Grade 10 Telangana Maths 2016 Paper - I

PART A

SECTION – I

Q1. Find the value of $\log_5 125$.

Solution:

Let $\log_5 125 = x$

$\Rightarrow 5^x = 125$

$\Rightarrow 5^x = 5^3$

$\Rightarrow x = 3$

Therefore, $\log_5 125 = 3$

Q2. If $A = \{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}\}$, then write A in set-builder form.

Solution:

$A = \{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}\}$

$= \left\{\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \frac{1}{5^2}\right\}$

The set-builder form: $A = \left\{\frac{1}{x^2} : x \leq 5, x \in N\right\}$

Q3. Write an example for a quadratic polynomial that has no zeroes.

Solution:

$x^2 + x + 11$ is one of the polynomials which do not have zeroes.

Q4. If $b^2 - 4ac > 0$ in $ax^2 + bx + c = 0$, then what can you say about roots of the equation? $(a \neq 0)$

Solution:

Given,

$ax^2 + bx + c = 0$

And

$b^2 - 4ac > 0$

Hence, the roots of the equation are real and distinct.

Q5. Find the sum of the first 200 natural numbers.

Solution:

First 200 natural numbers: 1, 2, 3, 4, ...,200

$n = 200$

We know that

Sum of first n natural numbers $= \frac{n(n+1)}{2}$

Sum of the first 200 natural numbers $= \frac{200(200+1)}{2}$

$= 100 \times 201$

$= 20100$

Q6. For what values of m, the pair of equations $3x + my = 10$ and $9x + 12y = 30$ have a unique solution.

Solution:

Given.

$3x + my = 10$

$9x + 12y = 30$

Comparing with the standard form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

$a_1 = 3,\ b_1 = m, c_1 = -10$

$a_2 = 9,\ b_2 = 12, c_2 = -30$

Condition for unique solution

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{3}{9} \neq \frac{m}{12}$

$\Rightarrow m \neq \frac{12}{3}$

$\Rightarrow m \neq 4$

Hence, m takes all the real values except 4.

Q7. Find the midpoint of the line segment joining the points $(-5, 5)$ and $(5, -5)$.

Solution:

Let the given points be:

$(x_1, y_1) = (-5, 5)$

$(x_2, y_2) = (5, -5)$

Midpoint $= \left[\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right]$

$= \left[\frac{-5+5}{2}, \frac{5-5}{2}\right]$

$= \left(\frac{0}{2}, \frac{0}{2}\right)$

$= (0,0)$

SECTION - II

Q8. If $x^2 + y^2 = 7xy$, then show that $2\log(x+y) = \log x + \log y + 2\log 3$.

Solution:

Given,

$x^2 + y^2 = 7xy$

Adding $2xy$ on both the sides,

$x^2 + y^2 + 2xy = 7xy + 2xy$

$(x+y)^2 = 9xy$

$(x+y)^2 = (3)^2 xy$

Taking log on both sides,

$\log(x+y)^2 = \log(3)^2 xy$

$2\log(x+y) = \log 3^2 + \log x + \log y$

$2\log(x+y) = 2\log 3 + \log x + \log y$

Q9. Length of a rectangle is 5 units more than its breadth. Express its perimeter in polynomial form.

Solution:

Let the breadth of a rectangle be x.

Length $= (x + 5)$ units

Perimeter of rectangle $=$ 2 (Length + Breadth)

$= 2(x + 5 + x)$

$= 2(2x + 5)$

$= 4x + 10$

Q10. Measures of sides of a triangle are in Arithmetic Progression. Its perimeter is 30 cm and the difference between the longest and shortest side is 4 cm, then find the measures of the sides.

Solution:

Let $a - d, a, a + d$ be the measures of three sides of a triangle.

According to the given,

Perimeter $= 30$ cm

$a - d + a + a + d = 30$

$3a = 30$

$a = \frac{30}{3}$

$a = 10$

Also,

$a + d - (a - d) = 4$

$2\,d = 4$

$d = \frac{4}{2}$

$d = 2$

Thus, $a - d = 10 - 2 = 8$

$a + d = 10 + 2 = 12$

Hence, the measures of the triangle are 8 cm, 10 cm and 12 cm.

