Grade 10 Mathematics Tamil Nadu 2023

PART – I

1. $A = \{a, b, p\}, B = \{2,3\}, C = \{p, q, r, s\}$ then $n\{(A \cup C) \times B\}$ is:

   (a) 8

   (b) 20

   (c) 1

   (d) 16

   Answer: (c)

   Solution:

   Here, $n(A) = 3,\ n(B) = 2,\ n(C) = 4$

   $n(AUC) = 6$

   Therefore, $n\{(A \cup C) \times B\} = 6 \times 2 = 12$

2. If $n(A) = p, n(B) = q$, then the total number of relations that exist from A to B is

   (a) 0

   (b) 1

   (c) $2^{pq} - 1$

   (d) 2^{pq}

   Answer: (d)

   Solution:

   The number of relations from a set A with m elements to a set B with n elements is given by 2^{mn}

   So given $n(A) = p$ and $n(B) = q$

   Then 2^{pq} relations are defined from A to B.

3. Given $F_1 = 1, F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then, F_5 is:

   (a) 3

   (b) 5

   (c) 8

   (d) 11

   Answer: (d)

   Solution:

   $F_1 = 1, F_2 = 3$

$F_n = F_{n-1} + F_{n-2}$

$F_5 = F_{5-1} + F_{5-2} = F_4 + F_3$

$= F_3 + F_2 + F_2 + F_1 = F_2 + F_1 + F_2 + F_2 + F_1$

$= 3 + 1 + 3 + 3 + 1 = 11$

4. If the sequence t_1, t_2, t_3 .... are in A.P., then the sequence t_6, t_{12}, t_{18} is:

   (a) a Geometric Progression

   (b) an Arithmetic Progression

   (c) neither an Arithmetic Progression nor a Geometric Progression

   (d) a constant sequence

   Answer: (b)

   Solution:

   Consider an AP, 1, 2, 3, 4, ...

   Here, $a = 1$ and $d = 1$

   where, t_1, t_2, t_3 ... ... are in A.P

   t_6 will be 6, t_{12} will be 12 and t_{18} will be 18

   Hence, the common difference is also the same.

5. $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is:

   (a) $\frac{9y}{7}$

   (b) $\frac{9y^3}{(21y-21)}$

   (C) $\frac{21y^2-42y+21}{3y^3}$

   (d) $\frac{7(y^2-2y+1)}{y^2}$

   Answer: (a)

   Solution:

   $$\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$$

   $$= \frac{3(y-1)}{y} \times \frac{3y^2}{7(y-1)}$$

   $$= \frac{9y}{7}$$

6. Graph of a Quadratic equation is a

(a) straight line

(b) circle

(c) parabola

(d) hyperbola

Answer: (c)

Solution:

Graph of a quadratic equation is a parabola.

So, the correct option is C.

7. If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$ then they will be similar, when:

(a) $\angle B = \angle E$

(b) $\angle A = \angle D$

(c) $\angle B = \angle D$

(d) $\angle A = \angle F$

Answer: (c)

Solution:

Given $\frac{AB}{DE} = \frac{BC}{FD}$

Now if the angle made by both the sides of each triangle are equal to each other then triangle $\triangle ABC$ and $\triangle DEF$ will become similar.

Hence $\angle B$ should be equal to $\angle D$

means $\angle B = \angle D$

8. A tangent of a circle is perpendicular to the radius at the:

(a) centre

(b) point of contact

(c) infinity

(d) chord

Answer: (b)

Solution:

Tangent at any point of a circle is perpendicular to the radius through the point of contact. This is the theorem. So, the correct option is b.

9. The slope of the straight line perpendicular to x-axis is:

(a) 1

(b) 0

(c) ∞

(d) -1

Answer: (c)

Solution:

A horizontal line has slope 0, because even though the value of x changes, the value of y does not change i.e. $\Delta y = 0$.

$$\frac{\Delta y}{\Delta x} = \frac{0}{\Delta x} = 0$$

Therefore, the slope of the x-axis is 0.

Now since the slope of the perpendicular line is the negative reciprocal to the slope of the x-axis.

Therefore, the slope of the line $(m) = \frac{1}{0} = \infty$ (undefined)

Hence, the slope of the line perpendicular to the x-axis is ∞ (undefined).

So, the correct option is c.

10. If $\sin\theta = \cos\theta$, then the value of $2\tan^2\theta + \sin^2\theta - 1$ is:

(a) $\frac{3}{2}$

(b) $\frac{-3}{2}$

(c) $\frac{2}{3}$

(d) $\frac{-2}{3}$

Answer: (a)

Solution:

$\sin\theta = \cos\theta$

$$\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$$

$\tan\theta = 1 = \tan 45^\circ$

$\theta = 45^\circ$

Therefore, $2\tan^2\theta + \sin^2\theta - 1 = 2\tan^2 45^\circ + \sin^2 45^\circ - 1$

$$= 2 \times (1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 + \frac{1}{2} - 1 = \frac{3}{2}$$

11. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be:

(a) 12 cm

(b) 10 cm

(c) 13 cm

(d) 5 cm

Answer: (a)

Solution:

Here $r = 5$ cm and $l = 13$ cm

$h = \sqrt{l^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12$ cm

12. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is:

(a) $1:2:3$

(b) $2:1:3$

(c) $1:3:2$

(d) $3:1:2$

Answer: (d)

Solution:

Volume of a cylinder $= \pi r^2 h$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Their ratio $V_1 : V_2 : V_3$

$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{4}{3}\pi r^3$

$= h : \frac{h}{3} : \frac{4r}{3}$

$= 3h : h : 4r$

$= 3h : h : 2(2r)$

$\therefore V_1 : V_2 : V_3 = 3:1:2 (\text{ where } 2r = h)$

13. If the sum and mean of a data are 407 and 11 respectively then the number of observations in the data are

(a) 37

(b) 4477

(c) 396

(d) 418

Answer: (a)

Solution:

Sum of the observations is 407.

