Grade 10 Telangana Maths 2018

PART A

SECTION - I

Q1. Find the distance between the points $(1, 5)$ and $(5, 8)$.

Solution:

Let the given points be:

$(x_1, y_1) = (1, 5)$

$(x_2, y_2) = (5, 8)$

Distance between two points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-1)^2 + (8-5)^2}$

$= \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$ units

Q2. Expand $\log_{10} 385$.

Solution:

$\log_{10} 385$

$= \log_{10} (5 \times 7 \times 11)$

We know that $\log abc = \log a + \log b + \log c$

$= \log_{10} 5 + \log_{10} 7 + \log_{10} 11$

Q3. Give one example each for a finite set and an infinite set.

Solution:

Finite set: $A = \{3, 5, 7, 9, 11, 13, 15, 17\}$

$n(A) = 8$

Infinite set: Z = Set of all integers

$= \{\ldots\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\ldots\}$

$n(Z) = \infty$

Q4. Find the sum and product of roots of the quadratic equation.

$x^2 - 4\sqrt{3}x + 9 = 0$

Solution:

Given quadratic equation is:

$x^2 - 4\sqrt{3}x + 9 = 0$

Comparing with the standard form $\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$,

$a = 1, b = -4\sqrt{3}$ and $c = 9$

Sum of the roots $= -\frac{b}{a} = -\frac{-4\sqrt{3}}{1} = 4\sqrt{3}$

Product of the roots $= \frac{c}{a} = \frac{9}{1} = 9$

Q5. Is the sequence $\sqrt{3},\ \sqrt{6},\ \sqrt{9},\ \sqrt{12}$, ... form an Arithmetic Progression? Give reason.

Solution:

Given,

$\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$

First term $= a = \sqrt{3}$

Second term - First term $= \sqrt{6} - \sqrt{3}$

Third term - Second term $= \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

Since the common difference is not the same throughout the sequence, it is not an AP.

Q6. If $x = a$ and $y = b$ is the solution for the pair of equations $x - y = 2$ and $x + y = 4$, then find the values of a and b.

Solution:

Given pair of equations are:

$x - y = 2$ .....(i)

$x + y = 4$ ......ii)

Adding (i) and (ii),

$x - y + x + y = 2 + 4$

$2x = 6$

$x = \frac{6}{2} = 3$

Substituting $x = 3$ in (i),

$3 - y = 2$

$y = 3 - 2$

$y = 1$

Therefore, $a = 3$ and $b = 1$.

Q7. Verify the relation between zeroes and coefficients of the quadratic polynomial $x^2 - 4$.

Solution:

Let the given quadratic polynomial $p(x) = x^2 - 4$

Let $\mathrm{p(x)} = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \sqrt{4}$

$\mathrm{x} = \pm 2$

Therefore, zeroes of the given polynomial are -2 and 2 .

Sum of the zeroes $= -2 + 2 = 0$

$= -\frac{0}{1} = -\frac{\text{Coefficient of } x}{(\text{Coefficient of } x^2)}$

Product of the zeroes $= (-2)(2) = -4$

$= -\frac{4}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, verified the relationship between the zeroes and coefficients of the given quadratic polynomial.

SECTION - II

Q8. Complete the following table for the polynomial $y = p(x) = x^3 - 2x + 3$.

x	1	0	1	2
x^3				
$-2x$				
3				
y				
(x, y)				

Solution:

Given,

$y = \mathrm{p(x)} = \mathrm{x}^3 - 2\mathrm{x} + 3$

x	-1	0	1	2
x^3	-1	0	1	8
$-2x$	2	0	-2	-4
3	3	3	3	3
y	4	3	2	7
(x, y)	$(-1,4)$	$(0,3)$	$(1,2)$	$(2,7)$

Q9. Show that $\log\left(\frac{162}{343}\right) + 2\log\left(\frac{7}{9}\right) - \log\left(\frac{1}{7}\right) = \log\ 2$

Solution:

$\mathrm{LHS} = \log\left(\frac{162}{343}\right) + 2\log\left(\frac{7}{9}\right) - \log\left(\frac{1}{7}\right)$

$= \log\left(\frac{162}{343}\right) + +\log\left(\frac{7}{9}\right)^2 - \log\left(\frac{1}{7}\right)$

$= \log\left(\frac{162}{343}\right) + \log\left(\frac{49}{81}\right) - \log\left(\frac{1}{7}\right)$

$$= \log\left[\frac{162\times 49}{343\times 81}\right] - \log\left(\frac{1}{7}\right)$$

$$= \log\left(\frac{2}{7}\right) - \log\left(\frac{1}{7}\right)$$

$$= \log\left[\frac{\frac{2}{7}}{\frac{1}{7}}\right]$$

$$= \log\ 2$$

= RHS

Therefore, $\log\left(\frac{162}{343}\right) + 2\log\left(\frac{7}{9}\right) - \log\left(\frac{1}{7}\right) = \log\ 2$

Q10. If the equation $kx^2 - 2kx + 6 = 0$ has equal roots, then find the value of k.

