# Grade 10 Maths Telangana 2017

## SECTION – I

Q1. If $\sin A = \frac{1}{\sqrt{2}}$ and $\cot B = 1$, prove that $\sin(A+B) = 1$, where $A$ and $B$ both are acute angles.

**Solution:**

Given,

$A$ and $B$ are acute angles.

$$\sin A = \frac{1}{\sqrt{2}}$$

$\sin A = \sin 45^\circ$

$\Rightarrow A = 45^\circ$

$\cot \mathrm{B} = 1$

$\cot B = \cot 45^\circ$

$\Rightarrow B = 45^\circ$

Now,

$\sin(\mathrm{A}+\mathrm{B}) = \sin(45^\circ + 45^\circ)$

$= \sin 90^\circ$

$= 1$

Hence proved.

Q2. The length of the minute hand of a clock is 3.5 cm. Find the area swept by a minute hand in 30 minutes. (use $\pi = \frac{22}{7}$)

**Solution:**

Given,

Length of a minute hand $= 3.5$ cm

The minute hand forms a semicircle in 30 min .

Thus, radius $= \mathrm{r} = 3.5$ cm

Area $= \frac{\pi \mathrm{r}^2}{2}$

$= \left(\frac{1}{2}\right) \times \left(\frac{22}{7}\right) \times 3.5 \times 3.5$

$= 19.25$

Hence, the area swept by the minute hand of a clock is 19.25 $cm^2$.

Q3. Express $\cos\ \theta$ in terms of $\tan\ \theta$.

**Solution:**

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Squaring on both sides,

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$\tan^2\theta = \frac{(1-\cos^2\theta)}{\cos^2\theta}$$

$$\tan^2\theta = \left(\frac{1}{\cos^2\theta}\right) - 1$$

$$\tan^2\theta + 1 = \frac{1}{\cos^2\theta}$$

$$\Rightarrow \cos^2\theta = \frac{1}{(1+\tan^2\theta)}$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{(1+\tan^2\theta)}}$$

Q4. From the first 50 natural numbers, find the probability of a randomly selected number is a multiple of 3.

**Solution:**

Total number of outcomes $= n(S) = 50$

i.e. the first 50 natural numbers.

Let E be the event of getting a number which is multiple of 3 .

E $= \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$

$n(E) = 16$

$$P(E) = \frac{n(E)}{n(S)}$$

$= \frac{16}{50}$

$= \frac{8}{25}$

Hence, the required probability is $\frac{8}{25}$.

Q5. Write the formula to find the curved surface area of a cone and explain each term in it.

**Solution:**

Curved surface area of a cone $= \pi r l$

Here,

$r =$ Radius of the circular base

$l$ = Slant height

Q6. "The median of observations $-2,5,3,-1,4,6$ is 3.5 ". Is it correct? Justify your answer.

**Solution:**

Given observations are:

$-2,5,3,-1,4,6$

Ascending order:

$-2,-1,3,4,5,6$

Number of observations $= n = 6$

Median $= \frac{1}{2}\left[\left(\frac{n}{2}\right)^{th} + \left\{\left(\frac{n}{2}\right)+1\right\}^{th}\right]$ observations

$= \frac{1}{2}$[3 rd +4 th observations ]

$= \frac{1}{2}[3+4]$

$= \frac{7}{2}$

$= 3.5$

Therefore, the median is 3.5.

Q7. If $\cos\ \theta = \frac{1}{\sqrt{2}}$, then find the value of $4 + \cot\ \theta$.

**Solution:**

Given,

$$\cos\ \theta = \frac{1}{\sqrt{2}}$$

$\cos\ \theta = \cos\ 45^\circ$

$\Rightarrow \theta = 45^\circ$

$4 + \cot\ \theta = 4 + \cot\ 45^\circ$

$= 4 + 1$

$= 5$

## SECTION - II

Q8. The diameter of a solid sphere is 6 cm. It is melted and recast into a solid cylinder of height 4 cm. Find the radius of the cylinder.

