Grade 10 Tamil Nadu Maths 2019

SECTION - I

Q1. The function $f: N \rightarrow R$ is defined by $f(n) = 2^n$. The range of the function is:

(a) the set of all even positive integers

(b) N

(c) R

(d) a subset of set of all even positive integers

Solution:

Correct answer: (d)

Given,

$\mathrm{f(n)} = 2^{\mathrm{n}}$

The value of 2^n is an even positive integer for all natural numbers.

Hence, the range of the function is a subset of all even possible integers.

Q2. If a, b, c, l, m are in AP, then the value of $a - 4b + 6c - 4l + m$ is:

(a) 1

(b) 2

(c) 3

(d) 0

Solution:

Correct answer: (d)

$a - 4b + 6c - 4l + m$

$= a + 6c + m - 4(b + l)$

$= a + m + 6c - 4(2c)$[since $b + l = 2c$]

$= 2\mathrm{c} + 6\mathrm{c} - 4(2\mathrm{c})$ [since $\mathrm{a} + \mathrm{m} = 2\mathrm{c}$]

$= 8\mathrm{c} - 8\mathrm{c}$

$= 0$

Q3. The common ratio of the GP a^{m-n}, a^m, a^{m+n} is:

(a) a^m

(b) a^{-m}

(c) a^n

(d) a^{-n}

Solution:

Correct answer: (c)

Given,

a^{m-n}, a^m, a^{m+n} are in GP.

Common ratio $= \frac{a^m}{a^{m-n}}$

$= a^{m-(m-n)}$

$= a^{m-m+n}$

$= a^n$

Q4. The number of polynomials having zeroes 2 and 1 is:

(a) 1

(b) 2

(c) 3

(d) more than 3

Solution:

Correct answer: (d)

Let $p(x) = ax^2 + bx + c$ be the quadratic polynomial.

Given, 2 and 1 are the zeroes of the polynomial.

Sum of zeroes $= -\frac{\mathrm{b}}{\mathrm{a}}$

$$2 + 1 = -\frac{\mathrm{b}}{\mathrm{a}}$$

$$\Rightarrow -\frac{b}{a} = \frac{3}{1}$$

$$\Rightarrow \frac{\mathrm{b}}{\mathrm{a}} = -\frac{3}{1}$$

Thus, $b = -3$ and $a = 1$

Product of zeroes $= \frac{c}{a}$

$2 \times 1 \ = \frac{c}{a}$

$\Rightarrow \frac{c}{a} = \frac{2}{1}$

Thus, $c = 2$ and $a = 1$

The possible expressions which satisfies the above conditions are:

$p(x) = x^2 - 3x + 2$

$p(x) = kx^2 - 3kx + 2k$, where k is a real number

$p(x) = \left(\frac{x^2}{k}\right) + \left(\frac{3}{k}\right)x + \left(\frac{2}{k}\right)$, where k is any non-zero real number and so on.

Therefore, there exist more than 3 polynomials with zeroes 2 and 1.

Q5. The common root of the equations $x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ is:

(a) $\frac{c+a}{2b}$

(b) $\frac{c-a}{2b}$

(c) $\frac{c+b}{2a}$

(d) $\frac{a+b}{2c}$

Solution:

Correct answer: (a)

$x^2 - bx + c = 0$ and $x^2 + bx - a = 0$ have a common root.

$\Rightarrow x^2 - bx + c = x^2 + bx - a$

$\Rightarrow c + a = bx + bx$

$\Rightarrow c + a = 2bx$

$\Rightarrow x = \frac{c + a}{2b}$

Q6. Which of the following statements is incorrect?

(a) A unit matrix is a scalar matrix

(b) A scalar matrix is a diagonal matrix

(c) A unit matrix is a diagonal matrix

(d) For any two matrices, the addition of matrices exists

Solution:

Correct answer: (d)

We know that,

The addition of two matrices exists, only if they have the same order.

Q7. If the line segment joining the points A(3,4) and B(14, −3) meets the x-axis at P, then the ratio in which P divides the segment AB is:

(a) 4: 3

(b) 3: 4

(c) 2: 3

(d) 4: 1

Solution:

Correct answer: (a)

Let $P(x, 0)$ divide the line segment joining the points $A(3,4)$ and $B(14, -3)$ in the ratio m: n.

Using section formula:

Y-coordinate of P is:

$$0 = \frac{[m(-3) + n(4)]}{m + n}$$

$\Rightarrow -3m + 4n = 0$

$\Rightarrow 4\text{n} = 3\text{ m}$

$\Rightarrow \frac{m}{n} = \frac{4}{3}$

Therefore, the required ratio is 4: 3.

Q8. The distance of the point $(-2, -3)$ from x-axis is:

(a) -2

(b) 2

(c) -3

(d) 3

Solution:

Correct answer: (d)

Given point is $(-2, -3)$.

We know that the distance of any point from the x-axis is the value of y-coordinate.

Hence, the required distance is 3 units.

Q9. The sides of two similar triangles are in the ratio $2:3$, then their areas are in the ratio:

(a) $9:4$

(b) $4:9$

(c) $2:3$

(d) $3:2$

Solution:

Correct answer: (b)

Given,

Ratio of sides of two similar triangles $= 2:3$

Ratio of their areas $=$ Square the ratio of sides

$= 2^2:3^2$

$= 4:9$

Q10. The perimeters of two similar triangles are 24 cm and 18 cm, respectively. If one side of the first triangle is 8 cm, then the corresponding side of the other triangle is:

(a) 4 cm

(b) 3 cm

(c) 9 cm

(d) 6 cm

Solution:

Correct answer: (d)

Given,

Perimeters of two similar triangles are 24 cm and 18 cm respectively.

