Grade 10 Telangana Maths 2022 - PART A

Time: 2 Hours 45 Minutes **Maximum Marks : 60**

SECTION – I

Note:

1. Answer any SIX questions choosing three from each of the following two groups, i.e., A and B.

2. Each question carries 2 marks.

GROUP - A

Q1. Expand $\log a^3b^2c^5$.

Solution:

Using the product rule:

$\log(AB) = \log A + \log B$

$\log(a^3b^2c^5) = \log a^3 + \log b^2 + \log c^5$

Using the power rule:

$\log a^m = m\log a$

$= 3\log a + 2\log b + 5\log c$

Q2. If $p(x) = x^2 + 3x + 4$, then find the values of $p(0)$ and $p(1)$.

Solution:

Given polynomial:

$p(x) = x^2 + 3x + 4$

To find $p(0)$:

$p(0) = (0)^2 + 3(0) + 4 = 4$

To find $p(1)$:

$p(1) = (1)^2 + 3(1) + 4 = 1 + 3 + 4 = 8$

Q3. Find the 10^{th} term of the arithmetic progression 3,5,7, … …..

Solution:

We need to find the 10th term (T_{10}).

The general formula for the nth term of an AP is:

$T_n = a + (n-1)d$

where:

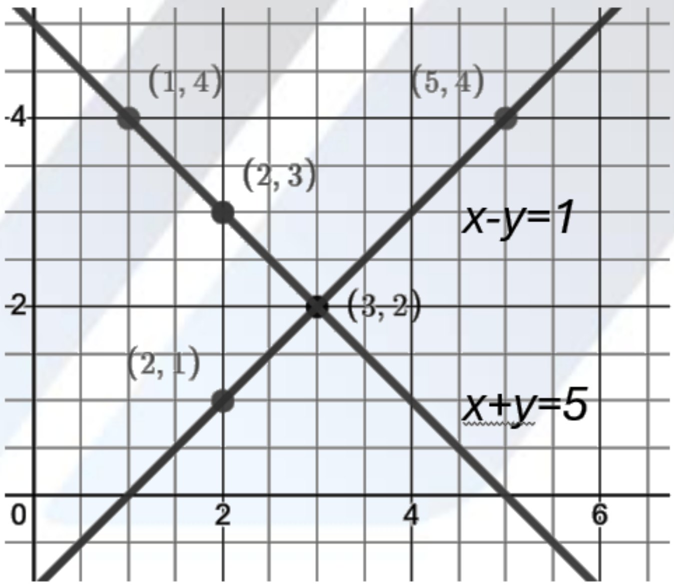
$a = 3$ (first term)

$d = 5 - 3 = 2$ (common difference)

$n = 10$
Substituting values:
$T_{10} = 3 + (10 - 1) \times 2$
$= 3 + 9 \times 2$
$= 3 + 18 = 21$

Q4. Is $(x+2)^2 = x^2 + 3$ a Quadratic equation? Justify.
Solution:
Expanding $(x+2)^2 = x^2 + 3$, we get:
$x^2 + 4x + 4 = x^2 + 3$
Rearranging:
$4x + 1 = 0$
Since the equation does not have an x^2 term, it is a linear equation, not a quadratic equation. Hence, the given equation is not a quadratic equation 0 .

Q5. The solutions of the linear equation $x + y = 5$ are (1,4), (2,3) and (3, 2). The solutions of another linear equation $x - y = 1$ are (3,2), (2,1) and (5, 4). Plot these points on a graph sheet and draw lines.
Solution:



Q6. Find the distance between the points (0,0) and $(\sin\ \theta, \cos\ \theta)$, where $0^\circ \leq \theta \leq 90^\circ$.
Solution:
Using the distance formula:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
For points (0,0) and $(\sin\ \theta, \cos\ \theta)$:

$d=\sqrt{\sin^2\ \theta+\cos^2\ \theta}=\sqrt{1}=1$

Thus, the distance between the given points is 1 unit.