**Solution:**

Given,

Diameter of sphere $= 6$ cm

Radius of sphere $= r = \frac{6}{2} = 3$ cm

Height of cylinder $= h = 4$ cm

Let $R$ be the radius of the cylinder.

Given that, the sphere is melted and recast into a cylinder.

Volume of sphere = Volume of cylinder

$$\left(\frac{4}{3}\right)\pi r^3 = \pi \mathrm{R}^2\ \mathrm{h}$$

$$\left(\frac{4}{3}\right) \times \left(\frac{22}{7}\right) \times 3 \times 3 \times 3 = \left(\frac{22}{7}\right) \times R^2 \times 4$$

$$\mathrm{R}^2 = \frac{36}{4}$$

$\mathrm{R}^2 = 9$

R $= 3$ cm

Hence, the radius of the cylinder is 3 cm .

Q9. Write the formula of mode for grouped data and explain each term in it.

**Solution:**

Mode for grouped data $= l + \left[\frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)}\right] \times h$

Here,

$l$ = Lower limit of the modal class

$f_1$ = Frequency of the modal class

$f_0$ = Frequency of the class preceding the modal class

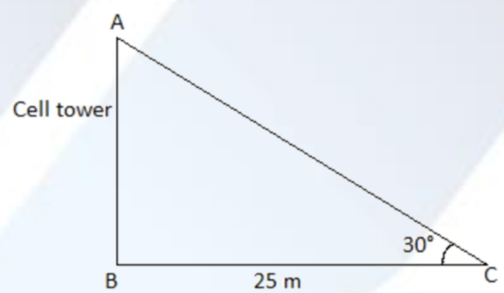
$f_2$ = Frequency of the class succeeding the modal class

$h$ = class size (or class height)

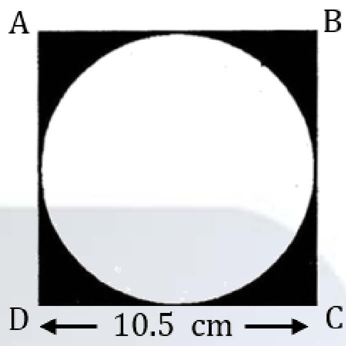
Q10. A person 25 m away from a cell tower observes the top of the cell tower at an angle of elevation 30°. Draw the suitable diagram for this situation.

**Solution:**

Let $AB$ be the cell tower and $C$ be the point of observation.




Q11. Find the area of the shaded region in the given figure. ABCD is a square with side 10.5 cm.




**Solution:**

Given,

Side of square $= a = 10.5$ cm

Diameter of circle $= 10.5$ cm

Radius of circle $= r = \frac{10.5}{2} = 5.25$ cm

Area of the shaded region = Area of square - Area of circle

$= a^2 - \pi r^2$

$= (10.5)^2 - \left(\frac{22}{7}\right) \times 5.25 \times 5.25$

$= 110.25 - 86.625$

$= 23.625 \text{ cm}^2$

Q12. One card is selected from a well shuffled deck of 52 cards. Find the probability of getting a red card with a prime number.

**Solution:**

Total number of outcomes $= n(S) = 52$

Let E be the event of getting a red card with a prime number.

Prime numbers on red card (on diamonds and hearts) $= 2,3,5,7$

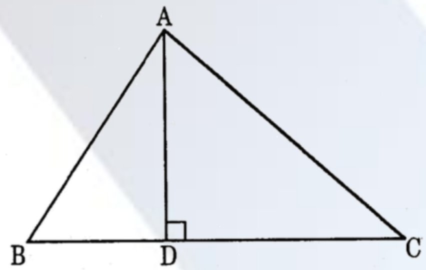
$n(E) = 8$

$P(E) = \frac{n(E)}{n(S)}$

$= \frac{8}{52}$

$= \frac{2}{13}$

Q13. In a $\triangle$ ABC, AD $\perp$ BC and $AD^2 = BD \times CD$, prove that $\triangle$ ABC is a right angled triangle.




