Grade 10 Telangana Maths 2019

PART A

SECTION - I

Q1. If $A = \{x : x \text{ is a factor of } 24\}$, then find $n(A)$.

Solution:

$A = \{1,2,3,4,6,8,12,24\}$

$n(A) = 8$

Q2. Find the HCF of 24 and 33 by using a division algorithm.

Solution:

$24 < 33$

```
24) 33 ( 1
    24
   ----
      9) 24 ( 2
         18
        ----
          6) 9 ( 1
             6
            ---
             3) 6 ( 2
                6
               ---
                0
```

So, HCF=3

Q3. Radha says " 1,1,1, ... are in AP and also in GP". Do you agree with Radha? Give reason.

Solution:

Given,

1,1,1, ….

If a, b, c are in AP, then $a + c = 2b$

Thus, $1 + 1 = 2(1)$

$2 = 2$

Therefore, 1, 1, 1 are in AP.

If a, b, a are in GP, then, $ac = b^2$

$(1)(1) = (1)^2$

$1 = 1$

Therefore, 1, 1, 1 are in GP.

Therefore, we can conclude that 1, 1, 1, ... form both an arithmetic progression (AP) and a geometric progression (GP).

Q4. If $P(x) = x^4 + 1$, then find $P(2) - P(-2)$.

Solution:

Given,

$P(x) = x^4 + 1$

$P(2) = (2)^4 + 1 = 16 + 1 = 17$

$P(-2) = (-2)^4 + 1 = 16 + 1 = 17$

$P(2) - \mathrm{P}(-2) = 17 - 17 = 0$

Q5. Find the roots of the quadratic equation $x^2 + 2x - 3 = 0$.

Solution:

Given quadratic equation is: $\mathrm{x}^2 + 2\mathrm{x} - 3 = 0$

$x^2 + 3x - x - 3 = 0$

$x(x + 3) - 1(x + 3) = 0$

$(x - 1)(x + 3) = 0$

$x - 1 = 0, x + 3 = 0$

$x = 1, x = -3$

Therefore, the roots of the given quadratic equation are 1 and -3.

Q6. Find the centroid of $\triangle$ PQR, whose vertices are $\mathrm{P}(1, 1), \mathrm{Q}(2, 2), \mathrm{R}(-3, -3)$.

Solution:

Let the given vertices of $\triangle$ PQR are:

$(x_1, y_1) = (1,1)$

$(x_2, y_2) = (2,2)$

$(x_3, y_3) = (-3,-3)$

Centroid of a triangle $= \left[\frac{(x_1+x_2+x_3)}{3}, \frac{(y_1+y_2+y_3)}{3}\right]$

$= \left[\frac{1+2-3}{3}, \frac{1+2-3}{3}\right]$

$= \left(\frac{0}{3}, \frac{0}{3}\right)$

$= (0,0)$

Q7. For what value of 't' the following pair of linear equations has a no solution?

$2x - ty = 5$ and $3x + 2y = 11$

Solution:

Given pair of linear equations are:

$2x - ty = 5$

$3x + 2y = 11$

On comparing with the standard form, i.e. $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$,

$a_1 = 2, b_1 = -t, c_1 = -5$

$\mathrm{a}_2 = 3,\ \mathrm{b}_2 = 2, \mathrm{c}_2 = -11$

Since the given pair of linear equations has no solution.

$\frac{\mathrm{a}_1}{\mathrm{a}_2} = \frac{\mathrm{b}_1}{\mathrm{b}_2} \neq \frac{\mathrm{c}_1}{\mathrm{c}_2}$

$\frac{2}{3} = -\frac{t}{2}$

$\Rightarrow t = -\frac{4}{3}$

SECTION - II

Q8. If $\mu = \{0,1,2,3,4,5,6,7,8,9,10\}, A = \{2,3,5,8\}$ and $B = \{0,3,5,7,10\}$. Then represent $A \cap B$ in the Venn diagram.