Length of side of the first triangle = 8 cm

Let x be the side of another triangle.

Ratio of perimeters = Ratio of the corresponding sides

$\Rightarrow \frac{24}{18} = \frac{8}{x}$

$\Rightarrow \frac{4}{3} = \frac{8}{x}$

$\Rightarrow x = \frac{24}{4}$

$\Rightarrow x = 6$

Therefore, the corresponding side of the second triangle is 6 cm .

Q11. $1 - \left[\frac{\sin^2 \theta}{1+\cos \theta}\right] =$

(a) $\cos \theta$

(b) $\tan \theta$

(c) $\cot \theta$

(d) $\operatorname{cosec}\theta$

Solution:

Correct answer: (a)

$1 - \left[\frac{\sin^2 \theta}{1 + \cos \theta}\right]$

$= 1 - \times \left[\frac{\sin^2\theta}{1 - \cos \theta}\right]\left[\frac{1 - \cos \theta}{1 - \cos \theta}\right]\}$

$= 1 - \left[\frac{\sin^2 \theta(1 - \cos \theta)}{(1 - \cos^2 \theta)}\right]$

$= 1 - \left[\frac{\sin^2 \theta(1 - \cos \theta)}{\sin^2 \theta}\right]$

$= 1 - (1 - \cos \theta)$

$= 1 - 1 + \cos\ \theta$

$= \cos\ \theta$

Q12. $\sin^2\ \theta + \frac{1}{(1+\tan^2\theta)} =$

(a) $\text{cosec}^2\theta + \cot^2\ \theta$

(b) $\text{cosec}^2\theta - \cot^2\ \theta$

(c) $\cot^2\ \theta - \text{cosec}^2\theta$

(d) $\sin^2\ \theta - \cos^2\ \theta$

Solution:

Correct answer: (b)

$\sin^2\ \theta + \frac{1}{(1+\tan^2\theta)}$

$= \sin^2\ \theta + \frac{1}{\sec^2\theta}$

$= \sin^2\ \theta + \cos^2\theta$

$= 1$

$= \text{cosec}^2\theta - \cot^2\theta$ [since $1 + \cot^2\ \theta = \text{cosec}^2\theta$]

Q13. If the total surface area of a solid right circular cylinder is $200\pi\ \text{cm}^2$ and its radius is 5 cm, then the sum of its height and radius is:

(a) 20 cm

(b) 25 cm

(c) 30 cm

(d) 15 cm

Solution:

Correct answer: (a)

Given,

Total surface area of cylinder $= 200\pi\ \text{cm}^2$

Radius $= r = 5$ cm

Let h be the height of the cylinder.

$\Rightarrow 2\pi r(r+h) = 200\pi$

$\Rightarrow 5(5+h) = 100$

$\Rightarrow 5+h = \frac{100}{5}$

$\Rightarrow 5+h = 20$

$\Rightarrow h = 20-5 = 15$ cm

Sum of the radius and height $= r+h = 5+15 = 20$ cm

Q14. Variance of the first 11 natural numbers is:

(a) $\sqrt{5}$

(b) $\sqrt{10}$

(c) $5\sqrt{2}$

(d) 10

Solution:

Correct answer: (d)

We know that variance of first n natural numbers $= V = \frac{(n^2-1)}{12}$

Variance for first 11 natural numbers $= \frac{[(11)^2-1]}{12}$

$= \frac{121-1}{12}$

$= \frac{120}{12}$

$= 10$

Q15. There are 6 defective items in a sample of 20 items. One item is drawn at random. The probability that it is a non-defective item is:

(a) $\frac{7}{10}$

(b) 0

(c) $\frac{3}{10}$

(d) $\frac{2}{3}$

Solution:

Correct answer: (a)

Given,

Total number of items $= 20$

Defective items = 6

Non-defective items = $20 - 6 = 14$

P(non-defective item) $= \frac{14}{20} = \frac{7}{10}$

SECTION - II

Q16. Define a set.

Solution:

A set is defined as the collection of well-defined objects which can be separated distinctly. For example, $S = \{1,3,5,7\}$ is a collection of the odd numbers from 1 to 7.

Q17. Find the 18th and 25th terms of the sequence defined by

$a_n = n(n+3)$, in $n \in N$ and n is even

$a_n = \frac{2n}{(n^2+1)}$, if $n \in N$ and n is odd

Solution:

18th term of the sequence:

$\mathrm{n} = 18$ (even)

$\mathrm{a_n} = \mathrm{n}(\mathrm{n}+3)$

$\mathrm{a}_{18} = 18(18+3)$

$= 18 \times 21$

$= 378$

25th term of the sequence:

$\mathrm{n} = 25$ (odd)

$$\mathrm{a_n} = \frac{2\mathrm{n}}{(\mathrm{n}^2+1)}$$

$$\mathrm{a}_{25} = \frac{2 \times 25}{[(25)^2+1]}$$

$= \frac{50}{625+1}$

$= \frac{50}{626}$

$= \frac{25}{313}$

Q18. Using cross multiplication rule, solve:

$3x + 5y = 25$

$7x + 6y = 30$

Solution:

Given,

$3x + 5y = 25$

$7x + 6y = 30$

Comparing with the standard form,

$a_1 = 3,\ b_1 = 5, c_1 = -25$

$a_2 = 7, b_2 = 6, c_2 = -30$

Using cross multiplication method,

$$\frac{x}{(b_1c_2 - b_2c_1)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1\ b_2 - a_2\ b_1)}$$

$$\frac{x}{-150 + 150} = \frac{y}{-175 + 90} = \frac{1}{18 - 35}$$

$$\frac{x}{0} = \frac{y}{-85} = \frac{1}{-17}$$

$$\frac{x}{0} = -\frac{1}{17}, \frac{y}{-85} = -\frac{1}{17}$$

$$x = 0, y = -\frac{85}{-17}$$

$x = 0, y = 5$

Q19. Form a quadratic equation whose roots are $\frac{4+\sqrt{7}}{2}, \frac{4-\sqrt{7}}{2}$.