Q11. Show that the points $A(-3, 3), B(0, 0), C(3, -3)$ are collinear.

Solution:

If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are collinear, then the area of the triangle formed by these vertices is 0.

That means $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

Given,

$A(-3, 3), B(0, 0), C(3, -3)$

Area of triangle ABC $= \frac{1}{2}[-3(0 + 3) + 0(-3 - 3) + 3(3 - 0)]$

$= \frac{1}{2}[-9 + 0 + 9]$

$= \frac{1}{2}(0)$

$= 0$

Hence, the given points are collinear.

Q12. Solve the following pair of linear equations by substitution method.

$2x - 3y = 19$

$3x - 2y = 21$

Solution:

Given,

$2x - 3y = 19$ ......(i)

$3x - 2y = 21$ ......ii)

From (i),

$2x - 3y = 19$

$2x = 3y + 19$

$x = \frac{3y+19}{2}$ ......iii)

Substituting (iii) in (ii),

$3\left[\frac{3y + 19}{2}\right] - 2y = 21$

$9y + 57 - 4y = 42$

$5y = 42 - 57$

$5y = -15$

$y = -\frac{15}{5}$

$y = -3$

Substituting y $= -3$ in (iii),

$x = \frac{[3(-3) + 19)]}{2}$

$= \frac{-9 + 19}{2}$

$= \frac{10}{2}$

$= 5$

Hence, $x = 5$ and $y = -3$ is the solution of the given pair of linear equations.

Q13. If $9x^2 + kx + 1 = 0$ has equal roots, find the value of k.

Solution:

Given,

$9x^2 + kx + 1 = 0$

Comparing with the standard form $ax^2 + bx + c = 0, a = 9, b = k, c = 1$

Condition for equal roots:

$b^2 - 4ac = 0$

$k^2 - 4(9)(1) = 0$

$k^2 - 36 = 0$

$k^2 = 36$

$k = \sqrt{36}$

$k = \pm 6$

SECTION - III

Q14. Use Euclid's division lemma to show that the cube of any positive integer is of the form $7m$ or $7m + 1$ or $7m + 6$.

Solution:

Let a be any positive integer and $b = 7$.

By Euclid's division lemma,

$a = bq + r, 0 \leq r < b$

$a = 7q + r; r = 0, 1, 2, 3, 4, 5, 6$

When $r = 0$,

$a = 7q$

$a^3 = (7q)^3$

$a^3 = 343q^3$

$a^3 = 7(49q^3)$

$a^3 = 7\,m$, where $m = 49q^3$

Also, in $(7q + r)^3$, consider r^3 and divide by 7. The remainder will give the result in each case.

When $r = 1$,

$1^3 = 1$ and $a^3 = 7\,m + 1$

When $r = 2$,

$2^3 = 8$, divided by 7, the remainder is 1. Therefore, $a^3 = 7\,m + 1$

When $r = 3$,

$3^3 = 27$ divided by 7, the remainder is 6. Therefore, $a^3 = 7\,m + 6$

When $r = 4$,

$4^3 = 64$ divided by 7, the remainder is 1. Therefore, $a^3 = 7\,m + 1$

When $\mathrm{r} = 5$,

$5^3 = 125$ divided by 7, the remainder is 6. Therefore, $a^3 = 7\,m + 6$

When $r = 6$,

$6^3 = 216$ divided by 7, the remainder is 6 . Therefore, $a^3 = 7\,m + 6$

Hence, the cube of any positive integer is of the form $7\,m$ or $7\,m + 1$ or $7\,m + 6$.

OR

Prove that $\sqrt{2} - 3\sqrt{5}$ is an irrational number.

Solution:

Let $\sqrt{2} - 3\sqrt{5}$ be a rational number.

$\sqrt{2} - 3\sqrt{5} = a$, where a is an integer.