**Solution:**

Given,

$AD \perp BC$

In $\triangle$ ADB and $\triangle$ ADC,

By Pythagoras theorem,

$AB^2 = AD^2 + BD^2$ ….. (i)

$AC^2 = AD^2 + DC^2$ ……(ii)

Adding (i) and (ii),

$AB^2 + AC^2 = 2AD^2 + BD^2 + DC^2$

$= 2BD \cdot CD + BD^2 + CD^2$ [given $AD^2 = BD \cdot CD$ ]

$= (BD + CD)^2$

$= BC^2$

In triangle ABC, $AB^2 + AC^2 = BC^2$

Hence, the triangle ABC is a right triangle.

**SECTION - III**

Q14. The length of cuboid is 12 cm, breadth and height are equal in measurements and its volume is 432 $cm^3$. The cuboid is cut into 2 cubes. Find the lateral surface area of each cube.

**Solution:**

Given,

Length of cuboid $= l = 12$ cm

Let $x$ be the breadth and height of cuboid.

Volume of cuboid $= 432 \text{ cm}^3$ (given)

$lbh = 432$

$12(x)(x) = 432$

$x^2 = \frac{432}{12}$

$x^2 = 36$

$x = 6$

Breadth $=$ Height $= 6$ cm

Given that, the cuboid is cut into 2 cubes along its length.

Side of each cube $= \frac{12}{2} = 6$ cm

Lateral surface of cube $= 4(\text{ side })^2$

$= 4 \times 6 \times 6$

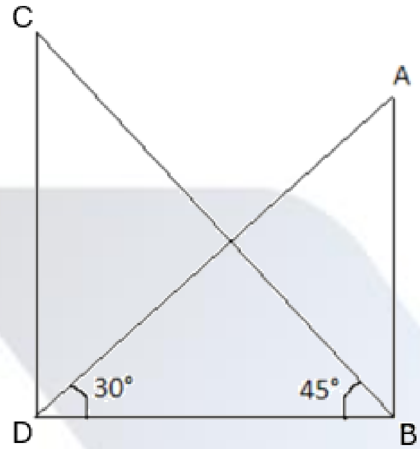
$= 144 \text{ cm}^2$

OR

Two poles are standing opposite to each other on either side of the road which is 90 feet wide. The angle of elevation from bottom of first pole to top of the second pole is $45^\circ$, the angle of elevation from bottom of second pole to top of first pole is $30^\circ$. Find the heights of poles. (use $\sqrt{3} = 1.732$ )

**Solution:**

Let AB be the first pole and CD be the second pole.




In right triangle BDC , tan $45^\circ = \frac{CD}{BD}$

$1 = \frac{CD}{90}$

$CD = 90ft$

In right triangle ABD, tan $30^\circ = \frac{AB}{BD}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{90}$$

$$AB = \frac{90}{\sqrt{3}}$$

$$AB = \left(\frac{90}{\sqrt{3}}\right) \times \left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$AB = \frac{90\sqrt{3}}{3}$$

$AB = 30\sqrt{3}$

$AB = 30 \times 1.732$

$AB = 51.96ft$

Hence, the heights of the two poles are 90 ft and 51.96 ft .

Q15. A bag contains some square cards. A prime number between 1 and 100 has been written on each card. Find the probability of getting a card that the sum of the digits of the prime number written on it is 8.

**Solution:**

Number of prime numbers between 1 and 100 = 25

Total number of outcomes = 25

i.e.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Prime numbers in which the sum of their digits is 8 are 17, 53, 71.

Number of favorable outcomes = 3

P( getting a card that the sum of the digits of the prime number written on it is 8) $= \frac{3}{25}$

OR

The daily wages of 80 workers of a factory.

| Daily wages (Rs) | 500 – 600 | 600 – 700 | 700 – 800 | 800 – 900 | 900 – 1000 |
|---|---|---|---|---|---|
| Number of workers | 12 | 17 | 28 | 14 | 9 |

Find the mean daily wages of the workers of the factory by using an appropriate method.