GROUP - B

Q7. Express $\tan\theta$ in terms of $\sin\theta$.

Solution:

We know that $\tan\ \theta=\frac{\sin\theta}{\cos\theta}$.

Using the identity $\cos\ \theta=\sqrt{1-\sin^2\theta}$, we substitute:

$$\tan\theta=\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$$

Thus, $\tan\theta$ in terms of $\sin\theta$ is $\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

Q8. If a dice is rolled once, then find the probability of getting an odd number.

Solution:

A fair die has six faces numbered 1 to 6 . The sample space is:

$S=\{1,2,3,4,5,6\}$

The favourable outcomes for getting an odd number are:

$\{1,3,5\}$

Thus, the probability is:

$$P(\text{ odd number })=\frac{\text{number of favourable outcomes}}{\text{total outcomes}}=\frac{3}{6}=\frac{1}{2}$$

So, the probability of rolling an odd number is $\frac{1}{2}$

Q9. A Joker's cap is in the form of a right circular cone, whose base radius is 7 cm and slant height is 25 cm . Find it's curved surface area.

Solution:

The curved surface area (CSA) of a cone is given by:

$\text{CSA}=\pi rl$

where $r=7$ cm (radius) and $l=25$ cm (slant height).

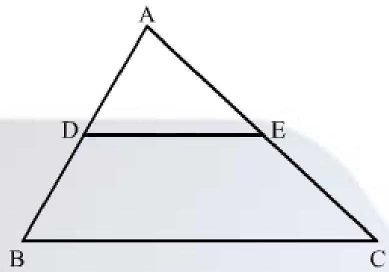
Substituting values:

$$\text{CSA}=\frac{22}{7}\times 7\times 25$$

$=22\times 25=550\ \text{cm}^2$

The curved surface area of the joker's cap is $550\ \text{cm}^2$

Q10. In the given figure, ABC is a triangle. AD = 3 cm, DB = 5 cm, AE = 6 cm and EC = 10 cm. Is DE||BC ? Justify.



Solution:

Consider

$$\frac{AD}{DB} = \frac{3}{5}$$

and

$$\frac{AE}{EC} = \frac{6}{10} = \frac{3}{5}$$

From these two, we concluded that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

We know,

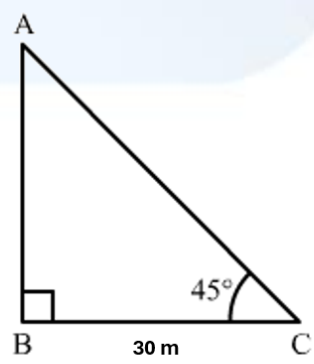
Converse of Basic Proportionality Theorem :- This theorem states that if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

So, using the converse of Basic Proportionality Theorem, we get

$\Longrightarrow DE \parallel BC$

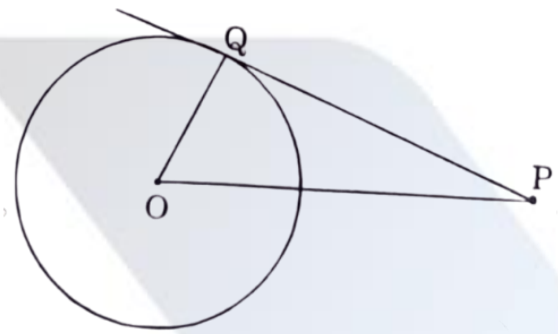
Q11. "The top of a tower is observed at an angle of elevation 45° and the foot of the tower is at a distance of 30 metres from the observer". Draw a suitable diagram for this data.

Solution:



AB- Height of the tower, BC- Distance between observer and foot of the tower.

Q12. In the given figure, ' O ' is the centre of a circle, OQ is the radius and $OQ = 5$ cm.The length of the tangent drawn from external point to radius and OQ = 5 cm,then find the distance between the points ' O ' and ' P '.



