Grade 10 Tamil Nadu Mathematics 2022

PART – I

Q1. If the ordered pairs $(a+2,4)$ and $(5,2a+b)$ are equal then (a,b) is:
(a) $(2,-2)$
(b) $(5,1)$
(c) $(2,3)$
(d) $(3,-2)$
Solution:
Correct answer: (d)
If ordered pairs are equal, then there domain and range are also equal.
So, $a+2=5$
So, $a=5-2$
$a=3$
Now $4=2a+b$
Putting the value of a, we get:
$4=3\times 2+b$
So, $4=6+b$
Therefore $\mathrm{b}=-2$.
$(a,b) = (3,-2)$

Q2. If the HCF of 65 and 117 is expressible in the form of $65\,m-117$, then the value of m is
(a) 4
(b) 2
(c) 1
(d) 3
Solution:
Correct answer: (b)
$65 = 5\times 13$
$117 = 3\times 3\times 13$
Therefore, HCF of 65 and 117 is 13.
So, $65m-117=13$
or, $65m=130$
Therefore, $m=2$

Q3. If t_n is the n^{th} term of an A.P., then $t_{8n}-t_n$ is:
(a) $(8n-1)d$

(b) $(8n-2)$d
(c) $(7\text{n}-2)$d
(d) $(7nd)$
Solution:
Correct answer: (d)
$t_n = a + (n-1)d = a + nd - d$
$t_{8n} = a + (8n-1)d = a + 8nd - d$
$t_{8n} - t_n = a + 8nd - d - a - nd + d = 7nd$

Q4. If $(x - 6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$, then the value of k is:
(a) 3
(b) 5
(c) 6
(d) 8
Solution:
Correct answer: (b)
Given that $(x-6)$ is the HCF of $x^2 - 2x - 2y$ and $x^2 - 4x - 6$ $(x-6)$ is factor of both expression
Let $f(x_1) = x_1^2 - 2x_1 - 24$
and $f(x_2) = x_2^2 - 4x_2 - 6$
Now $f(x_1) = f(x_2)$ at $(x_1 = x_2 = 6)$
$\Rightarrow (6)^2 - 2(6) - 24 = (6)^2 - 4(6) - 6$
$\Rightarrow 0 = 30 - 6\text{k} \Rightarrow 6\text{k} = 30 \therefore k = 5$

Q5. Which of the following should be added to make $x^4 + 64$ a perfect square?
(a) $4x^2$
(b) $16x^2$
(c) $8x^2$
(d) $-8x^2$
Solution:
Correct answer: (b)
$x^2 + 64 = (x^2)^2 + 8^2 - 2x(x^2) \times 8$
$= (x^2 - 8)^2$
$2 \times (x^2) \times 8$ must be added
i.e, $16x^2$ must be added to make $x^2 + 64$ a perfect square.

Q6. The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the x- axis of is:
(a) 0

(b) 1
(c) 0 or 1
(d) 2
Solution:
Correct answer: (b)
$(x+2)^2 = 0 \Rightarrow (x+2)(x+2) = 0$
$x+2 = 0$ or $x+2 = 0 \Rightarrow x = -2$ or $x = -2$
Number of points of intersection is 1 (both the values are same)

Q7. If $\triangle$ ABC is an isosceles triangle with $\angle C = 90^\circ$ and AC = 5 cm, then AB is:
(a) 2.5 cm
(b) 5 cm
(c) 10 cm
(d) $5\sqrt{2}$ cm
Solution:
Correct answer: (d)
$AB^2 = AC^2 + BC^2$
$AB^2 = 5^2 + 5^2$ ... (It is an isosceles triangle)
$AB = \sqrt{50}$
$= \sqrt{25 \times 2}$
$AB = 5\sqrt{2}$ cm

Q8. In a $\triangle$ ABC, AD is the bisector of $\angle BAC$. If AB = 8 cm, BD = 6 cm and DC = 3 cm, the length of the side AC is:
(a) 6 cm
(b) 4 cm
(c) 3 cm
(d) 8 cm
Solution:
Correct answer: (b)
Given: In a $\triangle$ ABC, AD is the bisector of angle BAC. AB = 8 cm, and DC = 3 cm and BD = 6 cm.
To find: AC
We know that the internal bisector of angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.
Hence,
$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{8}{AC} = \frac{6}{3}$$
$$AC = \frac{8 \times 3}{6} = 4\ cm$$

Q9. If $(5,7), (3, p)$ and $(6,6)$ are collinear, then the value of ' p ' is:

(a) 3

(b) 6

(c) 9

(d) 12

Solution:

Correct answer: (c)

If $(5,7), (3, p)$ and $(6,6)$ are collinear $\Delta = 0$

$(5p + 18 + 42) - (21 + 6p + 30) = 0$

$5p + 60 - (6p + 51) = 0$

$5p - 6p = -60 + 51$

$-1p = -9$

$p = 9$

Q10. The slope of the line which is perpendicular to a line joining the points $(0,0)$ and $(-8,8)$ is:

(a) -1

(b) 1

(c) $\frac{1}{3}$

(d) -8

Solution:

Correct answer: (a)

Slope of a line $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{8 - 0}{-8 - 0}$

$= \frac{8}{-8} = -1$

Slope of the Perpendicular $= 1$

Q11. A tower is 60 m high. Its shadow is x metres shorter when the sun's altitude is 45° than when it had been 30°, then ' x ' is equal to:

(a) 41.92 m

(b) 43.92 m

(c) 43 m
(d) 45.6 m

Solution:

Correct answer: (b)

$\tan 45^\circ = 1 = \frac{60}{y}$

$y = 60$ m

$\tan 30^\circ = \frac{60}{x+y} = \frac{1}{\sqrt{3}}$

$60 + x = 60\sqrt{3}$

$x = 60\sqrt{3} - 60$

$x = 60(\sqrt{3} - 1)$

$x = 43.92$ m.

Q12. If two solid hemispheres of same base radius ' r ' units are joined together along their bases, then curved surface area of this new solid is:
(a) $4\pi r^2$ sq.units
(b) $6\pi r^2$ sq.units
(c) $3\pi r^2$ sq.units
(d) $8\pi r^2$ sq.units

Solution:

Correct answer: (a)
Because curved surface area of a hemisphere is $2w^2$ and here, we join two solid hemispheres along their bases of radius r, from which we get a solid sphere. Hence, the curved surface area is $2\pi r^2 + 2\pi r^2 = 4\pi r^2$.

Q13. If the radius of the cylinder is doubled, the new volume of the cylinder will be____ times the original volume.
(a) same
(b) 3
(c) 4
(d) 2

Solution:

Correct answer: (c)
Volume of a cylinder $V_1 = \pi r^2 h$
If the radius is doubled r becomes $2r$
New volume of the cylinder $V_2 = \pi(2r)^2 h$
$= \pi 4r^2 h$
$= 4\pi r^2 h$

$= 4V_1$

Q14. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$, then the value of ' x ' is:

(a) 2

(b) 1

(c) 3

(d) 1.5

Solution:

Correct answer: (b)

Probability of getting a job $= \frac{x}{3}$

Probability of not getting a job $= 1 - \frac{x}{3}$

$$\frac{2}{3} = 1 - \frac{x}{3}$$

$$\Rightarrow \frac{x}{3} = 1 - \frac{2}{3}$$

$$\frac{x}{3} = \frac{3-2}{3}$$

$$\Rightarrow \frac{x}{3} = \frac{1}{3}$$

$$\Rightarrow x = 1$$

PART – II

Q15. Let $A = \{1,2,3\}, B = \{x \mid x \text{ is a prime number less than } 10\}$. Find $A \times B$ and $B \times A$.

Solution:

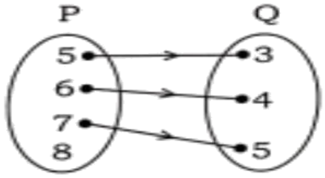
$A = \{1,2,3\}$

$B = \{2,3,5,7\}$

$\mathrm{A} \times \mathrm{B} = \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3)(2,5),(2,7),(3,2),(3,3),(3,5),(3,7)\}$

$\mathrm{B} \times \mathrm{A} = \{(2,1),(2,2)(2,3),(3,1),(3,2),(3,3),(5,1),(5,2)(5,3),(7,1),(7,2),(7,3)\}$

Q16. If The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) set builder form (ii) Roster form.



Solution:

i) Set builder form $R=\{(x,y) \mid y=x-2, x \in P, y \in Q\}$
ii) Roster form $R=\{(5,3),(6,4),(7,5)\}$

Q17. If $13824=2^{a} \times 3^{b}$, then find ' a ' and ' b '.

Solution:

$13824=512 \times 27$

$13824=2^{9} \times 3^{3}$

$a=9 ; b=3$

Q18. Which term of an A.P, 16,11,6,1, ... is -54 ?

Solution:

$t_{n}=-54, a=16, d=-5$

$t_{n}=a+(n-1) d$

$-54=16+(n-1)(-5)$

$-54=16-5 n+5$

$5 n=75$

$n=15$

Q19. Find the excluded values of the following expression $\frac{7 p+2}{8 p^{2}+13 p+5}$

Solution:

$8 p^{2}+13 p+5=0$

$8 p^{2}+8 p+5 p+5=0$

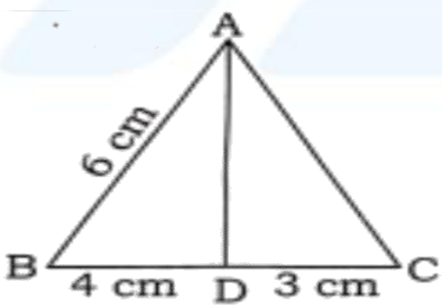
$8 p(p+1)+5(p+1)=0$

$(8 p+5)(p+1)=0$

$8 p+5=0 ; p+1=0$

Excluded values = $-\frac{5}{8},-1$

Q20. In the figure AD is the bisector of $\angle A$. If $BD=4$ cm, $DC=3$ cm and $AB=6$ cm, find AC.