Mean is 11.

We know,

$$Mean = \frac{Sum\ of\ the\ observations}{\text{Total number of observations}}$$

$$11 = \frac{407}{Total\ number\ of\ observations}$$

Total number of observations $= \frac{407}{11}$

Total number of observations $= 37$

Thus, the number of observations is 37.

14. If a letter is chosen at random from the English alphabets $\{a, b, \ldots \ldots, z\}$, then the probability that the letter chosen precedes x :

(a) $\frac{12}{13}$

(b) $\frac{1}{13}$

(c) $\frac{23}{26}$

(d) $\frac{3}{26}$

Answer: (c)

Solution:

Let S be the total number of alphabets in English.

$N(S) = 26$

Let A be the letter chosen precedes x.

$N(A) = 23$

So, $P(A) = \frac{N(A)}{N(S)} = \frac{23}{26}$

PART - II

15. If $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$ find A and B.

Solution:

$B \times A = \{(-2,3)(-2,4)(0,3)(0,4)(3,3)(3,4)\}$

$A = \{3,4\}$

$B = \{-2,0,3\}$ (By Ordered Pair)

16. Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$

Solution:

$f(f(k)) = 5$

$f(2k - 1) = 5$

$2(2k - 1) - 1 = 5$

$4k - 2 - 1 = 5$

$4k - 3 = 5$

$4k = 8$

$k = \frac{8}{4}$

$k = 2$

17. Find x so that $x + 6, x + 12$ and $x + 15$ are consecutive terms of a Geometric Progression.

Solution:

$G.P = x + 6, x + 12, x + 15$

In $G.P = \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{x + 12}{x + 6} = \frac{x + 15}{x + 12}$

$(x + 12)^2 = (x + 6)(x + 15)$

$x^2 + 24x + 144 = x^2 + 6x + 15x + 90$

$24x - 21x = 90 - 144$

$3x = -54$

$x = -\frac{54}{3} = -18$

$x = -18$

18. Simplify: $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$

Solution:

$= \frac{x + 2}{4y} \div \frac{x^2 - x - 6}{12y^2}$

$= \frac{x+2}{4y} \div \frac{(x-3)(x+2)}{12y^2}$

$= \frac{(x+2)}{4y} \times \frac{12y^2}{(x-3)(x+2)}$

$= \frac{3y}{x-3}$

19. Determine the nature of roots for the following quadratic equation.

$2x^2 - x - 1 = 0$

Solution:

The given quadratic equation is $2x^2 - x - 1 = 0$

Here, $a = 2, b = -1, c = -1$

$\therefore D = b^2 - 4ac$

$= (-1)^2 - (4 \times 2 \times -1)$

$= 1 - (-8) = 9$

Since $D > 0$, therefore roots of the given equation are real and distinct.

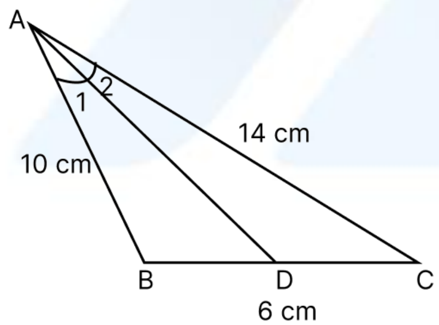
20. In the figure AD is the bisector of $\angle BAC$, if $AB = 10$ cm, $AC = 14$ cm and $BC = 6$ cm. Find BD and DC.

Solution:

It is given that AD bisects $\angle A$.

Applying angle - bisector theorem in $\triangle ABC$, we get:

$\frac{BD}{DC} = \frac{AB}{AC}$



Let BD be x cm.

Therefore, $DC = (6 - x)cm$

$\Rightarrow \frac{x}{6-x} = \frac{10}{14}$

$\Rightarrow 14x = 60 - 10x$

$\Rightarrow 24x = 60$

$\Rightarrow x = 2.5$ cm

Thus, $BD = 2.5$ cm

$DC = 6 - 2.5 = 3.5$ cm

21. A cat is located at the point $(-6, -4)$ in xy plane. A bottle of milk is kept at $(5, 11)$. The cat wishes to consume the milk travelling through the shortest possible distance. Find the equation of the path it needs to take the milk.

Solution:

Equation of the line joining the point is

Given points are $(-6, -4) = (x_1, y_1)$

$(5, 11) = (x_2, y_2)$

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\frac{y + 4}{11 + 4} = \frac{x + 6}{5 + 6}$$

$$\frac{y + 4}{15} = \frac{x + 6}{11}$$

$15(x + 6) = 11(y + 4)$

$15x + 90 = 11y + 44$

$15x - 11y + 90 - 44 = 0$

$15x - 11y + 46 = 0$

The equation of the path is $15x - 11y + 46 = 0$.