Solution:

By the Pythagoras theorem in $\triangle OQP$:

$OP^2 = OQ^2 + PQ^2$

Since PQ is the tangent, and tangent is perpendicular to the radius at the point of contact, we apply the theorem:

$OP^2 = 5^2 + PQ^2$

If PQ is also 5 cm , then:

$OP^2 = 5^2 + 5^2 = 25 + 25 = 50$

$OP = \sqrt{50} = 5\sqrt{2} \approx 7.07$ cm

The distance between points O and P is $5\sqrt{2}$ cm

SECTION – II

Note:

1. Answer any FOUR questions of the following Eight questions

2. Each question carries 4 marks.

Q13. Solve

$2x + y = 5$ and $5x + 3y = 11$

Solution:

We are given the pair of linear equations:

$2x + y = 5..(1)$

$5x + 3y = 11..(2)$

Step 1: Express y in terms of x

From the first equation:

$y = 5 - 2x$

Step 2: Substitute $y = 5 - 2x$ in the second equation

$5x + 3(5 - 2x) = 11$

Expanding:
$5x + 15 - 6x = 11$
Step 3: Solve for x
$-x + 15 = 11$
$-x = -4$
$x = 4$
Step 4: Substitute $x = 4$ back into $y = 5 - 2x$
$y = 5 - 2(4)$
$y = 5 - 8 = -3$
$x = 4,\ y = -3$

Q14. 5,8,11,14, … … is an arithmetic progression. Find the sum of first 20 terms of it.

Solution:

Given AP: 5, 8, 11, 14, ...
We need to find the sum of the first 20 terms.
Step 1: Identify the values
From the given AP,
First term $(a) = 5$
Common difference $(d) = 8 - 5 = 3$
Number of terms $(n) = 20$
Step 2: Use the formula terms of an AP

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Substituting the values:

$$S_{20} = \frac{20}{2}[2(5) + (20-1)(3)]$$

$S_{20} = 10[10 + 57]$
$S_{20} = 10 \times 67$
$S_{20} = 670$

Q15. If $A = \{1,2,3,4,5\}$ and $B = \{2,4,6,8\}$,
then show that $n(\text{A} \cup \text{B}) = n(\text{A}) + n(\text{B}) - n(\text{A} \cap \text{B})$.

Solution:

Step 1: Find $n(A)$ and $n(B)$
Number of elements in A, denoted as $n(A)$:
$n(A) = 5$
Number of elements in B, denoted as $n(B)$:
$n(B) = 4$
Step 2: Find $n(A \cap B)$
The intersection of A and B consists of the common elements:

$A \cap B = \{2,4\}$
Number of elements in $A \cap B$, denoted as $n(A \cap B)$:
$n(A \cap B) = 2$
Step 3: Find $n(A \cup B)$
The union of A and B consists of all unique elements from both sets:
$A \cup B = \{1,2,3,4,5,6,8\}$
Number of elements in $A \cup B$, denoted as $n(A \cup B)$:
$n(A \cup B) = 7$
Step 4: Verify the Formula
Using the formula:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
Substituting the values:
$7 = 5 + 4 - 2$
$7 = 7$

Q16. Write a Quadratic equation, whose roots are $3 + \sqrt{5}$ and $3 - \sqrt{5}$.

Solution:

Step 1: Use the standard quadratic equation formula
If α and β are the roots of a quadratic equation, then the equation is given by:
$x^2 - (\alpha + \beta)x + \alpha\beta = 0$
Step 2: Compute the sum and product of roots
Sum of roots:
$(3 + \sqrt{5}) + (3 - \sqrt{5}) = 6$
Product of roots:
$(3 + \sqrt{5})(3 - \sqrt{5}) = 3^2 - (\sqrt{5})^2 = 9 - 5 = 4$
Step 3: Write the quadratic equation
Substituting these values into the standard equation:
$x^2 - 6x + 4 = 0$

Q17. Write the formula of mode for a grouped data and explain each term of it.