Solution:

Let α and β be the roots of the quadratic equation.

Given, $\frac{4+\sqrt{7}}{2}, \frac{4-\sqrt{7}}{2}$ are the roots of the quadratic equation.

$$\alpha+\beta=\left[\frac{4+\sqrt{7}}{2}\right]+\left[\frac{4-\sqrt{7}}{2}\right]$$

$$=\left[\frac{4+\sqrt{7}+4-\sqrt{7}}{2}\right]$$

$$=\frac{8}{2}$$

$$=4$$

$$\alpha\beta=\left[\frac{4+\sqrt{7}}{2}\right]\times\left[\frac{4-\sqrt{7}}{2}\right]$$

$$=\frac{\left[(4)^2-(\sqrt{7})^2\right]}{4}$$

$$=\frac{16-7}{4}$$

$$=\frac{9}{4}$$

Therefore, the required quadratic equation is

$$x^2-4x+\left(\frac{9}{4}\right)=0$$

$$\Rightarrow 4x^2-16x+9=0$$

Q20. Define a diagonal matrix.

Solution:

Diagonal matrix is a square matrix in which every element except the principal diagonal elements is zero.

Example:

$$A=\begin{bmatrix}2 & 0\\ 0 & 4\end{bmatrix}$$

$$B=\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 5\end{bmatrix}$$

Q21. Find the product of the matrices $\begin{bmatrix} 6 \\ -3 \end{bmatrix}[2 \quad -7]$.

Solution:

$$\begin{bmatrix} 6 \\ -3 \end{bmatrix} \times [2 \quad -7]$$

$$= \begin{bmatrix} 6 \times 2 & 6 \times (-7) \\ -3 \times 2 & -3 \times (-7) \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -42 \\ -6 & 21 \end{bmatrix}$$

Q22. Find the equation of the straight line passing through the points $(-1,1)$ and $(2,-4)$.

Solution:

Let the given points be:

$(x_1, y_1) = (-1,1)$

$(x_2, y_2) = (2,-4)$

Equation of the line passing through the given points is:

$$\frac{(y - y_1)}{(y_2 - y_1)} = \frac{(x - x_1)}{(x_2 - x_1)}$$

$$\frac{y-1}{-4-1} = \frac{x+1}{2+1}$$

$$\frac{y-1}{-5} = \frac{x+1}{3}$$

$3(y-1) = -5(x+1)$

$3y - 3 = -5x - 5$

$5x + 3y - 3 + 5 = 0$

$5x + 3y + 2 = 0$

Q23. Prove the identity $\left(\frac{\sin\theta}{\operatorname{cosec}\theta}\right) + \left(\frac{\cos\theta}{\sec\theta}\right) = 1$.

Solution:

$$\text{LHS} = \left(\frac{\sin\theta}{\operatorname{cosec}\theta}\right) + \left(\frac{\cos\theta}{\sec\theta}\right)$$

$= \sin\ \theta \cdot \sin\ \theta + \cos\ \theta \cdot \cos\ \theta$

$= \sin^2\ \theta + \cos^2\ \theta$

$= 1$

= RHS

Hence proved.

Q24. A solid right circular cylinder has radius 7 cm and height 20 cm . Find its total surface area. (take $\pi = \frac{22}{7}$)

Solution:

Given,

Radius of right circular cylinder $= r = 7$ cm

Height $= h = 20$ cm

Total surface area $= 2\pi r(r + h)$

$= 2 \times \left(\frac{22}{7}\right) \times 7 \times (7 + 20)$

$= 2 \times 22 \times 27$

$= 1188 \text{ cm}^2$

Q25. The volume of a cone with a circular base is $216\pi \text{ cm}^3$. If the base radius is 9 cm, then find the height of the cone.

Solution:

Let h be the height of the cone.

Given,

Radius of cone $= r = 9$ cm

Volume $= 216\pi \text{ cm}^3$

$\left(\frac{1}{3}\right)\pi r^2 \text{ h} = 216\pi$

$\left(\frac{1}{3}\right) \times 9 \times 9 \times h = 216$

$27\ h = 216$

$h = \frac{216}{27}$

$h = 8$ cm

Therefore, the height of the cone is 8 cm .

Q26. The standard deviation of 20 observations is $\sqrt{5}$. If each observation is multiplied by 2, find the standard deviation and variance of the resulting observations.

Solution:

Given,

Standard deviation $= \sqrt{5}$

Each observation is multiplied by 2, then new standard deviation $= 2\sqrt{5}$

$Variance = (\text{ standard deviation })^2$

$= (2\sqrt{5})^2$

$= 4 \times 5 = 20$

Q27. A die is thrown twice. Find the probability of getting a total of 9.