Solution:

Given,

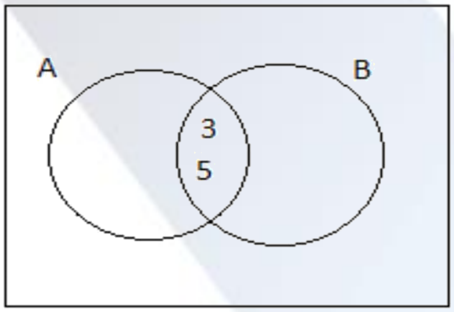
$\mu = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 5, 8\}$

$B = \{0, 3, 5, 7, 10\}$

$A \cap B = \{2, 3, 5, 8\} \cap \{0, 3, 5, 7, 10\} = \{3, 5\}$

Vien diagram for $A \cap B = \{3, 5\}$



Q9. Akhila says, "points A(1, 3), B(2, 2), C(5, 1) are collinear". Do you agree with Akhila? Why?

Solution:

Given points are $A(1, 3)$, $B(2, 2)$ and $C(5, 1)$.

If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are collinear, then the area of $\triangle$ ABC $= 0$

i.e. $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$= \frac{1}{2}[1(2 - 1) + 2(1 - 3) + 5(3 - 2)]$

$= 1 + 2(-2) + 5(1)$

$= 1 - 4 + 5$

$= 2 \neq 0$

Thus, the given points are not collinear.

Hence, we disagree with Akhila.

Q10. Write the quadratic equation, whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

Solution:

Let α and β be the roots of the quadratic equation.

Given,

$2+\sqrt{3}$ and $2-\sqrt{3}$ are the roots of the quadratic equation.

Sum of the roots $= \alpha+\beta = 2+\sqrt{3}+2-\sqrt{3} = 4$

Product of the roots $= \alpha\beta = (2+\sqrt{3})(2-\sqrt{3})$

$= (2)^2 - (\sqrt{3})^2$

$= 4-3$

$= 1$

Hence, the required quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta = 0$

$x^2-4x+1=0$

Q11. Divide x^3-4x^2+5x-2 by $x-2$.

Solution:

$$
\begin{array}{r|rrrr}
 & x^2 & -2x & +1 & \\
\hline
x-2 & x^3 & -4x^2 & +5x & -2 \\
- & & & & \\
 & x^3 & -2x^2 & & \\
\hline
 & & -2x^2 & +5x & -2 \\
 & & - & & \\
 & & -2x^2 & +4x & \\
\hline
 & & & x & -2 \\
 & & & - & \\
 & & & x & -2 \\
\hline
 & & & & 0
\end{array}
$$

Quotient $= q(x) = x^2-2x+1$

Q12. Write the formula of the nth term of GP and explain the terms in it.

Solution:

The formula for nth term of GP is

$a_n = ar^{n-1}$

Here,

$a_n = nth$ term of the sequence

$a =$ first term

$r =$ common ratio

Q13. Solve the pair of linear equations $2x + 3y = 8$ and $x + 2y = 5$ by elimination method.

Solution:

Given,

$2x + 3y = 8$ ....(i)

$x + 2y = 5$ ... (ii)