22. If the straight lines $12y = -(P + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find ' P '.

Solution:

$12y = -(p + 3)x + 12$

$$y = -\frac{p + 3}{12}x + \frac{12}{12}$$

$$y = -\frac{p + 3}{12}x + 1$$

$$\Rightarrow m_1 = -\frac{p + 3}{12}$$

$12x - 7y = 16$

$-7y = -12x + 16$

$$y = \frac{-12}{-7}x + \frac{16}{-7}$$

$$y = \frac{12}{7}x - \frac{16}{7}$$

$$\Rightarrow m_2 = \frac{12}{7}$$

Since (1) and (2) are perpendicular to each other

$$m_1 \times m_2 = -1$$

$$-\frac{p+3}{12} \times \frac{12}{7} = -1$$

$$-\left(\frac{p+3}{7}\right) = -1$$

$$\left(\frac{p+3}{7}\right) = 1$$

$$p + 3 = 7$$

$$p = 4$$

23. Prove that $\frac{\sec\ \theta}{\sin\ \theta} - \frac{\sin\ \theta}{\cos\ \theta} = \cot\ \theta$.

Solution:

$$\text{L.H.S} = \frac{\sec\ \theta}{\sin\ \theta} - \frac{\sin\ \theta}{\cos\ \theta}$$

$$= \frac{\frac{1}{\cos\theta}}{\sin\ \theta} - \frac{\sin\ \theta}{\cos\ \theta}$$

$$= \frac{1}{\sin\theta\ \cos\ \theta} - \frac{\sin\ \theta}{\cos\ \theta}$$

$$= \frac{1 - \sin^2\ \theta}{\sin\ \theta\cos\ \theta}$$

$$= \frac{\cos^2\ \theta}{\sin\ \theta\cos\ \theta}$$

$$= \frac{\cos\ \theta}{\sin\ \theta}$$

$$= \cot\ \theta$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence, proved

24. The radius of a conical tent is 7 m and height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m.

Solution:

A conical tent of radius 7 m and height 24 m is made from a rectangular canvas of width 4 m.

Here, C. S. A. (curved surface area) of the conical tent = Area of the rectangular canvas.

C. S. A. of the conical tent is given by, $\pi r l$

Where, r(radius) $= 7$ m, $h = 24$ m, l denotes the slant height.

For a cone, $l = \sqrt{h^2 + r^2}$, where l is the slant height.

Hence, $l = \sqrt{24^2 + 7^2}$

$\Rightarrow l = \sqrt{625}$

$\Rightarrow l = 25$ cm

So, the C. S. A. is $\pi r l$

$\Rightarrow \frac{22}{7} \times 7 \times 25$

$\Rightarrow 22 \times 25$

$\Rightarrow 550 \text{ m}^2$

Now, Area of the canvas = C. S. A. of the tent

Where, the canvas is rectangular so area will be length × breadth and the C. S. A. of the tent is 550 m^2

$\Rightarrow$ length $\times$ width $= 550 \text{ m}^2$

$\Rightarrow$ length $\times 4 = 550 \text{ m}^2$

$\Rightarrow$ length $= \frac{550}{4}$

$\Rightarrow$ length $= 137.5$ m

25. If the ratio of radii of two spheres is 4: 7, find the ratio of their volumes.

Solution:

Let the ratio of their radii is $r_1 : r_2$

$r_1 : r_2 = 4 : 7$

Ratio of their volumes $V_1 : V_2 = \frac{4}{3}\pi r_1^3 : \frac{4}{3}\pi r_2^3$

$= r_1^3 : r_2^3$

$= 4^3 : 7^3$

Ratio of their volumes $= 64 : 343$

26. Find the range and co-efficient of range of the following data.

63, 89, 98, 125, 79, 108, 117, 68

Solution:

Range $R = L - S$.

Co-efficient of range $= \frac{L-S}{L+S}$

L - Largest value,

S- Smallest value.

63, 89, 98, 125, 79, 108, 117, 68

Here $L = 125$

$S = 63$

$\therefore R = L - S = 125 - 63 = 62$

Co-efficient of range $= \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{188} = 0.33$

27. A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that probability of B being selected is at the most 0.8.

Solution:

Given, $P(A) = 0.5$ and $P(A \cap B) \leq 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow P(A) + P(B) - P(A \cup B) \leq 0.3$

$\Rightarrow P(B) \leq 0.3 + P(A \cup B) - P(A)$

$\Rightarrow P(B) \leq 0.3 + P(A \cup B) - 0.5$

$\Rightarrow P(B) \leq P(A \cup B) - 0.2$

[Since, $P(A \cup B) \leq 1 \Rightarrow P(A \cup B) - 0.2 \leq 0.8$]

$\therefore P(B) \leq 0.8 \Rightarrow P(B)$ cannot be at most 0.8.

28. If $p^2 \times q^1 \times r^4 \times s^3 = 3{,}15{,}000$ then find p, q, r and s.

Solution:

$3{,}15{,}000 = 5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 5^4 \times 2^3 \times 3^2 \times 7^1$

Therefore, the value of $p = 3, q = 7, r = 5, s = 2$

PART – III

29. Let $f: A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2,4,6,10,12\}$, $B = \{0,1,2,4,5,9\}$ Represent f by:

(i) set of ordered pairs

(ii) a table

(iii) an arrow diagram

(iv) a graph

Solution:

$A = \{2,4,6,10,12\}, B = \{0,1,2,4,5,9\}$

$f(x) = (\frac{x}{2}) - 1$

$f(2) = (\frac{2}{2}) - 1 = 1 - 1 = 0$

$f(4) = (\frac{4}{2}) - 1 = 2 - 1 = 1$

$f(6) = (\frac{6}{2}) - 1 = 3 - 1 = 2$

$f(10) = (\frac{10}{2}) - 1 = 5 - 1 = 4$

$f(12) = (\frac{12}{2}) - 1 = 6 - 1 = 5$

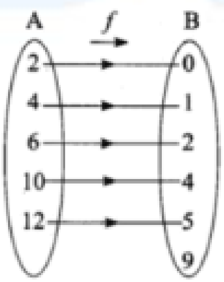
(i) Set of ordered pairs

$f = \{(2,0)(4,1)(6,2)(10,4)(12,5\}$

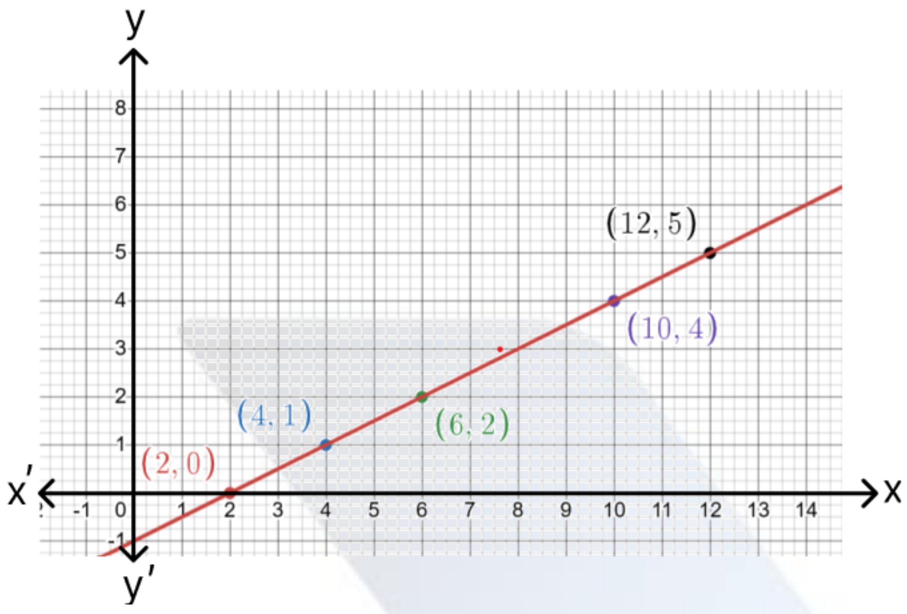
(ii) Table

x	2	4	6	10	12
(x)	0	1	2	4	5

(iii) Arrow diagram



(iv) Graph



30. The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number.

Solution:

Let there be a value of x such that the sum of the number of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it

House $H_1, H_2, H_3 \ldots \ldots .. H_{x-1}, H_{x+1} \ldots \ldots .. H_{49}$

House No. 1, 2, 3, ... $x-1,\ x+1$ 49

House number will form an A.P whose first term is 1 and the common difference is 1

Sum of n terms $S = \frac{n}{2}[2a + (n-1) \times d]$

$S_{x-1} = S_{49} - S_x$

$$\Rightarrow \frac{(x-1)}{2}[2(1) + (x-1-1)1] = \frac{49}{2}[2+48] - \frac{x}{2}[2(1) + (x-1)1]$$

$$\Rightarrow \frac{x-1}{2}[2 + (x-2)] = \frac{49}{2}[50] - \frac{x}{2}[2 + x - 1]$$

$$\Rightarrow \frac{x-1}{2}[x] = \frac{49}{2}[50] - \frac{x}{2}[x+1]$$

$$\Rightarrow \frac{x}{2}[x - 1 + x + 1] = \frac{49}{2}[50]$$

$$\Rightarrow \frac{x}{2}[2x] = 49 \times 25 \Rightarrow x^2 = 49 \times 25 \Rightarrow x = 7 \times 5 = 35$$

Therefore, Senthil's House's number is 35.

Since x is not a fraction hence the value of x satisfying the given condition exists and is equal to 35.

31. Find the sum to n terms of the series $5 + 55 + 555 + \cdots$

Solution:

$5 + 55 + 555 + \cdots$

$\Rightarrow S_n = \frac{5}{9}(9 + 99 + 999 + \cdots \text{ to } n \text{ terms })$

$\Rightarrow S_n = \frac{5}{9}\{(10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots + (10^n - 1)\}$

$\Rightarrow S_n = \frac{5}{9}\{(10 + 10^2 + 10^3 + \cdots + 10^n) - n\}$ ....(1)

We solve $(10 + 10^2 + 10^3 + \cdots n\ terms)$ separately,

We can observe that this is a GP, where $a = 10$ and $r = 10$

We know that sum of n terms = $\frac{a(r^n-1)}{r-1}$ (As r > 1)

Putting value of a and r, we get

$$\frac{10(10^n - 1)}{10 - 1}$$

Substitute $\frac{10(10^n-1)}{10-1}$ in (1), we get

$$\Rightarrow S_n = \frac{5}{9}\left\{10\left(\frac{10^n - 1}{10 - 1}\right) - n\right\}$$

$$\Rightarrow S_n = \frac{5}{9}\left\{\frac{10}{9}(10^n - 1) - n\right\}$$

$$= \frac{5}{81}(10^{n+1} - 9n - 10)$$

32. Solve the following system of linear equations in three variables.

$$x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$$

Solution:

$$x + 20 = \frac{3y}{2} + 10$$

Multiply by 2

$2x + 40 = 3y + 20$

$2x - 3y = -40 + 20$

$2x - 3y = -20$ ....(1)

$$\frac{3y}{2} + 10 = 2z + 5$$

Multiply by 2

$3y + 20 = 4z + 10$

$3y - 4z = 10 - 20$

$3y - 4z = -10$ ....(2)

$2z + 5 = 110 - (y + z)$

$2z + 5 = 110 - y - z$

$y + 3z = 110 - 5$

$y + 3z = 105$ ....(3)

$(3) \times (3) \Rightarrow 3y + 9z = 315$

$(2) \times (1) \Rightarrow 3y - 4z = -10$

$(3) - (2) \Rightarrow 13z = 325z = \frac{325}{13} = 25$

Substitute the value of $z = 25$ in (2)

$3y - 4(25) = -10$

$3y - 100 = -10$

$3y = -10 + 100$

$3y = 90\ y = \frac{90}{3} = 30$

Substitute the value of $y = 30$ in (1)

$2x - 3(30) = -20$

$2x - 90 = -20$

$2x = -20 + 90$

$2x = 70$

$x = \frac{70}{2} = 35$

∴ The value of $x = 35, y = 30$ and $z = 25$

33. If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix} \cdot B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ verify that $(AB)^T = B^T A^T$