Solution:

Total number of outcomes $= n(S) = 6^2 = 36$

Let E be the event of getting a sum 9 .

$E = \{(4,5), (5,4), (3,6), (6,3)\}$

Number of outcomes favourable to $E = n(E) = 4$

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{4}{36}$

$= \frac{1}{9}$

Hence, the required probability is $\frac{1}{9}$.

Q28. AB and CD are two chords of a circle which intersect each other externally at P. If AB $= 4$ cm, BP $= 5$ cm and PD $= 3$ cm, then find CD.

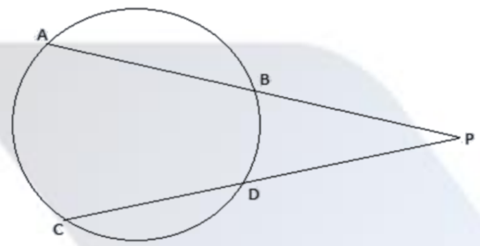
Solution:

Given,

AB and CD are two chords of a circle which intersect each other externally at P.

AB = 4 cm, BP = 5 cm and PD = 3 cm

Two chords AB and CD meet at P when produced.



$\Rightarrow \text{PA} \times \text{PB} = \text{PC} \times \text{PD}$

$\Rightarrow (PB + AB) \times PB = (PD + CD) \times PD$

$\Rightarrow (5 + 4) \times 5 = (3 + CD) \times 3$

$\Rightarrow 9 \times 5 = (3 + CD) \times 3$

$\Rightarrow (3 + CD) = \frac{45}{3}$

$\Rightarrow 3 + \text{CD} = 15$

$\Rightarrow CD = 15 - 3$

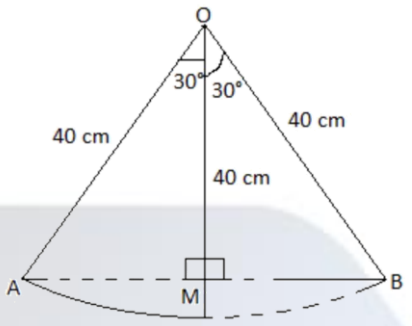
$\Rightarrow CD = 12 \text{ cm}$

Q29. A simple pendulum of length 40 cm subtends 60° at the vertex in one full oscillation. What will be the shortest distance between the initial position and the final position of the bob?

Solution:

Given,

Length of the pendulum = 40 cm



In triangle OMA,

$\sin\ 30^\circ = \frac{AM}{OA}$

$\frac{1}{2} = \frac{AM}{40}$

$AM = \frac{40}{2}$

$AM = 20$ cm

$AB = 2AM = 2(20) = 40$ cm

Hence, the required shortest distance is 40 cm.

Q30. (a) Show that the function $f: N \to N$ defined by $f(n) = n + 1$ is not onto, by drawing arrow diagrams.

Solution:

Given,

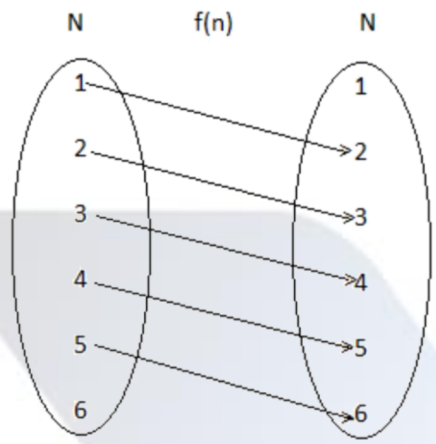
$f(n) = n + 1$

$f(1) = 1 + 1 = 2$

$f(2) = 2 + 1 = 3$

$f(3) = 3 + 1 = 4$

$f(4) = 4 + 1 = 5$



Hence, the given function is not on-to.

(b) Find the equation of the line perpendicular to the line $x = 5$, passing through the point (5,8).

Solution:

Given,

$x = 5$ is a vertical line.

Thus, it doesn't have a slope.

The line perpendicular to a vertical line will be a horizontal line. Hence, its slope is 0 (zero).

Therefore, the equation of the line is:

$y - y_1 = 0(x - x_1)$ (point slope form)

$y - 8 = 0(x - 5)$

$y - 8 = 0$

$y = 8$

SECTION - III

Q31. A function $f: [-7,6) \to R$ is defined as

$$f(x) = \begin{cases} x^2 + 2x + 1; & -7 \le x < -5 \\ x + 5; & -5 \le x < 2 \\ x - 1; & 2 \le x < 6 \end{cases}$$

Find $\frac{[4f(-3)+2f(4)]}{[f(-6)-3f(1)]}$.

Solution:

$f(-3) = -3 + 5 = 2$
$f(4) = 4 - 1 = 3$
$f(-6) = (-6)^2 + 2(-6) + 1$
$= 36 - 12 + 1$
$= 25$
$f(1) = 1 + 5 = 6$

$$\frac{[4f(-3) + 2f(4)]}{[f(-6) - 3f(1)]} = \frac{[4 \times 2 + 2 \times 3]}{[25 - 3 \times 6]}$$

$$= \frac{8 + 6}{25 - 18}$$

$$= \frac{14}{7}$$

$$= 2$$

Q32. Find the sum of the series $5^2 + 7^2 + 9^2 + \cdots. +39^2$.

