB}{AC}$

The sum of any two sides of a triangle is always greater than the third side..

Here, $\mathrm{BC} + \mathrm{AB} > \mathrm{AC}$

Thus, $\sin \mathrm{A} + \cos \mathrm{A} > 1$

Q13. Radius and height of a right circular cone and that of a right circular cylinder are respectively equal. If the volume of the cylinder is 120 cm^3, then the volume of the cone is equal to:

(a) 40 cm^3

(b) 1200 cm^3

(c) 90 cm^3

(d) 360 cm^3

Solution:

Correct answer: (a)

Given,

Radius and height of the right circular cone = Radius and height of the cylinder.

$\Rightarrow$ Volume of cone = $\left(\frac{1}{3}\right) \times$ Volume of cylinder

$= \left(\frac{1}{3}\right) \times 120$

$= 40\ \mathrm{cm}^3$

Q14. Standard deviation of a collection of data is $2\sqrt{2}$. If each value is multiplied by 3, then the standard deviation of the new data is:

(a) $6\sqrt{2}$

(b) $\sqrt{12}$

(c) $9\sqrt{2}$

(d) $4\sqrt{2}$

Solution:

Correct answer: (a)

Given,

Standard deviation $= 2\sqrt{2}$

If each observation in a dataset is multiplied by 3, the new standard deviation will also be multiplied by 3. Therefore, new standard deviation $= 3 \times (2\sqrt{2}) = 6\sqrt{2}$.

Q15. There are 6 defective items in a sample of 20 items. One item is drawn at random. The probability that it is a non-defective item is:

(a) $\frac{3}{10}$

(b) $\frac{7}{10}$

(c) $\frac{2}{3}$

(d) 0

Solution:

Correct answer: (b)

Given,

Total number of items $= 20$

Defective items $= 6$

Non-defective items = $20 - 6 = 14$

$P(\text{ non-defective item }) = \frac{14}{20} = \frac{7}{10}$

SECTION - II

Q16. Verify $A \subset B$ for the sets $A = \{a, b, c\}, B = \{1, \{a, b, c\}, 2\}$. If not justify your answer.

Solution:

A is not a subset of B.

Reason:

$\{\mathrm{a, b, c}\}$ is not an element in $\mathrm{P(B)}$

$P(B) = \{\{1\}, \{\{a, b, c\}\}, \{2\}, \{1, \{a, b, c\}\}, \{1,2\}, \{\{a, b, c\},2\}, \{1, \{a, b, c\},2\}, \{\}\}$

Q17. If $A = \{-2, -1,1,2\}$ and $f = \left\{\left(x, \frac{1}{x}\right) : x \in A\right\}$, write down the range of f. Is f a function from A to A ?

Solution:

Given,

$\mathrm{A} = \{-2, -1,1,2\}$

$f = \{\left(x, \frac{1}{x}\right) : x \in A\}$

$f = \left\{\left(-2, -\frac{1}{2}\right), \left(-1, -\frac{1}{1}\right), \left(1, \frac{1}{1}\right), \left(2, \frac{1}{2}\right)\right\}$

Thus, range of $f = \left\{-\frac{1}{2}, -1, 1, \frac{1}{2}\right\}$

$-\frac{1}{2}, \frac{1}{2}$ do not belong to A .

Therefore, it is not a function from A to A .

Q18. Three numbers are in the ratio 2: 5: 7. If the first number, the resulting number on subtraction of 7 from the second number and the third number from an arithmetic sequence, then find the numbers.

Solution:

Let $2x$, $5x$ and $7x$ be the numbers in the ratio 2: 5: 7.

Given,

$2x, 5x - 7$ and $7x$ are in AP.

$2(5x - 7) = 2x + 7x$

$10x - 14 = 9x$

$10x - 9x = 14$

$x = 14$

$2x = 2(14) = 28$

$5x = 5(14) = 70$

$7x = 7(14) = 98$

Therefore, the required numbers are 28,70 and 98 .

Q19. In the division algorithm of polynomials, the divisor is $(x + 2)$, quotient is $(x - 1)$ and the remainder is 4. Find the dividend.

Solution:

Given,

$g(x) = (x + 2)$

$q(x) = (x - 1)$

$r(x) = 4$

Using division algorithm of polynomials,

Dividend $p(x) = [g(x) \times q(x)] + r(x)$

$= (x + 2)(x - 1) + 4$

$= x^2 - x + 2x - 2 + 4$

$= x^2 + x + 2$

Q20. A matrix consists of 30 elements. What are the possible orders it can have?

Solution:

The possible orders of matrices containing 30 elements are those in which the number of rows and columns multiply to 30. These include:

15×2

3×10

10×3

5×6

6×5

Q21. If $A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$ then find AB and BA.