(ii) × 2 - (i),

$2x + 4y - (2x + 3y) = 10 - 8$

$2x + 4y - 2x - 3y = 2$

$y = 2$

Substituting $y = 2$ in (ii),

$x + 2(2) = 5$

$x = 5 - 4$

$x = 1$

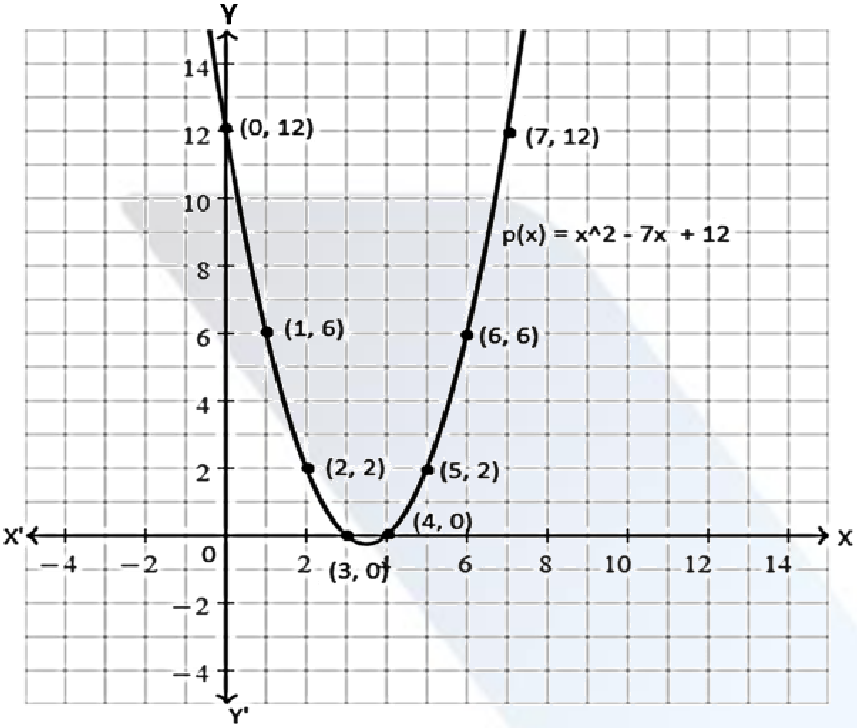
SECTION - III

Q14. (a) Draw the graph of the polynomial $p(x) = x^2 - 7x + 12$, then find its zeroes from the graph.

Solution:

Given polynomial is $y = p(x) = x^2 - 7x + 12$

x	0	1	2	3	4	5	6	7
y	12	6	2	0	0	2	6	12



The graph of the given polynomial intersects the x-axis at (3, 0) and (4, 0).

Therefore, x = 3 and x = 4 are the zeroes of the given polynomial.

OR

(b) Solve the equations graphically $3x + 4y = 10$ and $4x - 3y = 5$.

Solution:

Given,

$3x + 4y = 10$

$4x - 3y = 5$

Consider the first equation:

$3x + 4y = 10$

$4y = -3x + 10$

$y = \left(-\frac{3}{4}\right)x + \left(\frac{10}{4}\right)$

$y = \left(-\frac{3}{4}\right)x + \left(\frac{5}{2}\right)$

x	-2	0	2	6
y	4	$\frac{5}{2}$	1	-2

Now, consider the another equation,

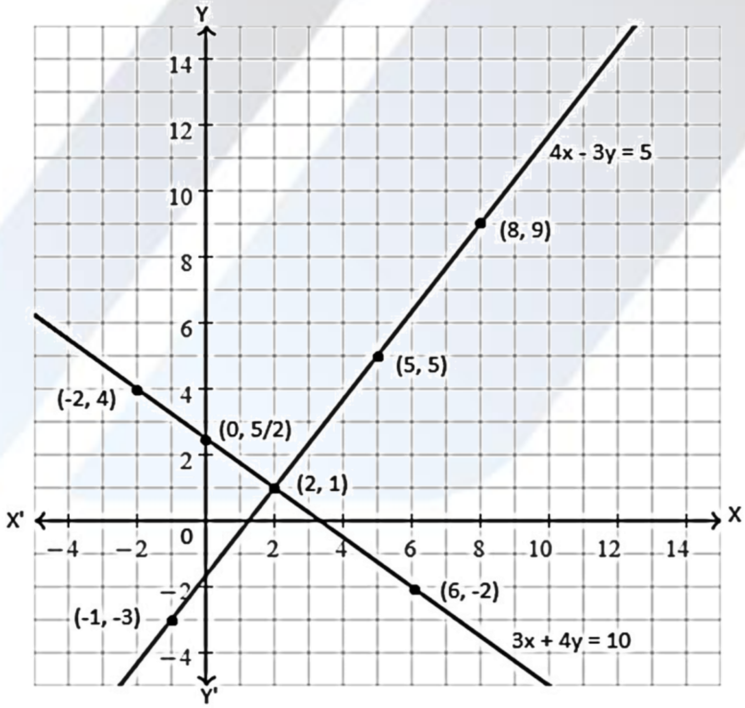
$4x - 3y = 5$

$3y = 4x - 5$

$y = \left(\frac{4}{3}\right)x - \left(\frac{5}{3}\right)$

x	-1	2	5	8
y	-3	1	5	9

Graph:



The lines representing the given pair of equations intersect each other at $(2,1)$.

Therefore, the solution of a given pair of linear equations is $x = 2$ and $y = 1$.

Q15. (a) Find the ratio in which X-axis divides the line segment joining the points $(2,-3)$ and $(5,6)$. Then find the intersection point on X-axis.

Solution:

Let $\mathrm{P}(x,0)$ divide the line segment joining the points $\mathrm{A}(2,-3)$ and $\mathrm{B}(5,6)$ in the ratio $m:n$.

Here,

$(x_1, y_1) = (2,-3)$

$(x_2, y_2) = (5,6)$

Using the section formula,

$$\mathrm{P}(x,0) = \left[\frac{(mx_2 + nx_1)}{m+n}, \frac{(my_2 + ny_1)}{m+n}\right]$$

$$(x,0) = \left[\frac{5\,m + 2n}{m+n}, \frac{6\,m - 3n}{m+n}\right]$$

$$\Rightarrow \frac{6m - 3n}{m+n} = 0$$

$$\Rightarrow 6\,m = 3n$$

$$\Rightarrow \frac{m}{n} = \frac{3}{6}$$

$$\Rightarrow \frac{m}{n} = \frac{1}{2}$$

Therefore, the required ratio is $1:2$.

Now,

$$x = \frac{5\,m + 2n}{m+n}$$

$$x = \frac{[5 + 2(2)]}{1+2}$$

$$x = \frac{9}{3}$$

$$x = 3$$

Therefore, the required point on the x-axis is $(3,0)$.

OR

(b) Find the sum of all two digit odd multiples of 3.

Solution:

Two-digit odd multiples of 3 are 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, and 99.

This is an AP with $a = 15$

Common difference $= d = 6$

$n = 15$

Sum of first n terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{15} = \left(\frac{15}{2}\right)[2(15) + (15-1)6]$

$= \left(\frac{15}{2}\right)[30 + 14(6)]$

$= \left(\frac{15}{2}\right)[30 + 84]$

$= \left(\frac{15}{2}\right) \times 114$

$= 855$

Therefore, the required sum is 855 .

Q16. (a) If $A = \{x: 2x + 1, x \in N, x \leq 5\}, B = \{x: x \text{ is a composite number}, x \leq 12\}$, then show that $(A \cup B)\ (A \cap B) = (A - B) \cup (B - A)$.

Solution:

$A = \{x: 2x + 1, x \in N, x \leq 5\}$

$= \{3, 5, 7, 9, 11\}$

$B = \{x: x \text{ is a composite number}, x \leq 12\}$

$= \{4, 6, 8, 9, 10, 12\}$

$A \cup B = \{3, 5, 7, 9, 11\} \cup \{4, 6, 8, 9, 10, 12\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A \cap B = \{3, 5, 7, 9, 11\} \cap \{4, 6, 8, 9, 10, 12\} = \{9\}$

$(A \cup B) - (A \cap B) = \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} - \{9\}$