Solution:

In $\triangle$ABC, AD is the bisector of $\angle$A.

$$\frac{BD}{DC}=\frac{AB}{AC}$$
$$\frac{4}{3}=\frac{6}{AC}$$
$$4AC=18$$
$$\text{AC}=\frac{9}{2}=4.5\text{ cm}$$

Q21. Show that the points $\text{P}(-1.5,3), \text{Q}(6,-2), \text{R}(-3,4)$ are collinear.

Solution:

When three points are collinear then the condition is

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

where, $x_1=-1.5, x_2=6, x_3=-3, y_1=3, y_2=-2, y_3=4$

Substituting the values,

$=(-1.5)(-2-4)+6(4-3)+(-3)(3-(-2))$

$=(-1.5)(-6)+6(1)+(-3)(5)$

$=9+6-15$

$=15-15=0$

The condition satisfied so the points are collinear.

Q22. The line ' p ' passes through the points $(3,-2),(12,4)$ and the line ' q ' passes through the points $(6,-2)$ and $(12,2)$. Is ' p ' parallel to ' q '?

Solution:

Slope of p:

$(x_1, y_1)=(3,-2)$

$(x_2, y_2)=(12,4)$

$$m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{4-(-2)}{12-3}=\frac{6}{9}=\frac{2}{3}$$
$$m_1=\frac{2}{3}$$

Slope of q:

$(x_1, y_1)=(6,-2)$

$(x_2, y_2)=(12,2)$

$$m_2=\frac{y_2-y_1}{x_2-x_1}=\frac{2-(-2)}{12-6}=\frac{4}{6}=\frac{2}{3}$$
$$m_2=\frac{2}{3}$$

$m_1=m_{2,}$

p is parallel to q

Q23. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point $(-1,2)$.

Solution:

$(x_1, y_1) = (-1,2)$

$m = -\frac{5}{4}$

$y - y_1 = m(x - x_1)$

$(y - 2) = -\frac{5}{4}(x - 2)$

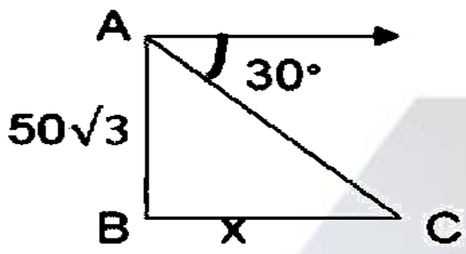
$4y - 8 = -5(x - 2)$

$4y - 8 = -5x + 10$

$5x + 4y - 3 = 0$

Q24. From the top of a rock $50\sqrt{3}$ m high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock.

Solution:



$\tan 30^\circ = \frac{AB}{BC}$

$\tan 30^\circ = \frac{50\sqrt{3}}{x}$

$\frac{1}{\sqrt{3}} = \frac{50\sqrt{3}}{x}$

$x = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 = 150$

Distance $= 150$ m

Q25. The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution:

$r_1 = 12\ cm, r_2 = 16\ cm$

Surface area of a sphere $= 4\pi r^2$ Sq.units

$4\pi r_1^2 : 4\pi r_2^2$

$12^2 : 16^2$

$144 : 256$

9: 16

Ratio of Surface area $= 9:16$

Q26. The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights.

Solution:

Volume of a cone $= \frac{1}{3}\pi r^2 h$ cu.units

$V_1 : V_2 = \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2 = 3600 : 5040$

$h_1 : h_2 = 5:7$

Q27. Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution:

$\mathrm{S} = \{\mathrm{HH}, \mathrm{HT}, \mathrm{TH}, \mathrm{TT}\}$

$n(s) = 4$

$A = \{HT, TH\}$

$P(A) = \frac{2}{4} = \frac{1}{2}$

Q28. If $P = \frac{x}{x+y}, Q = \frac{y}{x+y}$, then find $\frac{1}{P^2 - Q^2}$

Solution:

$P + Q = \frac{x+y}{x+y}$

$P - Q = \frac{x-y}{x+y}$

$\frac{1}{p^2 - Q^2} = \frac{1}{(P+Q)(P-Q)}$

$= \frac{x+y}{x+y} \times \frac{x+y}{x-y}$

$= \frac{x+y}{x-y}$

PART – III

Q29. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime numbers. Verify $A \times (B - C) = (A \times B) - (A \times C)$.