Solution:

Given,

$A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}, \mathrm{B} = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$

$\mathrm{AB} = \begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}$

$= \begin{pmatrix} 9+6 & 0+4 \\ 12+0 & 0+0 \end{pmatrix} = \begin{pmatrix} 15 & 4 \\ 12 & 0 \end{pmatrix}$

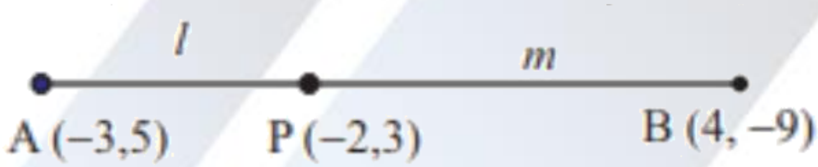
$\mathrm{BA} = \begin{pmatrix} 3 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix}$

$= \begin{pmatrix} 9+0 & 6+0 \\ 9+8 & 6+0 \end{pmatrix} = \begin{pmatrix} 9 & 6 \\ 17 & 6 \end{pmatrix}$

Q22. In what ratio does the point $P(-2,3)$ divide the line segment joining the points $A(-3,5)$ and $B(4,-9)$ internally?

Solution:

Let $P(-2,3)$ divide the line segment joining the points $A(-3,5)$ and $B(4,-9)$ internally in the ratio $1:m$.



Here,

$(x_1, y_1) = (-3,5)$

$(x_2, y_2) = (4,-9)$

Using section formula,

$P\left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}\right) = P(-2,3)$

$\Rightarrow \left(\frac{l(4) + m(-3)}{l+m}, \frac{l(-9) + m(5)}{l+m}\right) = (-2,3)$

x-coordinate of P:

$\frac{4l - 3m}{l+m} = -2$

$4l - 3m = -2l - 2m$

$4l + 2l = 3m - 2m$

$6\text{l} = \text{m}$

$\frac{1}{m} = \frac{1}{6}$

Therefore, the required ratio is $1\!:\!m = 1\!:\!6$

Q23. Find the equation of the straight line whose slope is $\frac{2}{3}$ and passing through $(5, -4)$.

Solution:

Equation of a line passing through the point (x_1, y_1) with slope m is

$y - y_1 = m(x - x_1)$

The required equation is:

$y - (-4) = \left(\frac{2}{3}\right)(x - 5)$

$3(y + 4) = 2(x - 5)$

$3y + 12 = 2x - 10$

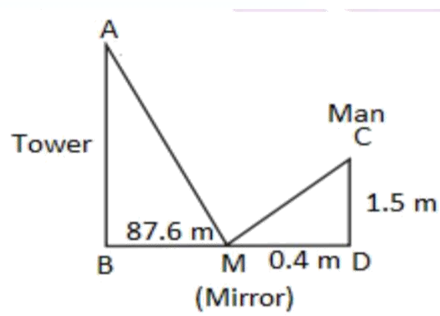
$\Rightarrow 2x - 10 - 3y - 12 = 0$

$\Rightarrow 2x - 3y - 22 = 0$

Q24. Draw the diagram for the given information.

A man sees the top of a tower in a mirror which is at a distance of 87.6 m from the tower. The mirror is on the ground, facing upward. The man is 0.4 m away from the mirror, and the distance of his eye level from the ground is 1.5 m. (The foot of man, the mirror and the foot of the tower lie along a straight line).

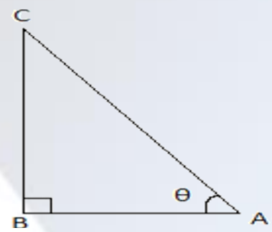
Solution:



Q25. Derive the identity $\cos^2\theta+\sin^2\theta=1$ for all θ such that $0^\circ\le\theta\le 90^\circ$.

Solution:

Let ABC be the right triangle, right angled at B .



By Pythagoras theorem,

$(AB)^2+(BC)^2=(AC)^2$

Dividing by AC^2 on both sides,

$\left(\frac{AB}{AC}\right)^2+\left(\frac{BC}{AC}\right)^2=\left(\frac{AC}{AC}\right)^2$

$\cos^2\theta+\sin^2\theta=1$

Hence proved.

Q26. Prove the identity $\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)=1$.

Solution:

$\text{LHS}=\sec\theta(1-\sin\theta)(\sec\theta+\tan\theta)$

$=(\sec\theta-\sec\theta\sin\theta)(\sec\theta+\tan\theta)$

$=\left[\sec\theta-\left(\frac{1}{\cos\theta}\right)\sin\theta\right](\sec\theta+\tan\theta)$

$=(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)$

$=\sec^2\theta-\tan^2\theta$

= 1

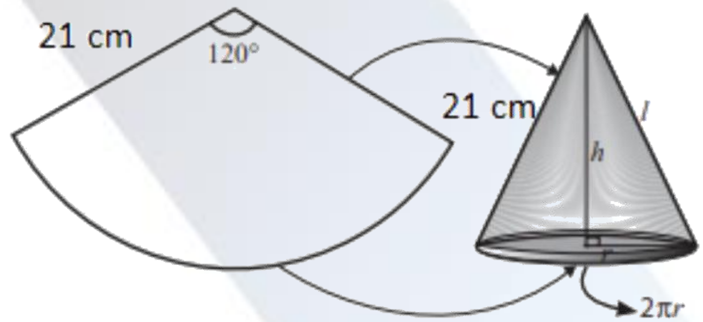
= RHS

Hence proved.

Q27. A sector containing an angle of $120°$ is cut off from a circle of radius 21 cm and folded into a cone. Find the curved surface area of the cone. ($\pi = \frac{22}{7}$)

Solution:

Given,



Radius of the sector = R = 21 cm

Angle of the sector = $\theta = 120°$

Let r be the radius and l be the slant height of the cone.