Solution:

$5^2 + 7^2 + 9^2 + \cdots. +39^2$

$= (1^2 + 2^2 + \cdots + 39^2) - (2^2 + 4^2 + 6^2 + \cdots + 38^2) - (1^2 + 3^2)$

$= (1^2 + 2^2 + \cdots + 39^2) - 4(1^2 + 2^2 + 3^2 + \cdots + 19^2) - (1^2 + 3^2)$

$$= \frac{39 \times 40 \times 79}{6} - \left[4 \times \frac{19 \times 20 \times 39}{6}\right] - 10$$

$= 20540 - 9880 - 10$

$= 10650$

Q33. The 4th term of a geometric sequence is $\frac{2}{3}$ and its seventh term is $\frac{16}{81}$. Find the geometric sequence.

Solution:

Let a be the first term and r be the common ratio of GP.

Given,

$$a_4 = \frac{2}{3}$$

$$a_7 = \frac{16}{81}$$

Now,

$$\frac{a_7}{a_4} = \frac{\frac{16}{81}}{\frac{2}{3}}$$

$$\frac{ar^6}{ar^3} = \frac{16 \times 3}{81 \times 2}$$

$$r^3 = \frac{8}{27}$$

$$r^3 = \left(\frac{2}{3}\right)^3$$

$$\Rightarrow r = \frac{2}{3}$$

Thus, $ar^3 = \frac{2}{3}$

$$a\left(\frac{2}{3}\right)^3 = \frac{2}{3}$$

$$a = \left(\frac{3}{2}\right)^2$$

$$a = \frac{9}{4}$$

Hence, the required GP is: $\frac{9}{4}, \left(\frac{9}{4}\right)\left(\frac{2}{3}\right), \left(\frac{9}{4}\right)\left(\frac{2}{3}\right)^2, \ldots.$

Q34. Show that the roots of the equation $x^2 + 2(a+b)x + 2(a^2+b^2) = 0$ are not real.

Solution:

Given,

$x^2 + 2(a+b)x + 2(a^2+b^2) = 0$

$\Delta = B^2 - 4AC$

$= [2(a+b)]^2 - 4(1)[2(a^2+b^2)]$

$= 4(a^2+b^2+2ab) - 8a^2 - 8b^2$

$= 4a^2 + 4b^2 + 8ab - 8a^2 - 8b^2$

$= -4a^2 + 8ab - 4b^2$

$= -4(a^2 - 2ab + b^2)$

$= -4(a-b)^2 < 0$

Therefore, the roots of the given equation are imaginary, i.e., not real.

Q35. The GCD of $x^4 + 3x^3 + 5x^2 + 26x + 56$ and $x^4 + 2x^3 - 4x^2 - x + 28$ is $x^2 + 5x + 7$. Find their LCM.

Solution:

$$
\begin{array}{r|rrrrr}
 & x^2 & -2x & +8 & & \\
\hline
x^2+5x+7 & x^4 & +3x^3 & +5x^2 & +26x & +56 \\
- & x^4 & +5x^3 & +7x^2 & & \\
\hline
 & & -2x^3 & -2x^2 & +26x & +56 \\
 & - & -2x^3 & -10x^2 & -14x & \\
\hline
 & & & 8x^2 & +40x & +56 \\
 & & - & 8x^2 & +40x & +56 \\
\hline
 & & & & & 0
\end{array}
$$

Quotient $= \mathrm{x}^2 - 2\mathrm{x} + 8$

LCM $= (x^2 - 2x + 8)(x^4 + 2x^3 - 4x^2 - x + 28)$

Q36. Solve:

$$(x \quad 1)\begin{pmatrix} 1 & 0 \\ -2 & -3 \end{pmatrix}\begin{pmatrix} x \\ 5 \end{pmatrix} = 0$$

Solution:

Given,

$$[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$[x \quad 1]\begin{bmatrix} x + 0 \\ -2x - 15 \end{bmatrix} = 0$$

$(x)(x) + (1)(-2x - 15) = 0$

$x^2 - 2x - 15 = 0$

$x^2 - 5x + 3x - 15 = 0$

$x(x - 5) + 3(x - 5) = 0$

$(x - 5)(x + 3) = 0$

$x = 5, -3$

Q37. If C is the midpoint of the line segment joining $A(4,0)$ and $B(0,6)$ and if O is the origin, then show that C is equidistant from all the vertices of $\triangle$ OAB.

Solution:

Given,

$A(4,0)$ and $B(0,6)$

$\mathrm{C} = \text{Midpoint of AB}$

$= \left[\frac{4+0}{2}, \frac{0+6}{2}\right]$

$= \left(\frac{4}{2}, \frac{6}{2}\right)$

$= (2,3)$

Let $\mathrm{O}(0,0)$ be the origin.

Now,

$OC = \sqrt{(2^2 + 3^2)} = \sqrt{(4+9)} = \sqrt{13}$

$AC = \sqrt{[(2-4)^2 + (3-0)^2]}$

$= \sqrt{(4+9)}$

$= \sqrt{13}$

$BC = \sqrt{[(2-0)^2 + (3-6)^2]}$

$= \sqrt{(4+9)}$

$= \sqrt{13}$

Therefore, $\mathrm{OC} = \mathrm{AC} = \mathrm{BC}$

Hence, C is equidistant from all the vertices of $\triangle$ OAB.