Solution:

$A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

To prove: $(AB)^T = B^T A^T$

$(AB) = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

$= \begin{bmatrix} (5 + 2 + 45) & (35 + 4 - 9) \\ (1 + 2 + 40) & (7 + 4 - 8) \end{bmatrix} = \begin{bmatrix} 52 & 30 \\ 43 & 3 \end{bmatrix}$

$$(AB)^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix} \cdot A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} (5+2+45) & (1+2+40) \\ (35+4-9) & (7+4-8) \end{bmatrix}$$

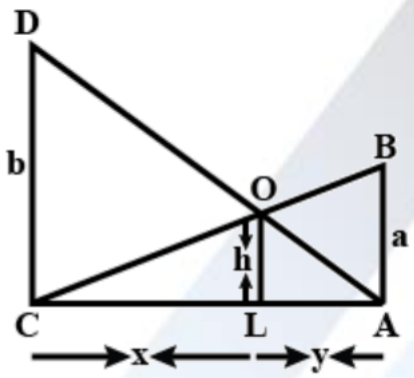
$$= \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix} \ldots (2)$$

$(1) = (2) \Rightarrow$ L.H.S. $=$ R.H.S. Verified.

34. Two poles of height ' a ' metres and ' b ' metres are ' p ' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres.

Solution:

Let AB and CD be two poles of heights a metres and b metres respectively such that the poles are p metres apart i.e. AC $=$ p metres. Suppose the lines AD and BC meet at O such that $OL = h$ metres.



Let $CL = x$ and $LA = y$. Then, $x + y = p$.

In $\triangle ABC$ and $\triangle LOC$, we have

$\angle CAB = \angle CLO$ [Each equal to 90°]

$\angle C = \angle C$ [Common]

$\therefore \triangle CAB \sim \triangle CLO$ [By AA-criterion of similarity]

$$\Rightarrow \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{p}{x} = \frac{a}{h} \Rightarrow x = \frac{ph}{a}$$

In $\triangle ALO$ and $\triangle ACD$, we have

$\angle ALO = \angle ACD$ [Each equal to 90°]

$\angle A = \angle A$[Common]

$\therefore \triangle ALO \sim \triangle ACD [ByAA -$ criterion of similarity]

$\Rightarrow \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{y}{p} = \frac{h}{b}$

$\Rightarrow y = \frac{ph}{b} [\because AC = x + y = p]$

From (i) and (ii), we have

$x + y = \frac{ph}{a} + \frac{ph}{b}$

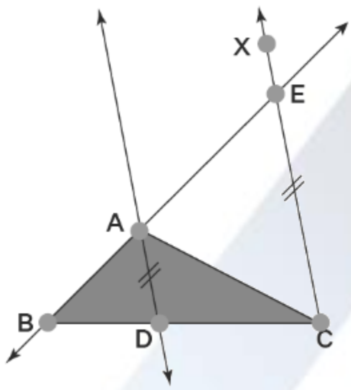
$\Rightarrow p = ph\left(\frac{1}{a} + \frac{1}{b}\right) [\because x + y = p] \Rightarrow 1 = h\left(\frac{a+b}{ab}\right) \Rightarrow h = \frac{ab}{a+b}$ metres

Hence, the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is $\frac{ab}{a+b}$ metres.

35. State and prove angle bisector theorem.

Solution:

Statement: In a triangle, the angle bisector of any angle will divide the opposite side in the ratio of the sides containing the angle. Let us see the proof of this.



Draw a ray CX parallel to AD, and extend BA to intersect this ray at E.

By the basic proportionality theorem, we have that if a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

In ΔCBE, DA is parallel to CE.

$\frac{BD}{DC} = \frac{BA}{AE} \cdots (1)$

Now, we are left with proving that $AE = AC$.

Let's mark the angles in the above figure.

Since DA is parallel to CE, we have

$\angle DAB = \angle CEA$ (corresponding angles) ----- (2)

$\angle DAC = \angle ACE$ (alternate interior angles) ----- (3)

Since AD is the bisector of $\angle BAC$, we have $\angle DAB = \angle DAC$ ---- (4).

From (2), (3), and (4), we can say that $\angle CEA = \angle ACE$. It makes ΔACE an isosceles triangle. Since sides opposite to equal angles are equal, we have $AC = AE$.

Substitute AC for AE in equation (1).

$\frac{BD}{DC} = \frac{BA}{AC}$

Hence proved.

36. Find the area of the quadrilateral formed by points (8,6), (5,11), (−5,12) and (−4,3).

Solution:

We have to find the area of the quadrilateral that is formed by the points (8,6), (5,11), (−5,12), and (−4,3)

Let the vertices of the quadrilateral be

$A(x_1, y_1) = A(8,6)$

$B(x_{2'}, y_2) = B(5,11)$

$C(x_3, y_3) = C(-5,12)$

$D(x_4, y_4) = D(-4,3)$

As we know that the area of the quadrilateral having vertices is given by the following formula:

Area of the quadrilateral

$$ABCD = \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\}$$

Now, putting these values in the formula we get;

Area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{((8)(11) + (5)(12) + (-5)(3) + (-4)(6)) - ((5)(6) + (-5)(11) + (-4)(12) + (8)(3))$$

$$= \frac{1}{2}\{(88 + 60 - 15 - 24) - (30 - 55 - 48 + 24)\}$$

$$= \frac{1}{2}\{(109) - (-49)\}$$

$$= \frac{1}{2} \times 158 = 79 \text{ sq. units}$$

Hence, the area of the quadrilateral is 79 sq. units.

37. Find the equation of a straight line parallel to x-axis and passing through the point of intersection of the lines $7x - 3y = -12$ and $2y = x + 3$.