Curved surface area of the cone = Area of the sector = $\left(\frac{\theta}{360°}\right) \times \pi R^2$

$= \left(\frac{120°}{360°}\right) \times \left(\frac{22}{7}\right) \times 21 \times 21$

$= 22 \times 21$

$= 462 \text{ cm}^2$

Q28. Draw the necessary table to find the standard deviation for the data 20,14,16,30,21, and 25.

Solution:

Given,

20,14,16,30,21, and 25

$$\text{AM} = \frac{20 + 14 + 16 + 30 + 21 + 25}{6}$$

$$= \frac{126}{6} = 21$$

To find the standard deviation, the following table format is used:

x	$d = x - \bar{x}$	d^2
14	-7	49
16	-5	25
20	-1	1
21	0	0
25	4	16
30	9	81
$\sum x = 126$	$\sum d = 0$	$\sum d^2 = 172$

Q29. A number is selected at random from integers 1 to 100. Find the probability that it is not a perfect cube.

Solution:

Sample space $= \mathrm{S} = \{1,2,3, \ldots\ldots, 100\}$

$\mathrm{n}(\mathrm{S}) = 100$

Let A be the event of obtaining a perfect cube number.

$\mathrm{A} = \{1,8,27,64\}$

$\mathrm{n}(\mathrm{A}) = 4$

$\mathrm{P}(\mathrm{A}) = \frac{\mathrm{n}(\mathrm{A})}{\mathrm{n}(\mathrm{S})}$

$= \frac{4}{100}$

$= \frac{1}{25}$

Probability that the number selected is not a perfect cube $= P(\text{ not } A)$

$= 1 - \mathrm{P}(\mathrm{A})$

$= 1 - \left(\frac{1}{25}\right)$

$= \frac{25 - 1}{25}$

$= \frac{24}{25}$

Q30. (a) Solve the equation $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$ by completing the square method.

Solution:

Given,

$x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

$x^2 - (\sqrt{3}+1)x = -\sqrt{3}$

Adding $\left[\frac{\sqrt{3}+1}{2}\right]^2$ on both sides,

$$x^2 - (\sqrt{3}+1)x + \left[\frac{\sqrt{3}+1}{2}\right]^2 = -\sqrt{3} + \left[\frac{\sqrt{3}+1}{2}\right]^2$$

$$\left[x - \frac{\sqrt{3}+1}{2}\right]^2 = \left[\frac{\sqrt{3}-1}{2}\right]^2$$

Thus,

$$x - \frac{\sqrt{3}+1}{2} = \pm\frac{\sqrt{3}-1}{2}$$

$$x - \frac{\sqrt{3}+1}{2} = \frac{\sqrt{3}-1}{2}; x - \frac{\sqrt{3}+1}{2} = -\frac{\sqrt{3}-1}{2}$$

$$x = \frac{\sqrt{3}+1+\sqrt{3}-1}{2}; x = \frac{\sqrt{3}+1-\sqrt{3}+1}{2}$$

$$x = \frac{2\sqrt{3}}{2}; x = \frac{2}{2}$$

$x = \sqrt{3}, 1$

(b) Find the total surface area of a hollow hemisphere whose outer and inner radii are 4.2 cm and 2.1 cm respectively.

Solution:

Given,

Outer radius of hollow hemisphere $= R = 4.2$ cm

Inner radius $= r = 2.1$ cm

Total surface area $= \pi(3R^2 + r^2)$

$= \pi \times [3(4.2)^2 + (2.1)^2]$

$= \pi \times [3 \times 17.64 + 4.41]$

$= \pi \times [52.92 + 4.41]$

$= 57.33\pi \text{ cm}^2$

$= 180.18 \text{ cm}^2$

SECTION - III

Q31. A radio station surveyed 190 students to determine the types of music they liked. The survey revealed that 114 liked rock music, 50 liked folk music and 41 liked classical music, 14 liked rock music and folk music, 15 liked rock music and classical music, 11 liked classical music and folk music, 5 liked all the three types of music. Find

(a) how many did not like any of the 3 types?

(b) how many liked any two types only?

(c) how many liked folk music but not rock music?

Solution:

Total number of students = 190

R = Rock music

F = Folk music

C = Classical music

From the given,

$n(R \cap F) = 14$

$n(R \cap C) = 15$

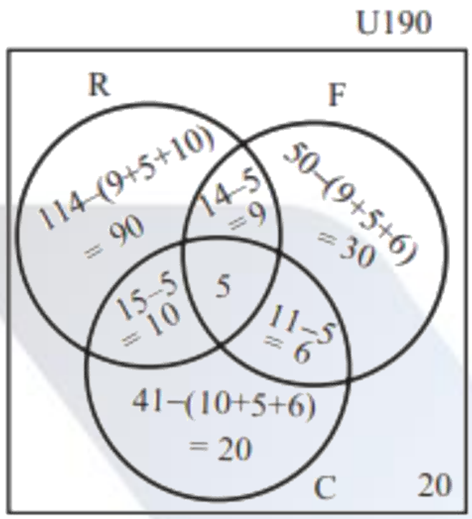
$n(C \cap F) = 11$

$n(R \cap F \cap C) = 5$

$n(R \cap F \cap C') = 14 - 5 = 9$

$n(R \cap C \cap F') = 15 - 5 = 10$

$n(F \cap C \cap R') = 11 - 5 = 6$



The number of students who liked any one of the types of music $= 90 + 9 + 30 + 10 + 5 + 6 + 20 = 170$