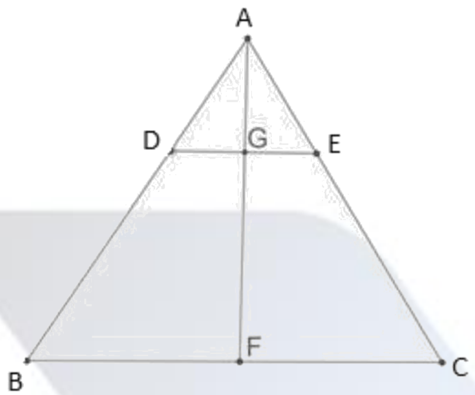
Q38. The points D and E are on the sides AB and AC of $\triangle ABC$ respectively, such that $DE \| BC$. If $AB = 3AD$ and the area of $\triangle$ ABC is 72 cm^2, then find the area of the quadrilateral DBCE.

Solution:

Given,

The points D and E are on the sides AB and AC of $\triangle$ ABC respectively, such that DE||BC.

$\mathrm{AB} = 3\mathrm{AD}$



In $\triangle$ ADE and $\triangle$ ABC

$\angle ADE = \angle ABC$ (corresponding angles)

$\angle DEA = \angle BCA$ (corresponding angles)

$\therefore \triangle AED \sim \triangle ACB$

Similarly,

$\triangle AGD \sim \Delta AFB$ (where $AF \perp BC$)

$\Rightarrow AF = 3AG (\because$ given $AB = 3AD) \ldots$ (i)

Also,

$BC = 3 \times DE$

Now,

$\left(\frac{1}{2}\right) \times BC \times AF = \left(\frac{1}{2}\right) \times (3 \times DE) \times (3 \times AG)$ [From (i) and (ii)]

$$\left(\frac{1}{2}\right) \times BC \times AF = 9 \times \left(\frac{1}{2}\right) \times DE \times AG$$

$\Rightarrow$ Area of triangle $ABC = 9 \times$ Area of triangle ADE

Thus, area of triangle ADE $= \frac{72}{9} = 8\ cm^2$

$Area\ of\ quadrilateral\ BCED$

$= Area\ of\ triangle\ ABC - Area\ of\ triangle\ ADE$

$= 72 - 8$

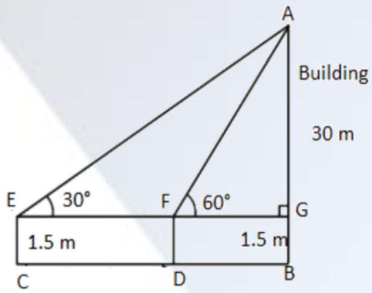
$= 64\ cm^2$

Q39. A boy is standing at some distance from a 30 m tall building and his eye level from the ground is 1.5 m. The angle of elevation from his eye to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Solution:

Let AB be the building.

C and D be the points of observation.



$AG = 30 - 1.5 = 28.5$ m

In right triangle AGF,

$$\tan\ 60^\circ = \frac{AG}{FG}$$

$$\sqrt{3} = \frac{28.5}{FG}$$

$$FG = \frac{28.5}{\sqrt{3}}$$

$$FG = 9.5\sqrt{3}\text{ m}$$

In right triangle AGE, $\tan\ 30^\circ = AG/GE$

$$(\frac{1}{\sqrt{3}}) = \frac{28.5}{EF + 9.5\sqrt{3}}$$

$$EF + 9.5\sqrt{3} = 28.5\sqrt{3}$$

$$EF = 28.5\sqrt{3} - 9.5\sqrt{3}$$

$$EF = 19\sqrt{3}\text{ m}$$

Hence, the boy walked $19\sqrt{3}$ m towards the building.

Q40. The diameter of a road roller of length 120 cm is 84 cm . If it takes 500 complete revolutions to level a playground, then find the cost of levelling it at the cost of 75 paise per square meter. (take $\pi = 22/7$)

Solution:

Given,

Diameter $= 84$ cm

Length $= h = 120$ cm

Radius $= r = 84/2 = 42$ cm

Surface area of roller $= 2\pi\, rh$

$= 2 \times \left(\frac{22}{7}\right) \times 42 \times 120$

$= 44 \times 6 \times 120$

$= 31680\ \text{cm}^2$

Area covered for 500 revolutions $= 31680 \times 500$

$= 15840000\ \text{cm}^2$

$= 1584$ sq.m

Cost of levelling 1 sq. m $= 75$ paise $=$ ₹ 0.75

Cost of levelling the ground $= 1584 \times$ ₹ 075 $=$ ₹ 1188

Q41. For a collection of data, if $\sum x = 35, n = 5, \sum(x-9)^2 = 82$, then find $\sum x^2$ and $\sum(x - \bar{x}\,)^2$.

Solution:

Given,

$$\sum x = 35, n = 5, \sum (x-9)^2 = 82$$

$\bar{x} = \sum \frac{x}{n} = \frac{35}{5} = 7$

$\sum(x-9)^2 = 82$

$\sum(x^2 - 18x + 81) = 82$

$\sum x^2 - 18\sum x + 81 = 82$

$\sum x^2 - 18(35) + 81 = 82$

$\sum x^2 - 630 + 81 = 82$

$\sum x^2 = 630 + 82 - 81$

$\sum x^2 = 631$

Similarly,

$\sum(x - \bar{x})^2$

$= \sum(x - 7)^2$

$= \sum x^2 + 49 - 2\sum 7x$ by expanding the above summation

$= 631 + 49 - 14 \times 35$ from above

=680−490=190

Q42. The probability that A, B and C can solve a problem are $\frac{4}{5}, \frac{2}{3}$ and $\frac{3}{7}$ respectively. The probability of the problem being solved by A and B is $\frac{8}{15}$, B and C is $\frac{2}{7}$, A and C is $\frac{12}{35}$. The probability of the problem being solved by all the three is $\frac{8}{35}$. Find the probability that the problem can be solved by at least one of them.