Solution:

$B - C = \{35,7\}$

$A \times (B - C) =$
$\{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7),$
$(5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$
$A \times B =$
$\{(1,2), (1,3), (1,5), (1,7), (2,2)(23), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7), (4,2), (4,3), (4,5), (4,7)$
$, (5,2), (53)(5,5)(5,7), (6,2), (6,3), (6,5), (6,7), (7,2)(7,3, (7,5), (7,7)\}$
$A \times C = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2), (7,2)\}$
$(A \times B) - (A \times C) =$
$\{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7),$
$(5,3), (5,5), (5,7), (6,3), (6,5), (6,7), (7,3), (7,5), (7,7)\}$
$A \times (B - C) = (A \times B) - (A \times C)$ Verified

Q30. If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z resp., then show that:
(i) $x(m-n) + y(n-l) + z(l-m) = 0$
(ii) $(x-y)n + (y-z)l + (z-x)m = 0$

Solution:

$t_n = a + (n-1)d$
$x = a + (l-1)d$
$y = a + (m-1)d$
$z = a + (n-1)d$
(i) $x(m-n) + y(n-l) + z(l-m)$
$= a + (l-1)d(m-n) + a + (m-1)d(n-l) + a + (n-1)d(l-m)$
$= a(0) + d(0) = 0$
(ii)$x - y = (l-m)d$
$y - z = (m-n)d$
$z - x = (n-l)d$
$(x-y)n + (y-z)l + (z-x)m$
$= (l-m)dn + (m-n)dl + (n-l)dm$
$= 0$

Q31. The ratio of 6^{th} and 8^{th} term of an A.P. is 7: 9. Find the ratio of 9^{th} term to 13^{th} term.

Solution:

$t_n = a + (n-1)d$
$t_6 : t_8 = 7 : 9$
$a + 5d : a + 7d = 7 : 9$
$a = 2d$
$t_9 : t_{13} = a + 8d : a + 12d$
$= 2d + 8d : 2d + 12d$
$= 10d : 14d$

$= 5d : 7d$
$= 5 : 7$

Q32. If $36x^4 - 60x^3 + 61x^2 - mx + n$ is a perfect square, find the values of m and n.

Solution:

$6x^2 \qquad \sqrt{36x^4 - 60x^3 + 61x^2 - mx + n} / 6x^2 - 5x + 6$

$. \qquad 36x^4$

$12x^2 - 5x| \qquad - 60x^3 + 61x^2$

$. \qquad - 60x^3 - 25x^2$

$.12x^2 - 10x + 3| \qquad 36x^2 - mx + n$

$. \qquad 36x^2 - 30x + 9$

$. \qquad - m + 30 \quad n - 9$

The reminder should be equal to 0.
Therefore, $m = -30$ and $n = 9$.

Q33. Solve: $pqx^2 - (p+q)^2 x + (p+q)^2 = 0$

Solution:

$a = pq; b = -(p+q)^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-(p+q)^2] \pm \sqrt{[-(p+)^2]^2 - 4(pq)(p+q)^2}}{2pq}$$

$$x = \frac{p+q}{q}, \frac{p+q}{p}$$

Q34. If α, β are the roots of $7x^2 + ax + 2 = 0$ and if $\beta - \alpha = \frac{-13}{7}$. Find the values of ' s

Solution:

$$x = \frac{-a \pm \sqrt{a^2 - 56}}{2 \times 7}$$

$$\alpha = \frac{-a + \sqrt{a^2 - 56}}{14}, \beta = \frac{-a - \sqrt{a^2 - 56}}{14}$$

$$\beta - \alpha = \frac{-a - \sqrt{a^2 - 56} + a - \sqrt{a^2 - 56}}{14}$$

$$= \frac{-2\sqrt{a^2 - 56}}{14} = \frac{-13}{7} \text{ (given)}$$

$$\Rightarrow \frac{-\sqrt{a^2 - 56}}{7} = \frac{-13}{7}$$

$$\Rightarrow -\sqrt{a^2 - 56} = -13$$

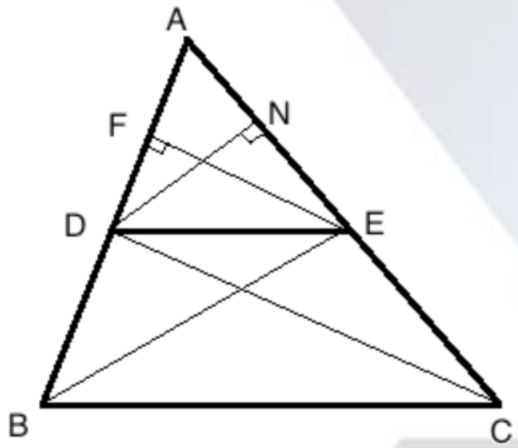
Squaring on both sides

$a^2 - 56 = 169$
$a^2 = 225$
$a = \pm 15$

Q35. State and prove Thales Theorem.

Solution:

Statement: If a line is drawn parallel to one side of a triangle, to interest the other two sides at distinct points, the other two sides are divided in the same ratio.



To prove: $\frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{CE}}$

Consider $\triangle$ ABC.Let DE $\parallel$ BC. Drop FE and DN perpendicular to sides AB and AC respectively.