(a) The number of students who didn't like any of the three types of music $= 190 - 170 = 20$

(b) The number of students who liked exactly two types of music $= 9 + 6 + 10 = 25$

(c) Number of students who liked folk music but not rock music $= 30 + 6 = 36$

Q32. A function $f: [-7,6) \rightarrow R$ is defined as follows

$$f(x) = \begin{cases} x^2 + 2x + 1; & -7 \leq x < -5 \\ x + 5; & -5 \leq x < 2 \\ x - 1; & 2 \leq x < 6 \end{cases}$$

Find:

(a) $f(-7) - f(-3)$

(b) $\frac{[4f(-3) + 2f(4)]}{[f(-6) - 3f(1)]}$

Solution:

(a)

$f(-7) = x^2 + 2x + 1$

$= (-7)^2 + 2(-7) + 1$

$= 49 - 14 + 1$

$= 36$

$f(-3) = x + 5 = -3 + 5 = 2$

$f(-7) - f(-3) = 36 - 2 = 34$

(b)

$f(4) = x - 1 = 4 - 1 = 3$

$f(-6) = x^2 + 2x + 1$

$= (-6)^2 + 2(-6) + 1$

$= 36 - 12 + 1$

= 25

$f(1) = 1 + 5 = 6$

$$\frac{[4f(-3) + 2f(4)]}{[f(-6) - 3f(1)]} = \frac{[4 \times 2 + 2 \times 3]}{[25 - 3 \times 6]}$$

$$= \frac{8 + 6}{25 - 18}$$

$$= \frac{14}{7}$$

$= 2$

Q33. Find the three consecutive terms in an AP whose sum is 18 and the sum of their squares is 140.

Solution:

Let $a - d, a, a + d$ be the three terms of an AP.

According to the given,

Sum of the three terms $= 18$

$a - d + a + a + d = 18$

$3a = 18$

$$a = \frac{18}{3}$$

$a = 6$

Sum of square of three terms $= 140$

$(a - d)^2 + a^2 + (a + d)^2 = 140$

$a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 140$

$3(6)^2 + 2d^2 = 140$

$2d^2 = 140 - 108$

$d^2 = 32/2$

$d^2 = 16$

$d = \pm 4$

If $a = 6$ and $d = 4$, then the three terms are $2, 6, 10$.

If $a = 6$ and $s = -4$, then the three terms are $10, 6, 2$.

Q34. Solve $3(2x + y) = 7xy; 3(x + 3y) = 11xy$ using elimination method.

Solution:

Given,

$3(2x + y) = 7xy$

$6x + 3y = 7xy$ .....i)

$3(x + 3y) = 11xy$

$3x + 9y = 11xy$ ......ii)

(i) $\times -$ (ii),

$18x + 9y - (3x + 9y) = 21xy - 11xy$

$15x = 10xy$

$\Rightarrow y = \frac{15}{10}$

$\Rightarrow \text{y} = \frac{3}{2}$

Substituting $y = \frac{3}{2}$ in (i),

$6x + 3\left(\frac{3}{2}\right) + 7x\left(\frac{3}{2}\right)$

$6x + \left(\frac{9}{2}\right) = \frac{21x}{2}$

$\frac{12x + 9}{2} = \frac{21x}{2}$

$12x + 9 = 21x$

$\Rightarrow 21x - 12x = 9$

$\Rightarrow 9x = 9$

$\Rightarrow x = 1$

Also, if $x = 0$, then $y = 0$

Hence, the solution of the given system of equations is $\left(1, \frac{3}{2}\right)$ and $(0, 0)$.

Q35. Find the square root of the polynomial $4 + 25x^2 - 12x - 24x^3 + 16x^4$ by division method.

Solution:

Given,

$$4 + 25x^2 - 12x - 24x^3 + 16x^4$$
$$= 16x^4 - 24x^3 + 25x^2 - 12x + 4$$

$$\begin{array}{r|l}
 & \quad 4x^2 - 3x + 2 \\
\hline
 & 16x^4 - 24x^3 + 25x^2 - 12x + 4 \\
4x^2 & 16x^4 \\
\cline{2-2}
 & -24x^3 + 25x^2 \\
8x^2 - 3x & -24x^3 + 9x^2 \\
 & \;(+) \qquad (-) \\
\cline{2-2}
8x^2 - 6x + 2 & \qquad 16x^2 - 12x + 4 \\
 & \qquad 16x^2 - 12x + 4 \\
 & (-) \quad (+) \qquad (-) \\
\cline{2-2}
 & \qquad\qquad 0 \\
\end{array}$$

Therefore, the square root of the polynomial $= |4x^2 - 3x + 2|$

Q36. The difference of the squares of two positive numbers is 45. The square of the smaller number is four times the larger number. Find the numbers.

Solution:

Let x and y be the two numbers, where $x > y$.