Solution:

Given,

$kx^2 - 2kx + 6 = 0$

Comparing with the standard form $ax^2 + bx + c = 0$,

$a = k, b = -2k, c = 6$

Given that, the equation has equal roots.

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-2\mathrm{k})^2 - 4(\mathrm{k})(6) = 0$

$\Rightarrow 4\mathrm{k}^2 - 24\mathrm{k} = 0$

$\Rightarrow 4\mathrm{k}(\mathrm{k} - 6) = 0$

$\Rightarrow k(k - 6) = 0$

$\Rightarrow k = 0, k = 6$

$k = 0$ is not possible.

Hence, the value of k is 6 .

Q11. Find the 7th term from the end of the arithmetic progression $7, 10, 13, \ldots, 184$.

Solution:

Given AP is:

$7,10,13, \ldots ,184$

First term = a = 184 (Reverse the given AP)

Common difference $= d = -3$

nth term of AP from the last term be l_n

$l_n = l + (n-1)d$

$l_7 = 184 + (7-1)(-3)$

$= 184 - (6)(3)$

$= 184 - 18$

$= 166$

Therefore, the 7th term of the AP is 166.

Q12. In the diagram on a Lunar eclipse, if the positions of Sun, Earth and Moon are shown by $(-4,6), (k,-2)$ and $(5,-6)$ respectively, then find the value of k.

Solution:

Given that the diagram on a Lunar eclipse, if the positions of Sun, Earth and Moon are shown by $(-4,6), (k,-2)$ and $(5,-6)$ respectively.

That means the points lie on a straight line.

Hence, the area of the triangle formed by these points will be 0 .

$\Rightarrow \frac{1}{2}[-4(-2+6) + k(-6-6) + 5(6+2)] = 0$

$\Rightarrow -4(4) + k(-12) + 5(8) = 0$

$\Rightarrow -16 - 12k + 40 = 0$

$\Rightarrow 12k = 24$

$\Rightarrow k = \frac{24}{12}$

$\Rightarrow k = 2$

Q13. Given the linear equation $3x + 4y = 11$, write linear equations in two variables such that their geometrical representations form parallel lines and intersecting lines.

Solution:

Given,

Equation of first line is $3x + 4y = 11$

Let $ax + by + c$ be the equation of the second line.

Condition for parallel lines:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{a} = \frac{4}{b} \neq \frac{11}{c}$$

Let us consider some values of a, b, c which satisfies the above condition.

$\mathrm{a} = 6, \mathrm{b} = 8, \mathrm{c} = 14$

Therefore, the equation of line which is parallel to the given line is $6x + 8y + 14 = 0$

Condition for intersecting lines:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\frac{3}{a} \neq \frac{4}{b}$$

Let us consider some values of a, b, c which satisfies the above condition.

$a = 8, b = 6, c = 11$

$8x + 6y + 11 = 0$

Therefore, the equation of line which is perpendicular to the given line is $8x + 6y + 11 = 0$

SECTION - III

Q14. Find the points of trisection of the line segment joining the points $(-2,1)$ and $(7,4)$.

Solution:

Let the given points be:

$\mathrm{A} = (x_1, y_1) = (-2,1)$

$B = (x_2, y_2) = (7,4)$

P divides AB in the ratio 1 : 2.

Here, $m : n = 1 : 2$

Using the section formula,

$$P(x, y) = \left[\frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}\right]$$

$$= \left[\frac{7-4}{1+2}, \frac{4+2}{1+2}\right]$$

$$= \left(\frac{3}{3}, \frac{6}{3}\right)$$

$$= (1, 2)$$

Q is the midpoint of PB .

$$Q = [\frac{1+7}{2}, \frac{2+4}{3}]$$

$$= \left(\frac{8}{2}, \frac{6}{2}\right)$$

$$= (4,3)$$

Hence, the required points are (1,2) and (4,3).

OR

Sum of squares of two consecutive even numbers is 580. Find the numbers by writing a suitable quadratic equation.

Solution:

Let x and $(x + 2)$ be the two consecutive even numbers.