$= \{3, 4, 5, 6, 7, 8, 10, 11, 12\}$

$A - B = \{3, 5, 7, 9, 11\} - \{4, 6, 8, 9, 10,12\} = \{3,5,7,11\}$

$B - A = \{4, 6, 8, 9, 10, 12\} - \{3, 5, 7, 9, 11\} = \{4, 6, 8, 10, 12\}$

$(A - B) \cup (B - A) = \{3, 5, 7, 11\} \cup \{4, 6, 8, 10, 12\} = \{3, 4, 5, 6, 7, 8, 10, 11, 12\}$

Therefore, $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

OR

(b) Prove that $\sqrt{2} + \sqrt{7}$ is an irrational number.

Solution:

Let $\sqrt{2} + \sqrt{7}$ be a rational number.

$\sqrt{2} + \sqrt{7} = a$, where a is an integer.

Squaring on both sides,

$(\sqrt{2} + \sqrt{7})^2 = a^2$

$(\sqrt{2})^2 + (\sqrt{7})^2 + 2(\sqrt{2})(\sqrt{7}) = a^2$

$2 + 7 + 2\sqrt{14} = a^2$

$9 + 2\sqrt{14} = a^2$

$2\sqrt{14} = a^2 - 9$

$$\sqrt{14} = \frac{(a^2 - 9)}{2}$$

$\frac{(a^2-9)}{2}$ is a rational number since a is an integer.

Therefore, $\sqrt{14}$ is also an integer.

We know that $\sqrt{14}$ not rational numbers.

Thus, our assumption that $\sqrt{2} + \sqrt{7}$ is a rational number is wrong.

Hence, $\sqrt{2} + \sqrt{7}$ is an irrational number.

Q17. (a) Sum of the areas of two squares is 850 m^2. If the difference of their perimeters is 40 m. Find the sides of the two squares.

Solution:

Let x and y be the sides of two squares.

According to the given,

Sum of the areas of two squares = 850 m^2

$x^2 + y^2 = 850$ .... (i)

Difference of their perimeters $= 40$ m

$4x - 4y = 40$

$4(x - y) = 40$

$x - y = 10$

$x = y + 10$ ....(ii)

Substituting (ii) in (i),

$(y + 10)^2 + y^2 = 850$

$y^2 + 100 + 20y + y^2 - 850 = 0$

$2y^2 + 20y - 750 = 0$

$2(y^2 + 10y - 375) = 0$

$y^2 + 10y - 375 = 0$

$y^2 + 25y - 15y - 375 = 0$

$y(y + 25) - 15(y + 25) = 0$

$(y - 15)(y + 25) = 0$

$y - 15 = 0, y + 25 = 0$

$y = 15, y = -25$

Measure cannot be negative.

Therefore, $y = 15$

Substitute $y = 15$ in (ii),

$x = 15 + 10 = 25$

Therefore, the sides of two squares are 25 cm and 15 cm .

OR

(b) Sum of the present ages of two friends are 23 years, five years ago the product of their ages was 42. Find their ages 5 years hence.

Solution:

Let x and $(23 - x)$ be the present ages (in years) of two friends.

According to the given

$(x - 5)(23 - 5 - x) = 42$

$(x - 5)(18 - x) = 42$

$18x - x^2 - 90 + 5x = 42$

$\Rightarrow x^2 - 5x - 18x + 90 + 42 = 0$

$\Rightarrow x^2 - 23x + 132 = 0$

$\Rightarrow x^2 - 11x - 12x + 132 = 0$

$\Rightarrow x(x - 11) - 12(x - 11) = 0$

$\Rightarrow (x - 11)(x - 12) = 0$

$\Rightarrow x = 12, x = 11$

If $x = 12$, then $23 - x = 23 - 12 = 11$

If $x = 11$, then $23 - x = 23 - 11 = 12$

Therefore, the present ages of the two friends are 11 and 12 years.

Hence, their ages after 5 years will be 16 and 17 years.