According to the given,

$x^2 - y^2 = 45$ ....(i)

$y^2 = 4x$ ....(ii)

From (i) and (ii),

$x^2 - 4x - 45 = 0$

$x^2 - 9x + 5x - 45 = 0$

$x(x-9)+5(x-9)=0$

$(x-9)(x+5)=0$

$x=9, x=-5$

x cannot be negative.

Therefore, $x=9$

$y^2=4(9)$

$y^2=36$

$y=6$

Hence, the required numbers are 9 and 6 .

Q37. If $\mathrm{A}=\begin{pmatrix}5 & 2\\ 7 & 3\end{pmatrix}$ and $\mathrm{B}=\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}$, then verify that $(\mathrm{AB})^{\mathrm{T}}=\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}$.

Solution:

Given,

$$\mathrm{A}=\begin{pmatrix}5 & 2\\ 7 & 3\end{pmatrix} \mathrm{B}=\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}$$

$$\mathrm{AB}=\begin{pmatrix}5 & 2\\ 7 & 3\end{pmatrix}\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}$$

$$=\begin{pmatrix}10 & -2\\ 14 & -5+2\\ 1 & -7+3\end{pmatrix}$$

$$=\begin{pmatrix}8 & -3\\ 11 & -4\end{pmatrix}$$

$$(\mathrm{AB})^{\mathrm{T}}=\begin{pmatrix}8 & 11\\ -3 & -4\end{pmatrix}$$

$$\mathrm{B}^{\mathrm{T}}=\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}, \mathrm{A}^{\mathrm{T}}=\begin{pmatrix}5 & 7\\ 2 & 3\end{pmatrix}$$

$$\mathrm{B}^{\mathrm{T}}\mathrm{A}^{\mathrm{T}}=\begin{pmatrix}2 & -1\\ -1 & 1\end{pmatrix}\begin{pmatrix}5 & 7\\ 2 & 3\end{pmatrix}$$

$$=\begin{pmatrix}10-2 & 14-3\\ -5+2 & -7+3\end{pmatrix}$$

$$=\begin{pmatrix}8 & 11\\ -3 & -4\end{pmatrix}$$

Therefore, $(AB)^T=B^T A^T$

Q38. Find the area of the quadrilateral whose vertices are $(-3,4),(-5,-6),(4,-1)$ and $(1,2)$.

Solution:

Given, vertices are $(-3,4)$, $(-5,-6)$, $(4,-1)$ and $(1,2)$.

Area of the quadrilateral

$$= \frac{1}{2}\left|\begin{array}{ccccc} -3 & -5 & 4 & 1 & -3 \\ 4 & -6 & -1 & 2 & 4 \end{array}\right|$$

$$= \left(\frac{1}{2}\right)|(18+5+8+4)-(20-24-1-6)|$$

$$= \left(\frac{1}{2}\right)|35-(-51)|$$

$$= \left(\frac{1}{2}\right)|86|$$

$= 43$ sq. units

Q39. The midpoints D, E, F of the sides of a triangle ABC are $(3,4)$, $(8,9)$ and $(6,7)$ respectively. Find the vertices of the triangle.

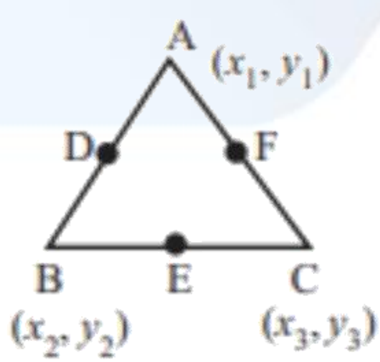
Solution:

D, E, F are the midpoints of the sides AB, BC and AC of triangle ABC.

$D = (3,4)$

$E = (8,9)$

$F = (6,7)$



Using the midpoint formula,

$$\left[\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right] = (3,4)$$

$x_1 + x_2 = 6$ ..... (i)

$y_1 + y_2 = 8$ ....(ii)

Similarly,

$x_2 + x_3 = 16$ ......(iii)

$y_2 + y_3 = 18$ ......iv)

And

$x_1 + x_3 = 12$

$y_1 + y_3 = 14$ ..... (vi)

Adding (i), (iii) and (v),

$2(x_1 + x_2 + x_3) = 6 + 16 + 12$

$x_1 + x_2 + x_3 = \frac{34}{2} = 17$ ..... (vii)

From (i), (iii), (v) and (vii),

$x_1 = 1, x_2 = 5, x_3 = 11$

Adding (ii), (iv) and (vi),

$2(y_1 + y_2 + y_3) = 8 + 18 + 14$

$y_1 + y_2 + y_3 = \frac{40}{2} = 20$ .... (viii)

From (ii), (iv), (vi) and (viiii),

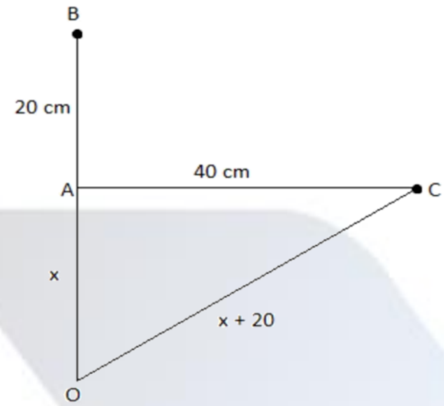
$y_1 = 2, y_2 = 6, y_3 = 12$

Hence, the coordinates of the vertices are $A(1,2), B(5,6)$ and $C(11,12)$.