According to the given,

$$x^2 + (x + 2)^2 = 580$$

$$x^2 + x^2 + 4x + 4 - 580 = 0$$

$$2x^2 + 4x - 576 = 0$$

$$2(x^2 + 2x - 288) = 0$$

$$x^2 + 2x - 288 = 0$$

$$x^2 + 18x - 16x - 288 = 0$$

$x(x+18)-16(x+18)=0$

$(x-16)(x+18)=0$

$x=16, x=-18$

If $x=16, x+2=18$

If $x=-18, x+2=-18+2=-16$

Hence, the two consecutive even numbers are 16 and 18 or -18 and -16.

Q15. Prove that $\sqrt{3}+\sqrt{5}$ is an irrational number.

Solution:

Let $\sqrt{3}+\sqrt{5}$ be a rational number.

$\sqrt{3}+\sqrt{5}=a$, where a is an integer.

Squaring on both sides,

$(\sqrt{3}+\sqrt{5})^2=a^2$

$(\sqrt{3})^2+(\sqrt{5})^2+2(\sqrt{3})(\sqrt{5})=a^2$

$3+5+2\sqrt{15}=a^2$

$8+2\sqrt{15}=a^2$

$2\sqrt{15}=a^2-8$

$\sqrt{15}=\frac{(a^2-8)}{2}$

$\frac{(a^2-8)}{2}$ is a rational number since a is an integer.

Therefore, $\sqrt{15}$ is also an integer.

We know that $\sqrt{15}$ is not rational numbers.

Thus, our assumption that $\sqrt{3}+\sqrt{5}$ is a rational number is wrong.

Hence, $\sqrt{3}+\sqrt{5}$ is an irrational number.

OR

Show that cube of any positive integer will be in the form of $8m$ or $8m+1$ or $8m+3$ or $8m+5$ or $8m+7$, where m is a whole number.

Solution:

Let a be the positive integer.

By Euclid's division lemma,

$a = bq + r$ where $0 \leq r < b$

Let $b = 8$ then $a = 8q + r$

Where, $r = 0, 1, 2, 3, 4, 5, 6, 7$

When r = 0

$a = 8q$

$a^3 = (8q)^3$

$= 512q^3$

$= 8(64q^3)$

$= 8m$ where $m = 64q^3$

When $r = 1$,

$a = 8q + 1$

$a^3 = (8q + 1)^3$

$a^3 = 512q^3 + 1 + 3(8q)(8q + 1)$

$= 512q^3 + 1 + 24q(8q + 1)$

$= 512q^3 + 1 + 192q^2 + 24q$

$= 8(64q^3 + 24q^2 + 3q) + 1$

$= 8m + 1$ where $m = 64q^3 + 24q^2 + 3q$

When r = 2,

$a = 8q + 2$

$a^3 = (8q + 2)^3$

$= 512q^3 + 8 + 48q(8q + 2)$

$= 512q^3 + 8 + 384q^2 + 96q$

$= 512q^3 + 384q^2 + 96q + 8$

$= 8(64q^3 + 48q^2 + 12q + 1)$

$= 8m$ where $m = 64q^3 + 48q^2 + 12q + 1$

When $r = 3$,

$a = 8q + 3$

$a^3 = (8q + 3)^3$

$= 512q^3 + 27 + 72q(8q + 3)$

$= 8(64q^3 + 3 + 72q^2 + 27q) + 3$

$= 8m + 3$ where $m = 64q^3 + 72q^2 + 27q + 3)$

When $r = 4$,

$a = 8q + 4$

$a^3 = (8q + 4)^3$

$= 512q^3 + 64 + 768q^2 + 384q$

$a^3 = 8(64q^3 + 8 + 96q^2 + 48q)$

$a^3 = 8m$ where $m = 64q^3 + 8 + 96q^2 + 48q$

when $r = 5$,

$a = 8q + 5$

$a^3 = (8q + 5)^3$

$= 512q^3 + 960q^2 + 600q + 125$

$= 8(64q^3 + 120q^2 + 75q + 15) + 5$

$= 8m + 5$

When $\mathrm{r} = 6$

$a = 8q + 6$

$a^3 = (8q + 6)^3$

$= 512q^3 + 1152q^2 + 864q + 216$

$= 8(64q^3 + 144q^2 + 108q + 27)$

$= 8m$ where $m = 64q^3 + 144q^2 + 108q + 27$

When $\mathrm{r} = 7$

$\mathrm{a} = 8q + 7$

$\mathrm{a}^3 = (8q + 7)^3$

$= 512q^3 + 343 + 1344q^2 + 1176q$

$= 8(64q^3 + 168q^2 + 147q + 42) + 7$

$= 8m + 7$ where $m = 64q^3 + 168q^2 + 147q + 42$

Therefore, the cube of any positive integer will be in the form of 8 m or 8 m + 1 or 8 m + 3 or 8 m + 5 or 8 m + 7, where m is a whole number.