Solution:

Formula for Mode of Grouped Data
The mode for grouped data is determined using the formula:

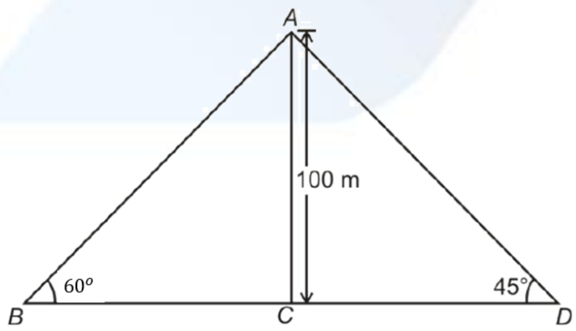
$$\text{Mode } = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Explanation of Each Term:
$l \rightarrow$ Lower boundary of the modal class (class with the highest frequency).
$h \rightarrow$ Size (width) of the modal class interval.
$f_1 \rightarrow$ Frequency of the modal class (highest frequency).

$f_0 \rightarrow$ Frequency of the class preceding the modal class.
$f_2 \rightarrow$ Frequency of the class succeeding the modal class.]

Q18. A box contains 20 cards which are numbered from 1 to 20 . If one card is selected at random from the box, find the probability that it bears
(i) a prime number, (ii) an even number.
Solution:
(i) A Prime Number
Step 1: Identify Prime Numbers from 1 to 20
Prime numbers within this range: 2,3,5,7,11,13,17,19
Total prime numbers = 8
Step 2: Apply Probability Formula
$$P(E)=\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$
$$P(\text{ Prime })=\frac{8}{20}=\frac{2}{5}$$
(ii) An Even Number
Step 1: Identify Even Numbers from 1 to 20
Even numbers: 2,4,6,8,10,12,14,16,18,20
Total even numbers = 10
Step 2: Apply Probability Formula
$$P(\text{ Even })=\frac{10}{20}=\frac{1}{2}$$

Q19. If two persons standing on either side of a tower of height 100 metres observes the top of it with angles of elevation of 60° and 45° respectively, then find the distance between the two persons.
[Note : Consider the two persons and the tower are on the same line.]
Solution:



Let the tower be $AC = 100$ m
Distance between the two persons is BD:

$BD = BC + CD$

Let BD be x and CD be y.

In $\triangle ACD$:

$$\cot 45^{\circ} = \frac{CD}{AC} = \frac{CD}{100} = 1$$

$y = CD = 100$ m

In $\triangle ACB$:

$$\cot 60^{\circ} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{\mathrm{x}}{100} = \frac{1}{\sqrt{3}}$$

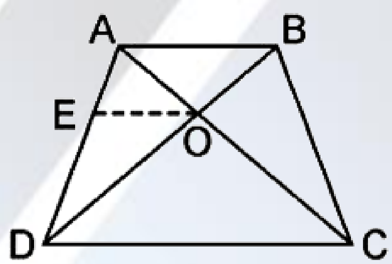
$$x = \frac{100}{\sqrt{3}}$$

$$\mathrm{BD} = \mathrm{BC} + \mathrm{CD} = \mathrm{x} + \mathrm{y} = \frac{100}{\sqrt{3}} + 100$$

$$\mathrm{BD} = 100(\frac{1}{\sqrt{3}} + 1) = 157.73 \text{ m}$$

Q20. In a trapezium ABCD, AB||DC. If diagonals intersect each other at point ' O ', then show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution:



Given: $ABCD$ is a trapezium in which $AB \parallel DC$ in which diagonals AC and BD intersect each other at O .