Q40. A lotus is 20 cm above the water surface in a pond and its stem is partly below the water surface. As the wind blew, the stem was pushed aside so that the lotus touched the water 40 cm away from the original position of the stem. How much of the stem was below the water surface originally?

Solution:

Let B is the lotus at a height of 20 cm from the surface of water.



$OC = OA + AB = (x + 20)cm$

$AC = 40$ cm

In right triangle OAC,

By Pythagoras theorem,

$\mathrm{OC}^2 = \mathrm{OA}^2 + \mathrm{AC}^2$

$(x + 20)^2 = x^2 + (40)^2$

$x^2 + 40x + 400 = x^2 + 1600$

$40x = 1600 - 400$

$40x = 1200$

$x = \dfrac{1200}{40}$

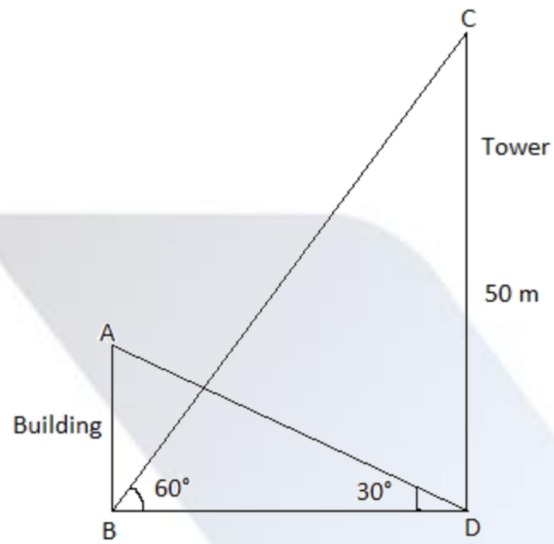
$x = 30$ cm

Hence, the stem was 30 cm below the water surface originally.

Q41. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Solution:

Let AB be the building and CD be the tower.



In triangle CDB, $\tan\ 60^\circ = \frac{\text{CD}}{\text{BD}}$

$\sqrt{3} = \frac{50}{BD}$

$\Rightarrow BD = \frac{50}{\sqrt{3}}$ m

In triangle ABD , $\tan\ 30^\circ = \frac{\text{AB}}{\text{BD}}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{BD}$$

$\Rightarrow BD = AB\sqrt{3}$

$$\Rightarrow \frac{50}{\sqrt{3}} = AB\sqrt{3}$$

$$\Rightarrow AB = \frac{50}{3}$$

$\Rightarrow AB = 16.67$ m

Therefore, the height of the building is 16.67 m.

Q42. The sum of the base radius and the height of a right circular solid cylinder is 37 cm. If the total surface area of the cylinder is 1628sq. cm, then find the volume of the cylinder.

Solution:

Let r be the base radius and h be the height of the right circular cylinder.

Given,

$r + h = 37\ cm$

Total surface area $= 1628\text{sq. cm}$

$2\pi r(r + h) = 1628$

$2 \times \left(\frac{22}{7}\right) \times r \times 37 = 1628$

$r = (1628 \times 7)/(2 \times 22 \times 37)$

$r = 7$ cm

Thus, $h = 37 - r = 37 - 7 = 30\ cm$

Volume of the cylinder $= \pi r^2\ h$

$= \left(\frac{22}{7}\right) \times 7 \times 7 \times 30$

$= 22 \times 7 \times 30$

$= 4620\ \text{cm}^3$

Q43. For a collection of data, if $\sum x = 35, n = 5, \sum(x - 9)^2 = 82$, then find $\sum x^2$ and $\sum(x - \bar{x})^2$.

Solution:

Given,

$$\sum\ x = 35, n = 5, \sum\ (x - 9)^2 = 82$$

$\bar{x} = \sum\ \frac{x}{n}\ = \frac{35}{5} = 7$

$\sum(x - 9)^2 = 82$

$\sum(x^2 - 18\text{x} + 81) = 82$

$\sum x^2 - 18\sum x + 81\sum \text{i} = 82$

$\sum x^2 - 18(35) + 81(5) = 82$

$\sum x^2 - 630 + 405 = 82$

$\sum x^2 = 630 + 82 - 405$

$\sum x^2 = 307$

Similarly,

$\sum(x - 9)^2 = 82$

$\sum(x - 7 - 2)^2 = 82$

$\sum[(x-7)-2]^2 = 82$

$\sum(x-7)^2 - 4\sum(x-7) + 4\sum \text{i} = 82$

$\sum_{\sum}(x-\bar{x})^2 - 4\sum(x-\bar{x}) + 4(5) = 82$

$\sum_{\sum}(x-\bar{x})^2 - 4(0) + 20 = 82$

$\sum(x-\bar{x})^2 = 82 - 20$

$\sum(x-\bar{x})^2 = 62$

Q44. Two dice are rolled simultaneously. Find the probability that the sum of the numbers on the faces is neither divisible by 3 nor by 4.