Solution:

The given equations are.

$7x-3y=12\ldots(i)$

$x-2y=-3\ldots(ii)$

Multiply in (i) by 2 and (ii) by 3

$14x-6y=24$

$-(3x-6y=-9)$

Solving these by eliminating y, we get, $\mathrm{x}=3$,

Putting in the original equation no(ii)

$3-2y=-3$

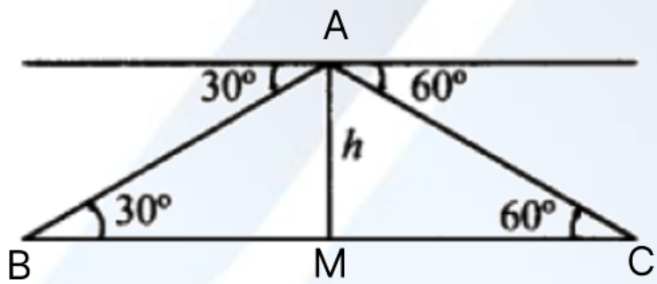
$2y=6$

$y=3$

Now the equation of the line is parallel to x-axis and passing through $(3,3)$ will be given by $y=3$.

38. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is ' h ' metres and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}$ m.

Solution:



Let A be the point of observation from the lighthouse B and C be the respective positions of two ships

According to figure, in $\triangle ABM$

$$\tan 30^{\circ}=\frac{h}{BM}$$

$$\frac{1}{\sqrt{3}}=\frac{h}{BM}$$

$$BM=\sqrt{3}h\ldots(i)$$

In $\triangle ACM$,

$$\tan\ 60^\circ = \frac{h}{MC}$$

$$\sqrt{3} = \frac{h}{MC}$$

$$MC = \frac{h}{\sqrt{3}} \cdots$$

The distance between two o ships is $BM + CM$

$$= \frac{\sqrt{3}h}{1} + \frac{h}{\sqrt{3}} = \frac{3h + h}{\sqrt{3}} = \frac{4h}{\sqrt{3}}m$$

So, the distance between the ships is $\frac{4h}{\sqrt{3}}m$.

39. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500sq, cm. Find its radius and height.

Solution:

Let the radius be $5x$ and the height be $7x$

Curved surface area of a cylinder $= 5500$ sq.cm .

Curved surface area of a cylinder of a cylinder $= 2\pi r$ h

$$2\pi rh = 5500$$

$$2 \times \frac{22}{7} \times 5x \times 7x = 5500$$

$$2 \times 22 \times 5 \times x^2 = 5500$$

$$x^2 = 25 \text{ cm}$$

$$x = 5 \text{ cm}$$

Radius of the cylinder $= 5 \times 5 = 25$ cm

Height of the cylinder $= 7 \times 5 = 35$ cm

40. Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person requires 4 sq.m. of the space on ground and 40 cu. meter of air to breathe. Find the height of the conical part of the tent if the height of cylindrical part is 8 m.

Solution:

Since, each person is required (occupies) 4 m^2 of space on ground.

$\therefore$ Area of base $= 150 \times$ Required (area for one person)

$$= 150 \times 4 \text{ m}^2 = 600 \text{ m}^2$$

$$\therefore \quad \pi r^2 = 600$$

Now, the volume of formed tent = volume of required air inside the tent $= 150 \times$ volume of required air to breath for each person

$$= 150 \times 40\, m^3 = 6000 m^3$$

$$\therefore \pi r^2 \left(8 + \frac{h}{3}\right) = 6000$$

$$\Rightarrow 8 + \frac{h}{3} = \frac{6000}{\pi r^2} = \frac{6000}{600} = 10$$

$$\Rightarrow \frac{h}{3} = 10 - 8 = 2$$

$$\Rightarrow h = 6 \text{ m}$$

Hence, the height of the conical part of the tent is 6 m.

41. Two unbiased dice are rolled once. Find the probability of getting :

(i) a doublet (equal numbers on both dice)

(ii) the product as a prime number

(iii) the sum as a prime number

(iv) the sum as 1

Solution:

(i) Doublet $= \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}$

Total number of outcomes $= 6 \times 6$

$n(S) = 36$

Number of favourable outcomes $= 6$

$$P(\text{ doublet }) = \frac{6}{36} = \frac{1}{6}$$

(ii) Favourable outcomes $= (1,2), (2,1), (1,3), (3,1), (1,5)$, and $(5,1)$

Number of favourable outcomes $= 6$

P(prime number as product) $= \frac{6}{36} = \frac{1}{6}$

(iii) Sum as prime numbers $= \{(1,1), (1,2), (2,3), (1,4), (1,6), (4,3), (5,6)\}$

Number of favourable outcomes $= 7$

$\Rightarrow$ Probability $= \frac{7}{36}$

(iv) With two dice, minimum sum possible $= 2$

$\therefore$ Prob (sum as 1) $= 0$ [Impossible event]

42. Let $A = \{x \in W / x < 3\}, B = \{x \in N / 1 < x \leq 5\}$ and $C = \{3,5,7\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Solution:

$A = \{0,1,2\}, B: \{2,3,4,5\}, C = \{3,5,7\}$
$B \cup C = \{2,3,4,5,7\}$
$A \times (B \cup C) =$
$\{(0,2)(0,3)(0,4)(0,5)(0,7)(1,2)(1,3)(1,4)(1,5)(1,7)(2,2)(2,3)(2,4)(2,5)(2,7)\}$
... (i)
$(A \times B) = \{(0,2)(0,3)(0,4)(0,5)(1,2)(1,3)(1,4)(1,5)(2,2)(2,3)(2,4)(2,5)\}$
$(A \times C) = \{(0,3)(0,5)(0,7)(1,3)(1,5)(1,7)(2,3)(2,5)(2,7)\}$
$(A \times B) \cup (A \times C) =$
$\{(0,2)(0,3)(0,4)(0,5)(0,7)(1,2)(1,3)(1,4)(1,5)(1,7)(2,2)(2,3)(2,4)(2,5)(2,7)\}$