Solution:

Given,

$\mathrm{P(A)} = \frac{4}{5}$

$\mathrm{P(B)} = \frac{2}{3}$

$\mathrm{P(C)} = \frac{3}{7}$

$P(A \cap B) = \frac{8}{15}$

$P(B \cap C) = \frac{2}{7}$

$P(A \cap C) = \frac{12}{35}$

$P(A \cap B \cap C) = \frac{8}{35}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{4}{5} + \frac{2}{3} + \frac{3}{7} - \frac{8}{15} - \frac{2}{7} - \frac{12}{35} + \frac{8}{35}$

$= \frac{84 + 70 + 45 - 56 - 30 - 36 + 24}{105}$

$= \frac{101}{105}$

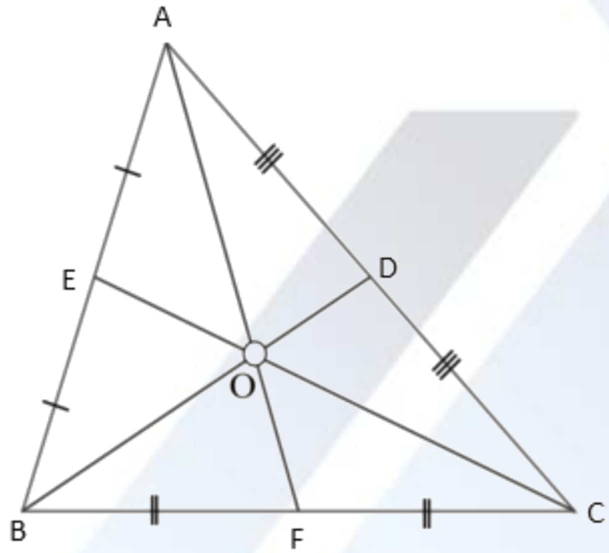
Hence, the probability of the problem can be solved by at least one of them is $\frac{101}{105}$.

Q43. A triangle has vertices at $(6,7), (2,-9)$ and $(-4,1)$. Find the slopes of its medians.

Solution:

Let $A(6,7), B(2,-9)$ and $C(-4,1)$ be the vertices of triangle ABC.

BD, CE and AF be the medians of AC, AB and BC respectively.



D = Midpoint of AC

$= \left[\frac{6-4}{2}, \frac{7+1}{2}\right]$

$= \left(\frac{2}{2}, \frac{8}{2}\right)$

$= (1,4)$

$E =$ Midpoint of AB

$= \left[\frac{6+2}{2}, \frac{7-9}{2}\right]$

$= \left(\frac{8}{2}, -\frac{2}{2}\right)$

$= (4, -1)$

F = Midpoint of BC

$= \left[\frac{2-4}{2}, \frac{-9+1}{2}\right]$

$= \left(-\frac{2}{2}, -\frac{8}{2}\right)$

$= (-1, -4)$

Slope of median AF $= \frac{-4-7}{-1-6} = \frac{11}{7}$

Slope of median BD $= \frac{4+9}{1-2} = -13$

Slope of median CE $= \frac{-1-1}{4+4} = -\frac{2}{8} = -\frac{1}{4}$

Q44. A hollow cylinder pipe is of length 40 cm . Its internal and external radii are 4 cm and 12 cm respectively. It is melted and cast into a solid cylinder of length 20 cm . Find the radius of the new cylinder.

Solution:

Given,

Length of hollow cylinder $= H = 40$ cm

Internal radius $= r = 4$ cm

External radius $= R = 12$ cm

Length of the solid cylinder $= h = 20$ cm

Let r_2 be the radius of the solid cylinder.

Volume of solid cylinder = Volume of hollow cylinder

$\pi r_2^2 \, h = \pi \mathrm{h}(\mathrm{R}^2 - \mathrm{r}^2)$

$\mathrm{r_2}^2 \times 20 = 40[(12)^2 - (4)^2]$

$\mathrm{r_2}^2 = 2[144 - 16]$

$\mathrm{r_2}^2 = 2 \times 128$

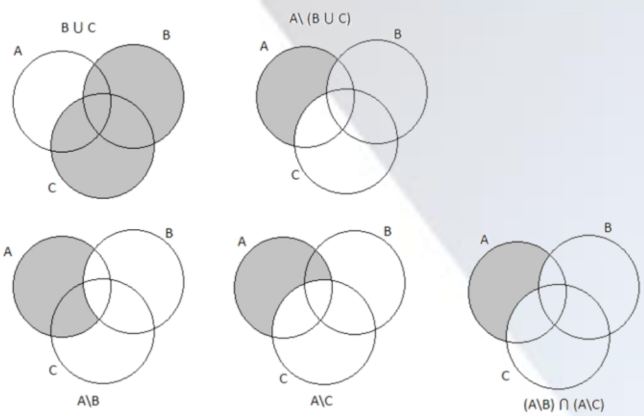
$\mathrm{r_2}^2 = 256$

$\mathrm{r_2} = 16$ cm

Hence, the required radius is 16 cm.

Q45. (a) Prove that $A\backslash(B \cup C) = (A \setminus B) \cap (A\backslash C)$ by using Venn diagrams.