Solution:

Total number of outcomes $= n(S) = 36$

Let A be the event that the sum of numbers on dice is divisible by 3 .

$\mathrm{A} = \{(1,2),(2,1),(2,4),(4,2),(1,5),(5,1),(3,3),(3,6),(6,3),(4,5),(5,4),(6,6)\}$

$\mathrm{n(A)} = 12$

$\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(S)}} = \frac{12}{36}$

Let B be the event that the sum of numbers on dice is divisible by 4 .

$\mathrm{B} = \{(1,3),(3,1),(2,2),(2,6),(6,2),(3,5),(5,3),(4,4),(6,6)\}$

$\mathrm{n(B)} = 9$

$P(B) = \frac{n(B)}{n(S)} = \frac{9}{36}$

$A \cap B = \{(6,6)\}$

$n(A \cap B) = 1$

$P(A \cap B) = \frac{1}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \left(\frac{12}{36}\right) + \left(\frac{9}{36}\right) - \left(\frac{1}{36}\right)$

$= \frac{12+9-1}{36}$

$= \frac{20}{36}$

$= \frac{5}{9}$

Probability that the sum of the numbers on the faces is neither divisible by 3 nor by 4 = 1 − P(A ∪ B)

$= 1 - \left(\frac{5}{9}\right)$

$= \frac{9-5}{9}$

$= \frac{4}{9}$

Q45. (a) The first term of a geometric series is 375 and the fourth term is 192. Find the common ratio and the sum

of the first 14 terms.

Solution:

Given,

First term of GP $= a = 375$

Fourth term $= t_4 = ar^{n-1}$

$ar^3 = 192$

$r^3 = \frac{192}{375}$

$r_3 = \left(\frac{4}{5}\right)^3$

$r = \frac{4}{5}$

Sum of first n terms of GP

$S_n = \frac{a(1-r^n)}{1-r}$

Sum of the first 14 terms is

$S_{14} = \frac{375\left[1-\left(\frac{4}{5}\right)^{14}\right]}{\left[1-\left(\frac{4}{5}\right)\right]}$

$= 375 \times 5\left[1-\left(\frac{4}{5}\right)^{14}\right]$

$= 1875\left[1 - \left(\frac{4}{5}\right)\right]$

(b) Water in a cylindrical tank of diameter 4 m and height 10 m is released through a cylindrical pipe of diameter 10 cm at the rate of 2.5 km/hr. How much time will it take to empty half of the tank? (Assume that the tank is full of water to begin with)

Solution:

Given,

Diameter of tank = 4 m

Radius of the tank $= R = 2$ m

Height of the tank $= H = 10$ m

Diameter of pipe = 10 cm

Radius of pipe $= r = \frac{10}{2} = 5$ cm

$$\mathrm{r} = \frac{5}{100}\ \mathrm{m} = \frac{1}{20}\ \mathrm{m}$$

Speed = 2.5 km/hr = 2500 m/hr

Let t be the time required to empty half of the tank.

Volume of water released through the pipe $= \frac{1}{2}$ (volume of tank)

$$\pi r^2 \times t \times \text{speed} = \left(\frac{1}{2}\right)\pi R^2 H$$

$$\left(\frac{1}{20}\right)^2 \times t \times 2500 = \left(\frac{1}{2}\right) \times (2)^2 \times 10$$

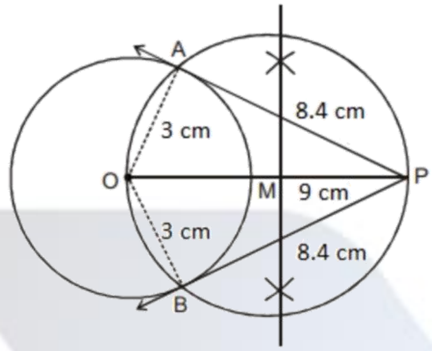
$$\mathrm{t} = \frac{20 \times 400}{2500}$$

t = 3.2 hours = 3 hours 12 min

SECTION - IV

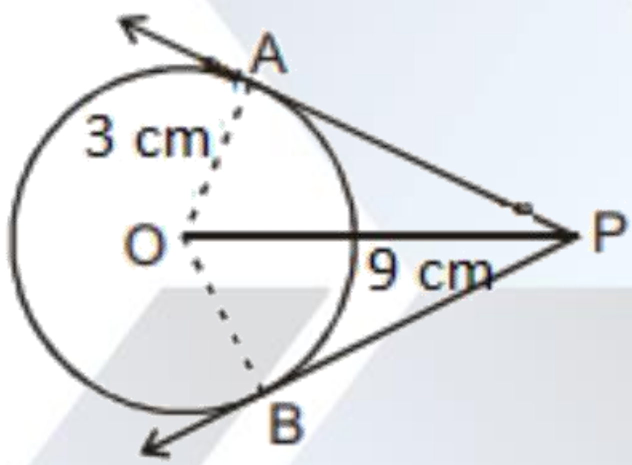
Q46. (a) Take a point which is 9 cm away from the centre of a circle of radius 3 cm, and draw two tangents to the circle from that point and calculate their lengths.