From (i) and (ii)

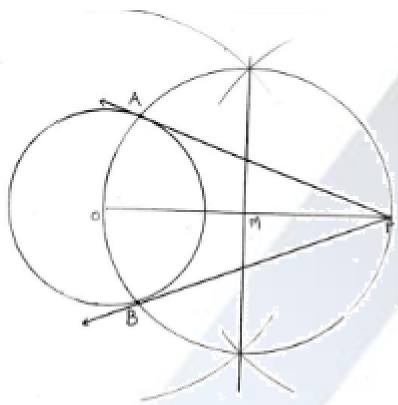
$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence Proved

PART - IV

43. (a) Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw two tangents to the circle from that point.

Solution:



Steps of construction:

1. With O as centre, draw a circle of radius 4 cm.

2. Draw a line $OP = 11$ cm.

3. Draw a perpendicular bisector of OP, which cuts OP at M.

4. With M as centre and MO as radius, draw a circle which cuts the previous circles A and B.

5. Join AP and BP. AP and BP are the required tangents. This the length of the tangents $PA = PB = 10.2$ cm

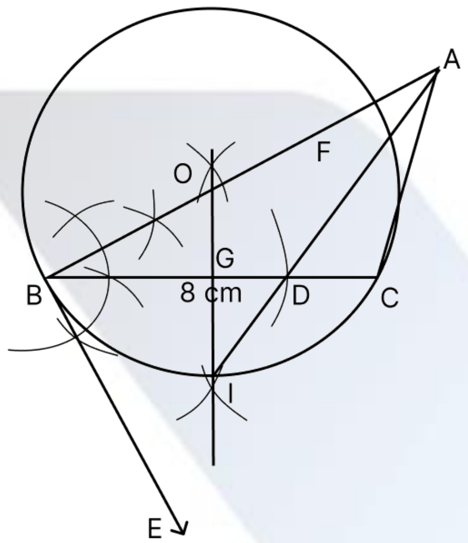
Verification: In the right-angle triangle OAP

$PA^2 = OP^2 - OA^2 = 11^2 - 4^2 = 121 - 16 = 105$
$PA = \sqrt{105} = 10.2$ cm

Length of the tangents $= 10.2$ cm

(b) Draw a triangle ABC of base $BC = 8$ cm, $\angle A = 60°$ and the bisector of $\angle A$ meets BC at D such that $BD = 6$ cm.

Solution:



Steps of construction:

1 Draw a line segment $BC = 8$ cm.

2 At B draw BE such that $\angle CBE = 40°$.

3 At B draw BF such that $\angle EBF = 90°$.

4 Draw the perpendicular bisector to BC which intersects BF at O and BC at G.

5 With O as centre and OC as radius draw a circle.

6 From B mark an arc of 6 cm on CB at D.

7 The perpendicular bisector intersects the circle at I. Joint ID.

8 ID produced meets the circle at A. Now Join AB and AC.

This $\triangle ABC$ is the required triangle.

44. (a) Varshika drew 6 circles with different sizes. 'Draw a graph for the relationship between the diameter and circumference (approximately related) of each circle as shown in the table and use it to find the circumference of a circle when its diameter is 6 cm.

Diameter (x)cm	1	2	3	4	5
Circumference (y)cm	3.1	6.2	9.3	12.4	15.5

Solution:

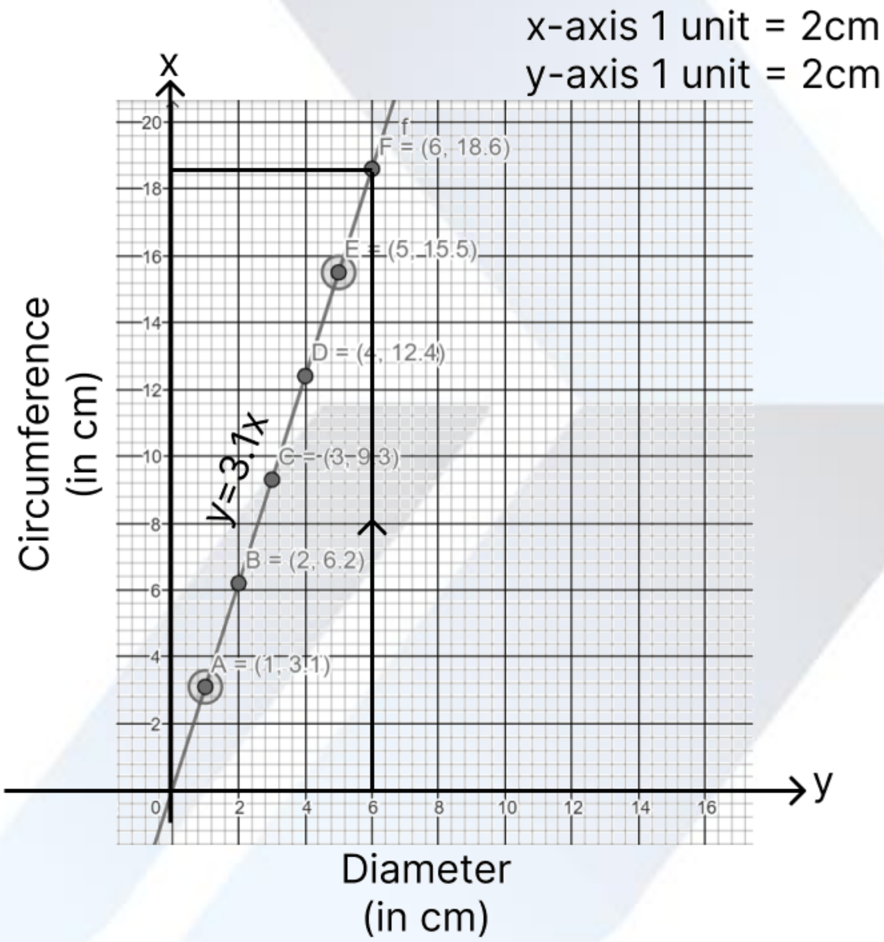
(a) From the table, we found that as x increses, y also increases. Thus, the variation is a direct variation.