Q16. Find the solution of $x + 2y = 10$ and $2x + 4y = 8$ graphically.

Solution:

Given,

$x + 2y = 10$

$2x + 4y = 8$

Consider the first equation,

$x + 2y = 10$

$2y = -x + 10$

$y = -\left(\frac{1}{2}\right)x + 5$

x	-2	0	2	4
y	6	5	4	3

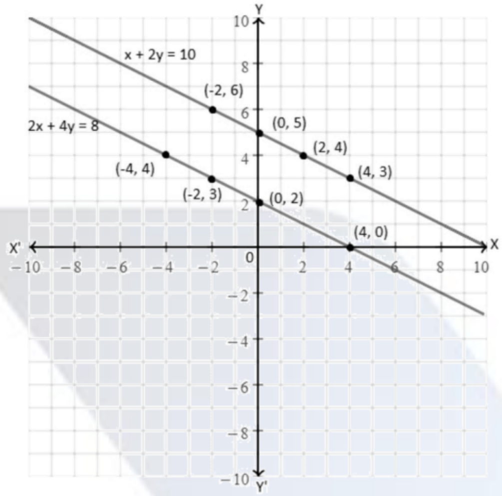
Now, consider the another equation,

$2x + 4y = 8$

$4y = -2x + 8$

$y = -\left(\frac{1}{2}\right)x + 2$

x	-4	-2	0	4
y	4	3	2	0



The lines representing the given pair of equations are parallel to each other.

Hence, there is no solution for the given pair of equations.

OR

$A = \{x: x \text{ is a perfect square}, x < 50, x \in N\}$

$B = \{x: x = 8m + 1, \text{where } m \in W, x < 50, x \in N\}$

Find $A \cap B$ and display it with a Venn diagram.

Solution:

Given,

$A = \{x: x \text{ is a perfect square}, x < 50, x \in N\}$

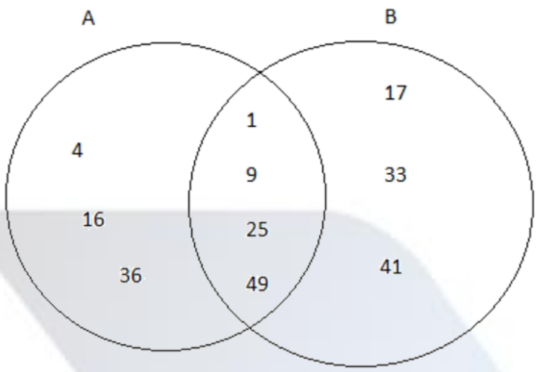
$A = \{1, 4, 9, 16, 25, 36, 49\}$

$B = \{x: x = 8m + 1, \text{where } m \in W, x < 50, x \in N\}$

$B = \{1, 9, 17, 25, 33, 41, 49\}$

$A \cap B = \{1, 4, 9, 16, 25, 36, 49\} \cap \{1, 9, 17, 25, 33, 41, 49\}$

$= \{1, 9, 25, 49\}$



Q17. Find the sum of all two digit positive integers which are divisible by 3 but not by 2 .

Solution:

Two digit positive numbers which are divisible by 3 but not by 2 are

15, 21, 27, 33, ..., 99

This is an AP with $a = 15$ and $d = 6$.

Last term = l = 99

$n = 15$

$S_n = \frac{n}{2}(a + l)$

$= \left(\frac{15}{2}\right) \times (15 + 99)$

$= \left(\frac{15}{2}\right) \times 114$

$= 15 \times 57$

$= 855$

Hence, the required sum is 855 .

OR

Total number of pencils required are given by $4x^4 + 2x^3 - 2x^2 + 62x - 66$. If each box contains $x^2 + 2x - 3$ pencils, then find the number of boxes to be purchased.

Solution:

Given,

Total number of pencils $= 4x^4 + 2x^3 - 2x^2 + 62x - 66$

Number of pencils in each box $= x^2 + 2x - 3$

Number of boxes required to be purchased

$= 4x^4 + 2x^3 - 2x^2 + 62x - 66 \div x^2 + 2x - 3$

$= \ 4x^2 - 6x + 22$

Hence, the total number of boxes required to be purchased $= 4x^2 - 6x + 22$