Solution:



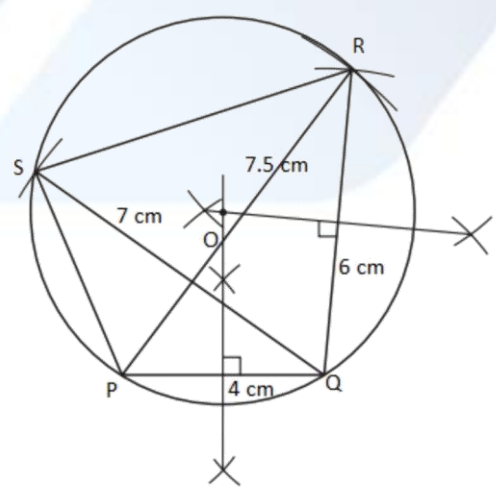
Length of the tangents $= 8.4$ cm

Rough diagram

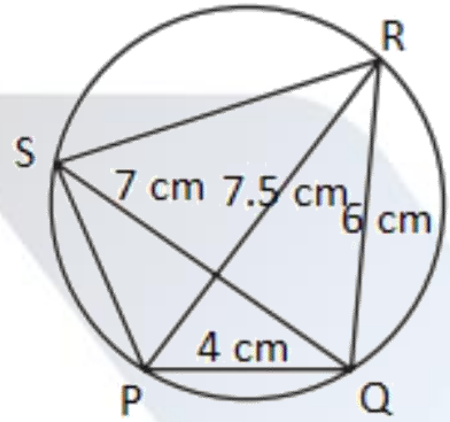


(b) Construct a cyclic quadrilateral PQRS with $PQ = 4$ cm, $QR = 6$ cm, $PR = 7.5$ cm, $QS = 7$ cm.

Solution:



Rough diagram



Q47. (a) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve the equation $x^2 + 2x + 4 = 0$.

Solution:

Given,

$y = x^2 + 3x + 2$

x	-4	-3	-2	-1	0	1	2	3
x^2	16	9	4	1	0	1	4	9
$3x$	-12	-9	-6	-3	0	3	6	9
2	2	2	2	2	2	2	2	2
y	6	2	0	0	2	6	12	20

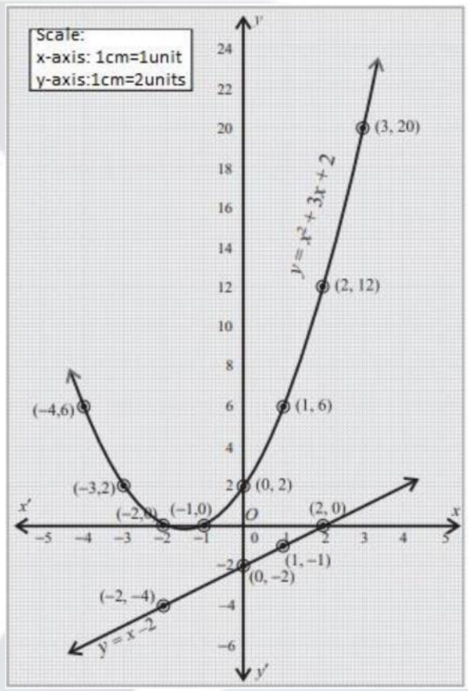
Now,

Subtracting $x^2 + 2x + 4 = 0$ from $y = x^2 + 3x + 2$

$y - 0 = x^2 + 3x + 2 - (x^2 + 2x + 4)$

$y = x - 2$

x	-2	0	1	2
y	-4	-2	-1	0



From the graph, the solution set does not have real roots, i.e. imaginary.

(b) The cost of milk per litre is ₹ 15. Draw the graph for the relation between the quantity and cost. Hence, find

(i) the proportionality constant

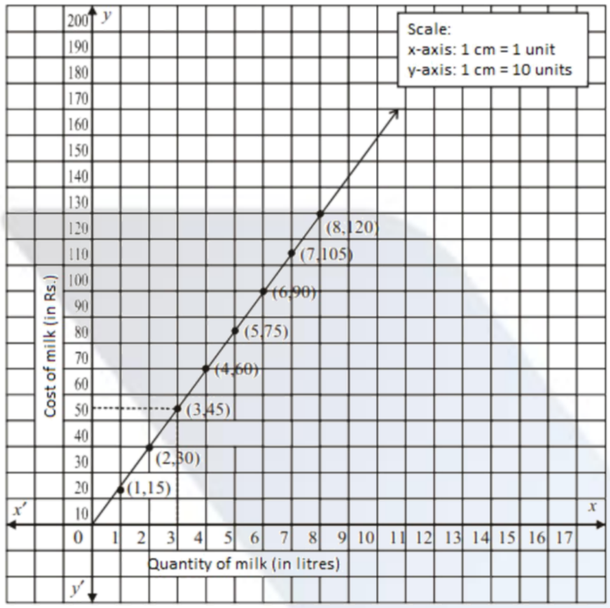
(ii) the cost of 3 litres of milk

Solution:

Let x be the quantity of milk (in litres) and y be the cost of milk (in ₹).

x	1	2	3	4	5	6
y	15	30	45	60	75	90

(i) The proportionality constant $= k = 15$



(ii) The cost of 3 litres of milk = ₹ 45