Now,

Area of $\triangle$ ADE $= \frac{1}{2} \times$ FE $\times$ AD.......(i)

Area of $\triangle$ ADE $= \frac{1}{2} \times$ AE $\times$ DN........(ii)

Also,

Area of $\triangle$ AEB $= \frac{1}{2} \times$ FE $\times$ AB ....(iii)

Area of $\triangle$ ADC $= \frac{1}{2} \times$ AC $\times$ DN.........(iv)

Now, since $\triangle$ BDE and $\triangle$ CED are on the same base DE and between two parallel lines DE and BC , therefore,

Area of $\triangle$ BDE = Area of $\triangle$ CED

Adding area of $\triangle$ ADE on both the sides, we get,

Area of $\triangle$ BDE +$\triangle$ ADE = Area of $\triangle$ CED +$\triangle$ ADE

$\Rightarrow$ Area of $\triangle$ AEB = Area of $\triangle$ ADC ....(v)

Now, (i) $\div$ (iii), we get,

$\frac{\text{ar}\Delta\text{ADE}}{\text{ar}\Delta\text{ADC}} = \frac{\frac{1}{2}\times\text{FE}\times\text{AD}}{\frac{1}{2}\times FE\times AB} = \frac{\text{AD}}{\text{AB}}$ ....(vi)

Now, (ii) $\div$ (iv), we get,

$\frac{\text{ar } \Delta ADE}{\text{ar } \Delta AEB} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times AC \times DN} = \frac{AE}{AC}$........(vii)

From (v), (vi) and (vii), we get,

$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC}$ or $\frac{AB}{AD} = \frac{AC}{AE}$

Subtracting 1 from both sides, we get,

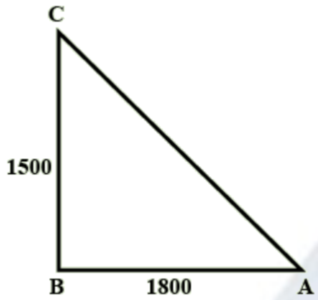
$\Rightarrow \frac{AB - AD}{AD} = \frac{AC - AE}{AE}$

$\Rightarrow \frac{BD}{AD} = \frac{CE}{AE}$

Thus, $\frac{AD}{BD} = \frac{AE}{CE}$

Q36. An aeroplane after take off from an airport, flies due north at a speed of 1000 km/hr. At the same time, another aeroplane takes off from the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Solution:



The first aeroplane leaves an airport and flies due north at a speed of 1000 km per hr.

∴ Distance travelled in 1.5 hrs .

$= 1000 \times 1.5 = 1500$ km

Similarly, Distance traveled by second aeroplane,

$= 1200 \times 1.5 = 1800$ km

In $\triangle$ ABC,

BC is distance travelled by first aeroplane and BA is the distance traveled by second aeroplane.

Hence, applying Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

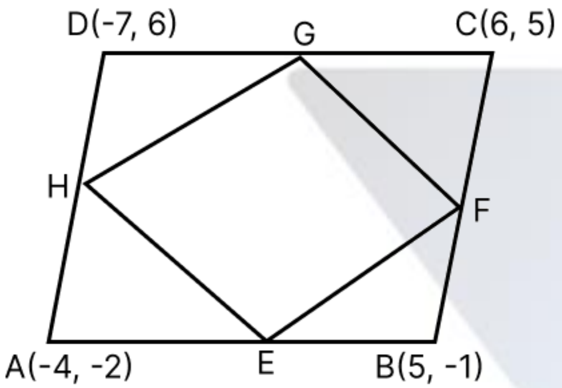
$\therefore AC^2 = 1500^2 + 180^2$

$\therefore AC = 300\sqrt{61}$ km

Hence, two planes are $300\sqrt{61}$ km apart.

Q37. A quadrilateral has vertices at $A(-4,-2), B(5,-1), C(6,5)$ and $D(-7,6)$. Show that the mid-points of its sides form a parallelogram.

Solution:



Mid point of a line $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Mid point of AB(E) $= \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right) = \left(\frac{1}{2}, \frac{-3}{2}\right)$

Mid point of BC(F) $= \left(\frac{5+6}{2}, \frac{-1+5}{2}\right) = \left(\frac{11}{2}, \frac{4}{2}\right) = \left(\frac{11}{2}, 2\right)$

Mid point of CD(G) $= \left(\frac{6-7}{2}, \frac{5+6}{2}\right) = \left(\frac{-1}{2}, \frac{11}{2}\right)$

Mid point of AD(H) $= \left(\frac{-4-7}{2}, \frac{-2+6}{2}\right) = \left(\frac{-11}{2}, \frac{4}{2}\right) = \left(\frac{-11}{2}, 2\right)$

The midpoint of ABCD are E $\left(\frac{1}{2}, -\frac{3}{2}\right)$, F $\left(\frac{11}{2}, 2\right)$, G $\left(-\frac{1}{2}, \frac{11}{2}\right)$ and H $\left(-\frac{11}{2}, 2\right)$

Slope of a line $= \frac{y_2-y_1}{x_2-x_1}$

Slope of EF $= \left(\frac{-\frac{3}{2}-2}{\frac{1}{2}-\frac{11}{2}}\right) = \left(\frac{\frac{-3-4}{2}}{\frac{1-11}{2}}\right) = \left(\frac{\frac{-7}{2}}{\frac{-10}{2}}\right)$

$= \frac{-7}{2} \times \frac{-1}{5} = \frac{7}{10}$

Slope of FG $= \frac{\frac{11}{2}-2}{-\frac{1}{2}-\frac{11}{2}} = \frac{\frac{11-4}{2}}{\frac{-1-11}{2}} = \frac{\frac{7}{2}}{-6}$