Let $y = kx$, where k is a constant of proportionality. From the given values, we have,

$$k = \frac{3.1}{1} = \frac{6.2}{2} = \frac{9.3}{3} = \frac{12.4}{4} = \cdots = 3.1$$

When you plot the points $A(1, 3.1), \mathrm{B}(2, 6.2), \mathrm{C}(3, 9.3), \mathrm{D}(4, 12.4), \mathrm{E}(5, 15.5)$, you find the relation $y = (3.1)x$ which forms a straight-line graph.

Clearly, from the graph, when diameter is 6 cm, its circumference is 18.6 cm.



(b) Draw the graph of $y = x^2 - 5x - 6$ and hence solve $x^2 - 5x - 14 = 0$.

Solution:

(i) Values of the y for various values of x are shown in the following table:

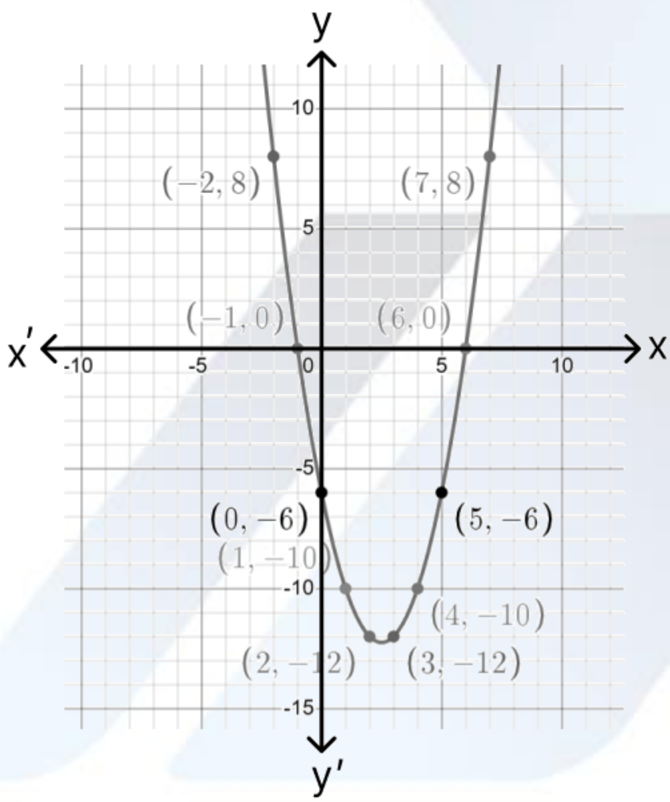
x	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
x^2	16	9	4	1	0	1	4	9	16	25	36	49	64

$-5x$	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40
-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6	-6
y	30	18	8	0	-6	-10	-12	-12	-10	-6	0	8	18

(ii) Plot the points

$(-3,18), (-2,8), (-1,0), (0,-6), (1,-10), (2,-12), (3,-12), (4,-10), (5,-6), (6,0)$ and $(7,8)$.

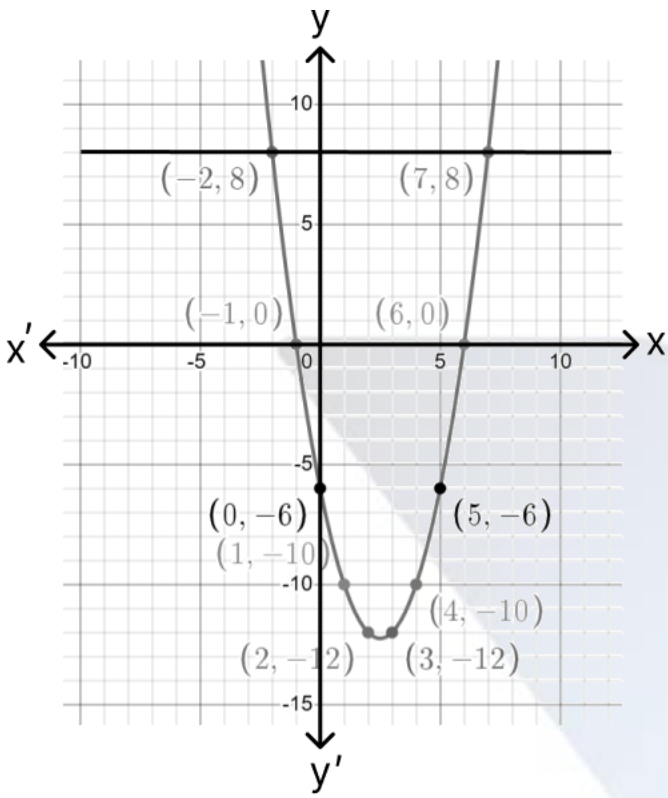
(iii) Join the points by a free hand to get smooth curve.



(iv) To solve $x^2 - 5x - 14 = 0$, subtract $x^2 - 5x - 14 = 0$ from $y = x^2 - 5x - 6$

The equation $y = 8$ represent a straight line draw a straight line through $y = 8$ intersect the curve at two places.

From the two points draw a perpendicular line to the X-axis it will intersect at -2 and 7.



The solution is $x = -2$ and 7