**Solution:**

Assumed mean method:

| CI | $x_i$ | Number of workers ($f_i$) | $d_i = x_i - A$ | $f_i d_i$ |
|---|---|---|---|---|
| 500 – 600 | 550 | 12 | -200 | -2400 |

| | | | | |
|---|---|---|---|---|
| 600 – 700 | 650 | 17 | -100 | -1700 |
| 700 – 800 | 750 = A | 28 | 0 | 0 |
| 800 – 900 | 850 | 14 | 100 | 1400 |
| 900 – 1000 | 950 | 9 | 200 | 1800 |
| | | $\sum f_i = 80$ | | $\sum f_i d_i = -900$ |

$\text{Mean} = A + \frac{(\sum f_i d_i)}{\sum f_i}$

$= 750 + \left(-\frac{900}{80}\right)$

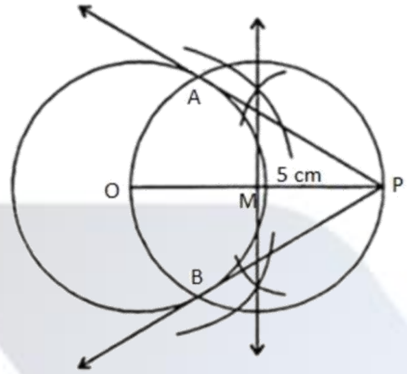
$= 750 - \left(\frac{90}{8}\right)$

$= 750 - 11.25$

$= 738.75$

Q16. Draw a circle of diameter 6 cm from a point 5 cm away from its centre. Construct the pair of tangents to the circle and measure their length.

**Solution:**




Hence, PA and PB are the required tangents to the circle.

PA = PB = 4 cm (by measurement)

OR

The following data gives the information on the observed life span (in hours) of 90 electrical components.

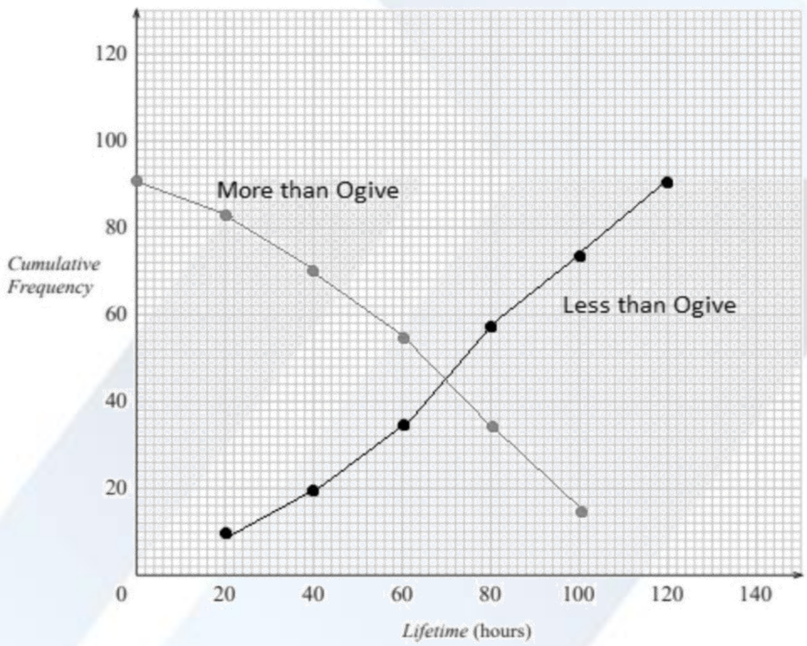
| Life span (in hours) | 0 – 20 | 20 – 40 | 40 – 60 | 60 – 80 | 80 – 100 | 100 – 120 |
|---|---|---|---|---|---|---|
| Frequency | 8 | 12 | 15 | 23 | 18 | 14 |

Draw both Ogives for the above data.