$= \frac{7}{2} \times \frac{-1}{6}$

$= \frac{-7}{12}$

Slope of GH $= \frac{2-\frac{11}{2}}{\frac{-11}{2}+\frac{1}{2}} = \frac{\frac{4-11}{2}}{\frac{-11+1}{2}}$

$= \frac{\frac{-7}{2}}{\frac{-10}{2}}$

$= \frac{-7}{2} \times \frac{-2}{10}$

$$= \frac{7}{10}$$

Slope of EH $= \frac{2+\frac{3}{2}}{-\frac{11}{2}-\frac{1}{2}} = \frac{\frac{4+3}{2}}{\frac{-11-1}{2}}$

$$= \frac{\frac{7}{2}}{-6}$$

$$= \frac{7}{2} \times \frac{-1}{6}$$

$$= -\frac{7}{12}$$

Slope of EF = Slope of GH $= \frac{7}{10}$

$\therefore$ EF $\parallel$ GH . . (1)

Slope of FG = Slope of EH $= -\frac{7}{12}$

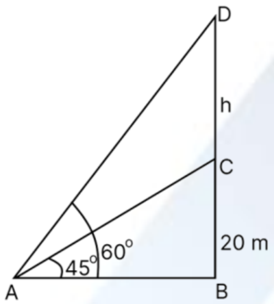
$\therefore$ FG $\parallel$ EH. . . (2)

From (1) and (2) we get EFGH is a parallelogram.

The midpoint of the sides of the Quadrilateral ABCD is a Parallelogram.

Q38. From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are 45° and 60° respectively. Find the height of the tower. $(\sqrt{3} = 1.732)$

Solution:



Let DC be the tower and BC be the building. Then, $\angle CAB = 45°, \angle DAB = 60°$, BC = 30 m Let height of the tower, DC = h m.

In right $\triangle$ ABC,

$$\tan\ 45° = \frac{BC}{AB}$$

$$1 = \frac{30}{AB}$$

AB = 30 m

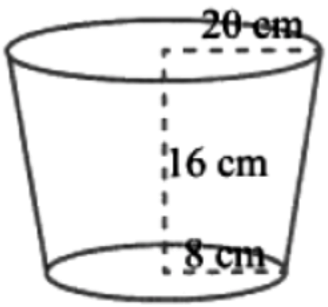
In right $\triangle$ ABD,

$$\tan\ 60° = \frac{BD}{AB}$$

$$\sqrt{3} = \frac{h + 30}{30}$$
$$h = 30(\sqrt{3} - 1)\text{m}$$
$$h = 21.96 \text{ m}$$

Q39. A container open at the top is in the form of frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of ₹40 per litre.

Solution:



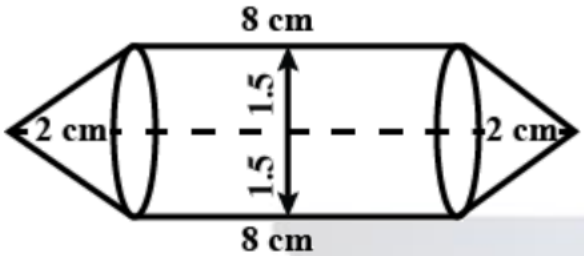
Volume of a frustum is

$$= \frac{1}{3}\pi(\text{R}^2 + \text{R}r + r^2)h \text{ cubic units}$$
$$= \frac{1}{3} \times \frac{22}{7}(20^2 + 20 \times 8 + 8^2) \times 16$$
$$= \frac{1}{3} \times \frac{22}{7}(400 + 160 + 64) \times 16$$
$$= \frac{1}{3} \times \frac{22}{7} \times 624 \times 16$$
$$= \frac{73216}{7}$$
$$= 10459.428 \text{ cm}^3$$
$$10459.428 \text{ cm}^3 = 10.459 \text{ litres}$$

The cost of milk ₹40 per litre = ₹418.36

Q40. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The length of the model is 12 cm and its diameter is 3 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.

Solution:



Height (h_1) of each conical part = 2 cm

Height (h_2) of cylindrical part $12-2-2=8\ cm$

Radius *(r)* of cylindrical part = Radius of conical part =$\frac{3}{2}$ cm

Volume of air present in the model

$=$ Volume of cylinder $+2\times$ Volume of a cone

$=\pi r^2 h_2+2\times\frac{1}{3}\pi r^2 h_1$

$=\pi\left(\frac{3}{2}\right)^2\times 8+2\times\frac{1}{3}\pi\left(\frac{3}{2}\right)^2(2)$

$=\pi\times\frac{9}{4}\times 8+\frac{2}{3}\pi\times\frac{9}{4}\times 2$

$=18\pi+3\pi=21\pi$

$=66\ cm^3$

Q41. In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the student is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The student opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

Solution:

Let A be the event of students opting for NCC

Let B be the event of students opting for NSS.