Solution:



Hence, proved that: $A\backslash(B \cup C) = (A\backslash B) \cap (A\backslash C)$

(b) If the remainder on division $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the value of k and hence find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Solution:

Given,

$\text{p(x)} = x^3 + 2x^2 + kx + 3$

$g(x) = x - 3$

$r(x) = 21$

Thus, $p(3) = 21$

$(3)^3 + 2(3)^2 + \text{k}(3) + 3 = 21$

$27 + 18 + 3k + 3 = 21$

$48 + 3k = 21$

$3k = 21 - 48$

$k = -27/3$

$k = -9$

Therefore, the cubic polynomial is $f(x) = x^3 + 2x^2 + (-9)x - 18 = x^3 + 2x^2 - 9x - 18$

$$
\begin{array}{r|llll}
 & x^2 & +5x & +6 & \\
\hline
x-3 & x^3 & +2x^2 & -9x & -18 \\
- & x^3 & -3x^2 & & \\
\hline
 & & 5x^2 & -9x & -18 \\
 & - & 5x^2 & -15x & \\
\hline
 & & & 6x & -18 \\
 & & - & 6x & -18 \\
\hline
 & & & & 0
\end{array}
$$

Consider,

$x^2 + 5x + 6 = 0$

$x^2 + 2x + 3x + 6 = 0$

$x(x + 2) + 3(x + 2) = 0$

$(x + 2)(x + 3) = 0$

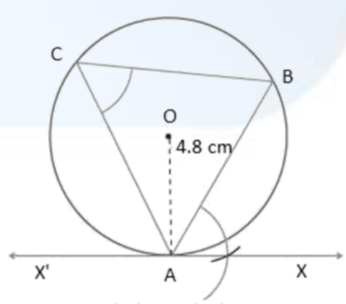
$x = -2, x = -3$

Hence, the required zeroes of the cubic polynomial are $3, -2, -3$.

SECTION - IV

Q46. (a) Draw a circle of radius 4.8 cm. Take a point on the circle, draw the tangent at that point using the tangent chord theorem.

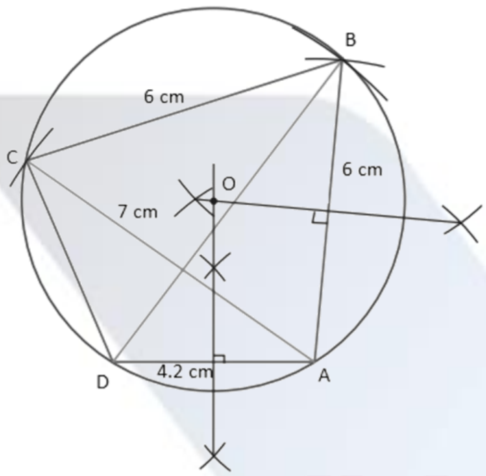
Solution:



Hence, XAX′ ' is the required tangent at A on the circle of radius 4.8 cm.

(b) Construct a cyclic quadrilateral ABCD in which $AB = 6$ cm, $AC = 7$ cm, $BC = 6$ cm and $AD = 4.2$ cm.

Solution:



Q47. (a) Draw the graph of $y = -3x^2$.

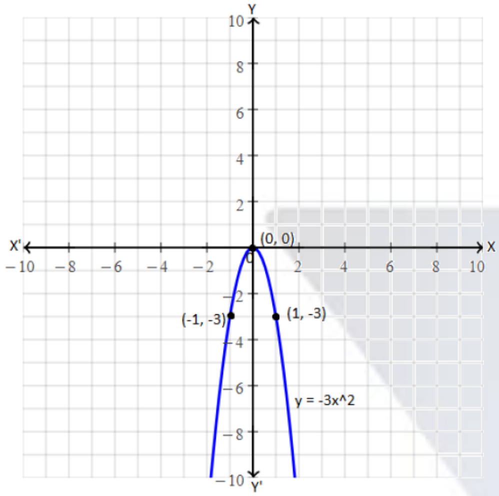
Solution:

Given,

$y = -3x^2$

Table for the corresponding x and y values:

x	-1	0	1
y	-3	0	-3



(b) A bus travels at a speed of 40 km/hr. Write the distance - time formula and draw the graph of it. Hence, find the distance travelled in 3 hours.

Solution:

Given,

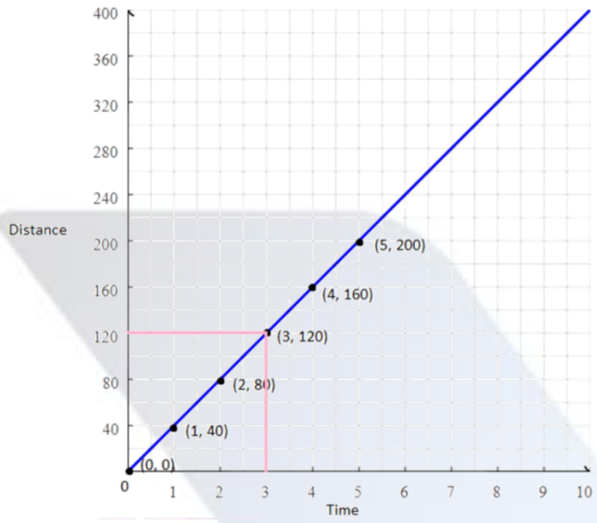
Speed $= 40$ km/hr

Let x be the time taken.

Distance $=$ Speed $\times$ time

$y = 40x$

x	0	1	2	3	4	5
y	0	40	80	120	160	200



From the graph,

Distance travelled in 3 hours is 120 km.