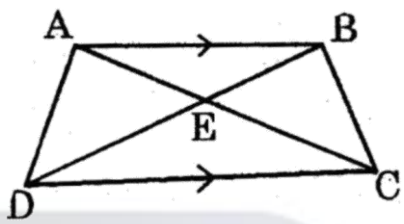
**Solution:**

| CI | Less than cumulative frequency | CI | More than cumulative frequency |
|---|---|---|---|
| Less than 20 | 8 | More than 0 | 90 |
| Less than 40 | 20 | More than 20 | 82 |

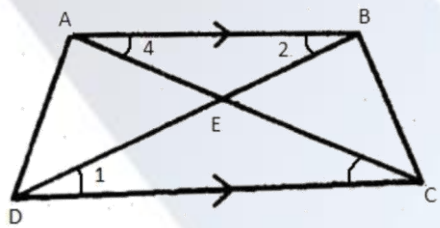
| | | | |
|---|---|---|---|
| Less than 60 | 35 | More than 40 | 70 |
| Less than 80 | 58 | More than 60 | 55 |
| Less than 100 | 76 | More than 80 | 32 |
| Less than 120 | 90 | More than 100 | 14 |




Q17. ABCD is a trapezium with AB||DC, the diagonals AC and BD are intersecting at E. If $\triangle$ AED is similar to $\triangle$ BEC, then prove that AD = BC.




**Solution:**




In $\triangle$ EDC and $\triangle$ EBA,

$\angle 1 = \angle 2$ (alternate angles)

$\angle 3 = \angle 4$ (alternate angles)

and $\angle$ CED $= \angle$AEB (vertically opposite angles)

$\triangle \text{EDC} \sim \triangle \text{EBA}$

$\Rightarrow \frac{\text{ED}}{\text{EB}} = \frac{\text{EC}}{\text{EA}}$

$\Rightarrow \frac{\text{ED}}{\text{EC}} = \frac{\text{EB}}{\text{EA}}$ ....(i)

Given that, $\triangle \text{AED} \sim \Delta\text{BEC}$

$\frac{ED}{EC} = \frac{EA}{EB} = \frac{AD}{BC}$ ....(ii)

From (i) and (ii),

$$\frac{\text{EB}}{\text{EA}} = \frac{\text{EA}}{\text{EB}}$$

$\Rightarrow \text{EB}^2 = \text{EA}^2$

$\Rightarrow \text{EB} = \text{EA}$

Substituting EB $=$ EA in (ii),

$$\frac{\text{EA}}{\text{EA}} = \frac{\text{AD}}{\text{BC}}$$

$$\Rightarrow \frac{\text{AD}}{\text{BC}} = 1$$

$\Rightarrow \text{AD} = \text{BC}$

Hence proved.

OR

Prove that $(1+\tan^2\ \theta)+\left[1+\left(\frac{1}{\tan^2\theta}\right)\right]=\frac{1}{(\sin^2\theta-\sin^4\theta)}$

**Solution:**

We know that,

$\sec^2\ \theta-\tan^2\ \theta=1$

$1+\tan^2\ \theta=\sec^2\ \theta$

$1+\tan^2\ \theta=(\frac{1}{\cos^2\theta})$

Similarly,

$1+\cot^2\ \theta=\text{cosec}^2\theta=(\frac{1}{\sin^2\theta})$

$\text{LHS}=(1+\tan^2\ \theta)+\left[1+\left(\frac{1}{\tan^2\theta}\right)\right]$

$=(1+\tan^2\ \theta)(1+\cot^2\ \theta)$

$=\frac{1}{(\sin^2\theta\cos^2\theta)}$

$=\frac{1}{\sin^2\theta(1-\sin^2\theta)}$

$=\frac{1}{(\sin^2\theta-\sin^4\theta)}$

$=$ RHS

Hence proved.