$n(S)=50$

$P(A)=\frac{28}{50}$

$P(B)=\frac{30}{50}$

$P(A\cap B)=\frac{18}{50}$

i) $P(A\cap\bar{B})=\frac{10}{50}=\frac{1}{5}$

ii) $P(\bar{A}\cap B)=\frac{12}{50}=\frac{6}{25}$

iii) $P[(A\cap\bar{B})\cup(\bar{A}\cap B)]=\frac{11}{25}$

Q42. Find the equation of the line passing through $(22, -6)$ and having intercept on x-axis exceeds the intercept on y-axis by 5 units.

Solution:

Let equation of line be $\frac{x}{a} + \frac{y}{b} = 1$

Here y - intercept is b

x - intercept is a

Also again $a = b + 5$

So $\frac{x}{b+5} + \frac{y}{b} = 1$, it pass through $(22, -6)$

$\frac{22}{b+5} - \frac{6}{b} = 1 \Rightarrow 22b - b(b+5) = 6(b+5)$

$\Rightarrow b^2 + 5b = 22b - 6b - 30$

$\Rightarrow b^2 + 5b = 16b - 30$

$\Rightarrow b^2 - 11b + 30 = 0 \Rightarrow (b-5)(b-6) = 0$

$b = 5,6$

so $a = 10$ for $b = 5$

$a = 11$ for $b = 6$

equation of lines are $\frac{x}{10} + \frac{y}{5} = 1$

$\frac{x}{11} + \frac{y}{6} = 1$

$x + 2y = 10$ and $6x + 11y = 66$

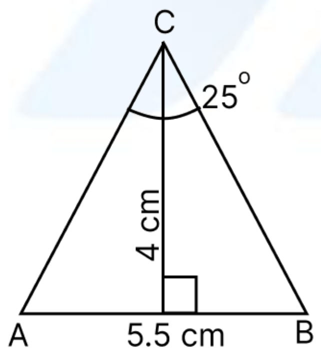
$x + 2y - 10$ and $6x + 11y - 66 = 0$

PART – IV

Q43. (a) Construct a $\triangle$ ABC such AB $= 5.5$ cm, C $= 25°$ and the altitude from C to AB is 4 cm.

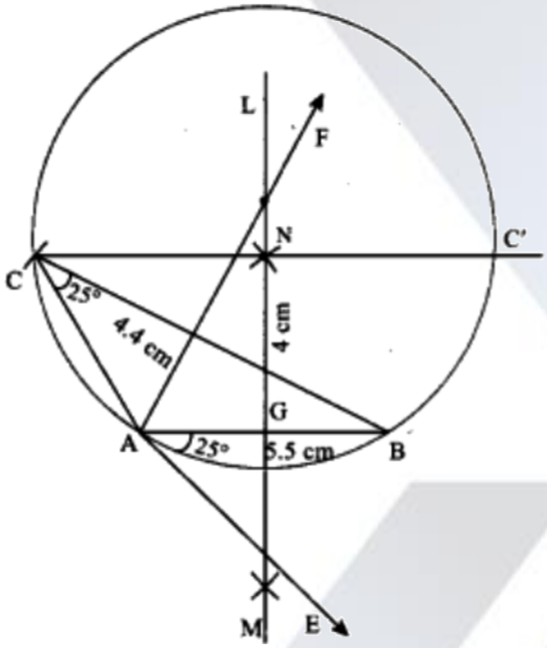
Solution:

Rough diagram:



Construction:

1. Draw $\overline{AB} = 5.5$ cm
2. Draw $\angle BAE = 25^{\circ}$
3. Draw $\angle FAE = 90^{\circ}$
4. Draw perpendicular bisector to AB.
5. The perpendicular bisector meets AF at O.
6. Draw a circle with O as centre and OA as radius.



(b) Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

Solution:

Steps of construction:

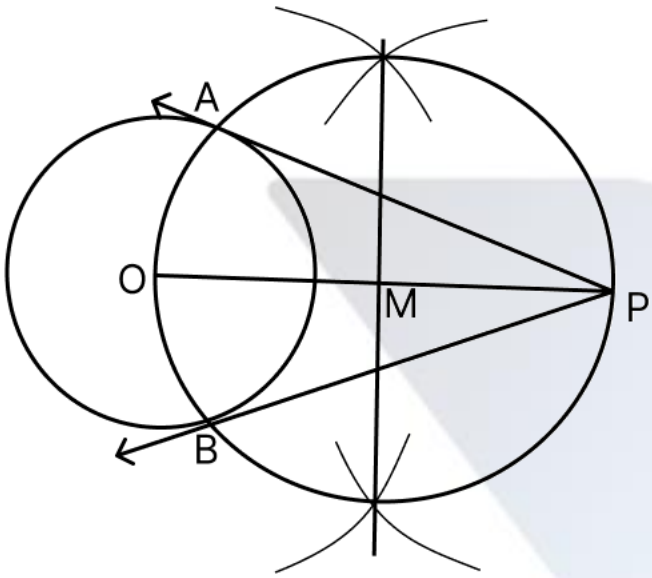
1. With O as centre, draw a circle of radius 3 cm.
2. Draw a line $OP = 5$ cm.
3. Draw a perpendicular bisector of OP, which cuts OP at M.
4. With M as centre and MO as radius, draw a circle which cuts the previous circles A and B.
5. Join AP and BP. AP and BP are the required tangents. Length of the tangents $PA = PB = 4$ cm

Verification: In the right-angle triangle OAP

$PA^2 = OP^2 - OA^2 = 5^2 - 3^2 = 25 - 9 = 16$

$PA = \sqrt{16} = 4$ cm

Length of the tangents $= 4$ cm.