Squaring on both sides,

$(\sqrt{2}-3\sqrt{5})^2 = a^2$

$(\sqrt{2})^2 + (3\sqrt{5})^2 - 2(\sqrt{2})(3\sqrt{5}) = a^2$

$2 + 45 - 6\sqrt{10} = a^2$

$47 - 6\sqrt{10} = a^2$

$-6\sqrt{10} = a^2 - 47$

$$\sqrt{10} = \frac{(47 - \mathrm{a}^2)}{6}$$

$\frac{(47-a^2)}{6}$ is a rational number since a is an integer.

Therefore, $\sqrt{10}$ is also rational.

We know that $\sqrt{10}$ is not rational numbers.

Thus, our assumption that $\sqrt{2} - 3\sqrt{5}$ is a rational number is wrong.

Hence, $\sqrt{2} - 3\sqrt{5}$ is an irrational number.

Q15. Draw the graph for the polynomial $p(x) = x^2 - 3x + 2$ and find the zeroes from the graph.

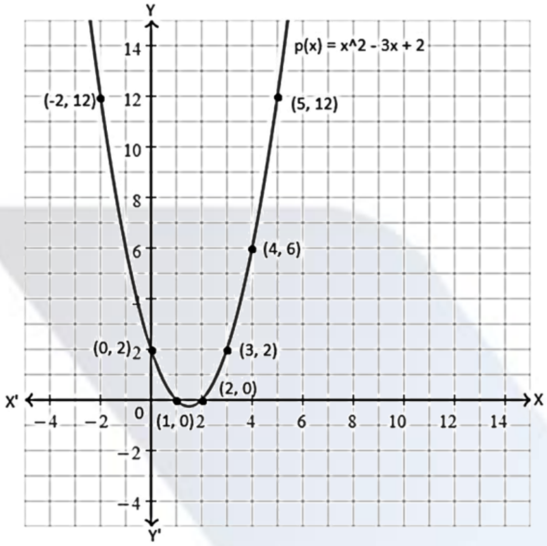
Solution:

Given,

$p(x) = x^2 - 3x + 2$

Let $y = x^2 - 3x + 2$

x	-2	0	1	2	3	4	5
$Y = p(x)$	12	2	0	0	2	6	12



OR

Draw the graph for the following pair of linear equations in two variables and find their solution from the graph. $3x - 2y = 2$ and $2x + y = 6$

Solution:

Given,

$3x - 2y = 2$

$2x + y = 6$

Consider the first equation:

$3x - 2y = 2$

$2y = 3x - 2$

$y = \left(\frac{3}{2}\right)x - 1$

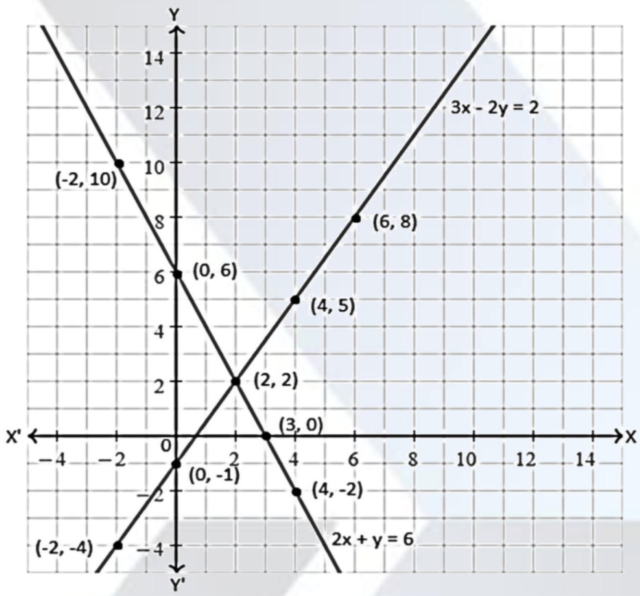
x	-2	0	2	4	6
y	-4	-1	2	5	8

Now, consider another equation:

$2x + y = 6$

$y = -2x + 6$

x	-2	0	2	3	4
y	10	6	2	0	-2



The lines representing the given pair of equations intersecting each other at $(2, 2)$.