Q44. (a) Draw the graph of $y = x^2 - 4x + 3$ and hence solve $x^2 - 6x + 9 = 0$.

Solution:

Step 1: Draw the graph of $y = x^2 - 4x + 3$ by preparing the table of values given below.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
x^2	16	9	4	1	0	1	4	9	16	25	36	49
$-4x$	16	12	8	4	0	-4	-16	-12	-16	-20	-24	-28
$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$	$+3$
y	35	24	15	8	$+3$	0	-9	0	3	8	15	24

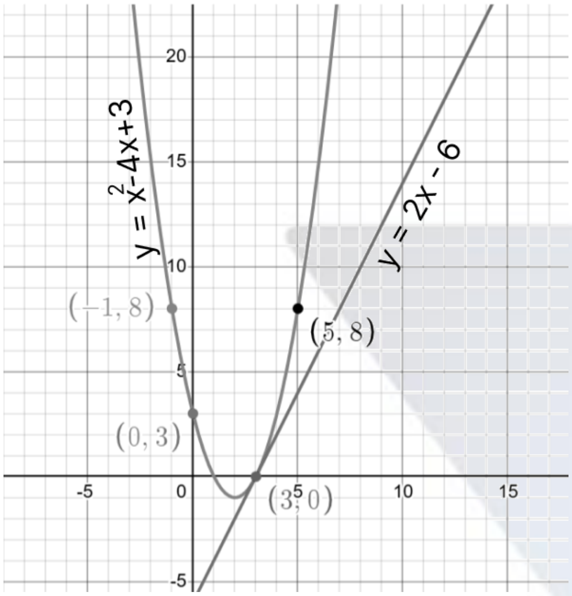
Step 2: Plot the points $(-3,24), (-2,15), (-1,8), (0,3), (1,0), (2,-9), (3,0), (4,8), (5,15)$ on the graph sheet using suitable scale.

Step 3: To solve $x^2 - 6x + 9 = 0$ subtract $x^2 - 6x + 9 = 0$ from $y = x^2 - 4x + 3$.

We get $y = 2x - 6$.

Step 4: $y = 2x - 6$ is a straight line.

x	-4	-2	0	2	3	4
y	-14	-10	-6	-2	0	2



Step 5: The straight line intersects the curve at (3,0).

Step 6: The solution set is 3.

(b) Draw the graph of $x^2 - 4x + 4 = 0$ and state the nature of their solution.

Solution:

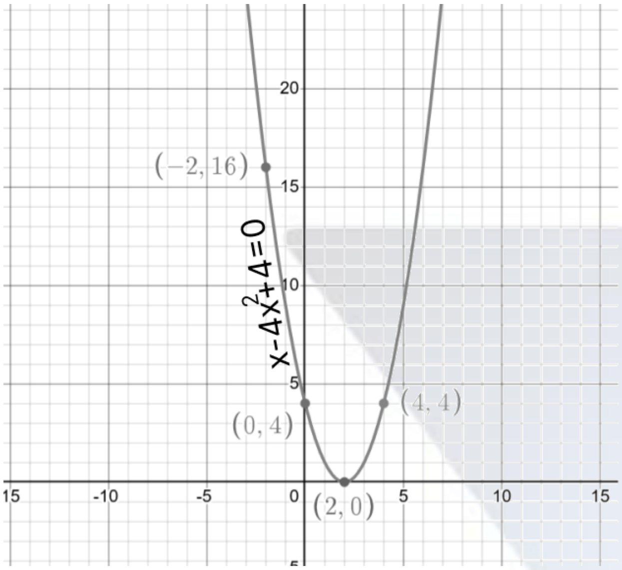
1. Prepare the table of values for $y = x^2 - 4x + 4$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-4	16	12	8	4	0	-4	-8	-12	-16
4	4	4	4	4	4	4	4	4	4
y	36	25	16	9	4	1	0	1	4

2. Plot the points $(-3,25)(-2,16)(-1,9)(0,4)(1,-1)(2,0)$, $(3,1)$ and $(4,4)$
3. Join the points by a free hand smooth curve.
4. The roots of the equation are the x-coordinates of the intersecting points of the curve with x-axis (2,0) which is 2.
5. Since there is only one point of intersection with the x-axis (2,0).

∴ The solution set is 2.

The Quadratic equation $x^2 - 4x + 4 = 0$ has real and equal roots.



x-4x^2+4=0
(−2, 16)
(0, 4)
(4, 4)
(2, 0)
20
15
10
5
-5
15
-10
-5
0
5
10
15