Hence, $x = 2$ and $y = 2$ is the solution of the given pair of linear equations.

Q16. Sum of the squares of two consecutive positive even integers is 100, find those numbers by using quadratic equations.

Solution:

Let x and $x + 2$ be the two consecutive even integers.

According to the given,

$x^2 + (x + 2)^2 = 100$

$x^2 + x^2 + 4 + 4x - 100 = 0$

$2x^2 + 4x - 96 = 0$

$2(x^2 + 2x - 48) = 0$

$x^2 + 2x - 48 = 0$

$x^2 + 8x - 6x - 48 = 0$

$x(x + 8) - 6(x + 8) = 0$

$(x+8)(x-6)=0$

$x=-8, x=6$

The value of x cannot be negative.

Thus, $x=6$

$x+2=6+2=8$

Hence, 6 and 8 are the required numbers.

OR

X is a set of factors of 24 and Y is a set of factors of 36,

then find sets $X \cup Y$ and $X \cap Y$ by using Venn diagram and comment on the answer.

Solution:

X = factors of 24

$= \{1, 2, 3, 4, 6, 8, 12, 24\}$

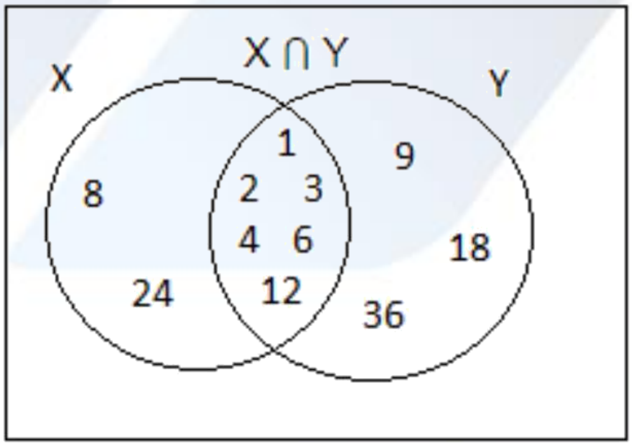
Y = factors of 36

$= \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$X \cup Y = \{1, 2, 3, 4, 6, 8, 12, 24\} \cup \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$= \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36\}$

$X \cap Y = \{1, 2, 3, 4, 6, 8, 12, 24\} \cap \{1, 2, 3, 4, 6, 9, 12, 18, 36\} = \{1, 2, 3, 4, 6, 12\}$



Q17. Find the sum of all the three digit numbers, which are divisible by 4.

Solution:

Three digit numbers which are divisible by 4 are:

$100, 104, 108, \ldots, 996$

This forms an AP with $a = 100$ and $d = 4$

nth term of AP

$a_n = a + (n-1)d$

$996 = 100 + (n-1)4$

$(n-1)4 = 996 - 100 = 896$

$n - 1 = \frac{896}{4}$

$n - 1 = 224$

$n = 225$

Sum of first n terms

$S_n = \frac{n}{2}(a + a_n)$

$S_{225} = \left(\frac{225}{2}\right) \times (100 + 996)$

$= \left(\frac{225}{2}\right) \times 1096$

$= 123300$

Hence, 123300 is the required sum.

OR

Find the coordinates of the points of trisection of the line segment joining the points $(-3, 3)$ and $(3, -3)$.

Solution:

Let the given points be:

$A(-3, 3)$ and $B(3, -3)$

Let C and D be the points of trisection of line joining the points A and B.

C divided AB in the ratio $1:2$.

$m : n = 1 : 2$

Using section formula,

$$C = \left[\frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}\right]$$

$$= [\frac{3-6}{1+2}, \frac{-3+6}{1+2}]$$

$$= \left(-\frac{3}{3}, \frac{3}{3}\right)$$

$$= (-1, 1)$$

D is the midpoint of BC .

$$D = \left[\frac{-1+3}{2}, \frac{1-3}{2}\right]$$

$$= \left(\frac{2}{2}, -\frac{2}{2}\right)$$

$$= (1, -1)$$

Hence, $(-1,1)$ and $(1,-1)$ are the required points.