To Prove: $\frac{\mathrm{AO}}{\mathrm{BO}} = \frac{\mathrm{CO}}{\mathrm{DO}}$

Construction: Through O, draw EO || DC || AB

Proof: In $\triangle ADC$, we have

OE || DC (By Construction)

$\therefore \frac{\mathrm{AE}}{EA} = \frac{\mathrm{AO}}{\mathrm{CO}}$ ... (i) [By using Basic Proportionality Theorem]

In $\triangle ABD$, we have

OE || AB (By Construction)

$\therefore \frac{\mathrm{DE}}{\mathrm{EA}} = \frac{\mathrm{DO}}{\mathrm{BO}}$ ...... (ii) [By using Basic Proportionality Theorem]

From equation (i) and (ii), we get

$$\frac{AO}{CO} = \frac{BO}{DO}$$
$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

SECTION – III

Note:

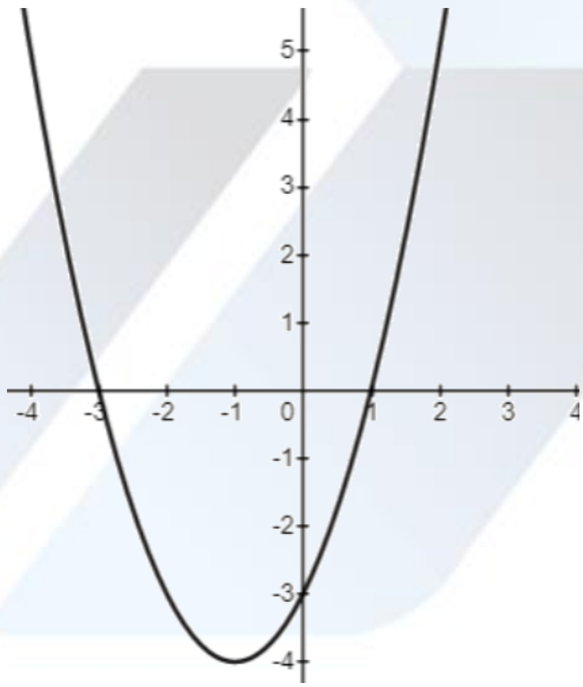
1. Answer any FOUR questions choosing Two from each of the following two groups, i.e., A and B.

2. Each question carries 8 marks.

GROUP - A

Q21. Draw the graph of the polynomial $p(x) = x^2 + 2x - 3$ and find the zeroes of the polynomial from the graph.

Solution:



As the graph intersects x-axis at -3 and 1 So, the zeroes are -3 and 1 .

Q22. Prove that $\sqrt{5} + \sqrt{7}$ is an irrational number.

Solution:

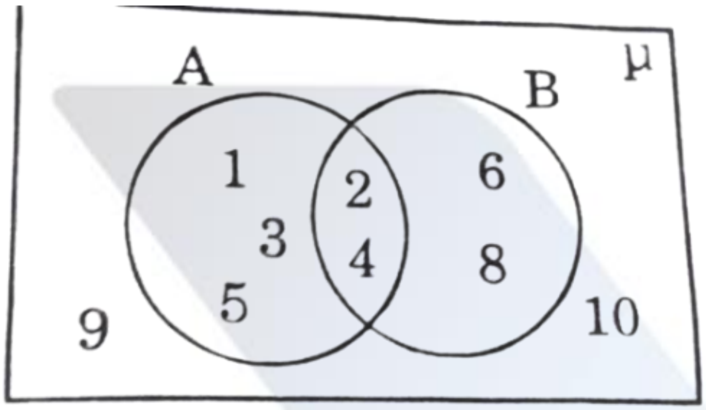
Step 1: Assume $\sqrt{5}+\sqrt{7}$ is Rational
Let us assume, for contradiction, that $\sqrt{5}+\sqrt{7}$ is a rational number.
That means, we can write:
$\sqrt{5}+\sqrt{7}=r$
where r is a rational number.
Step 2: Express One Root in Terms of r
Rearranging the equation:
$\sqrt{7}=r-\sqrt{5}$
Since r is rational and $\sqrt{5}$ is irrational, then $r-\sqrt{5}$ is the difference of a rational and an irrational number.
From the properties of irrational numbers, this difference must be irrational.
However, we know that $\sqrt{7}$ is irrational, which leads to a contradiction.
Step 3: Conclusion
The contradiction arises because of our incorrect assumption that $\sqrt{5}+\sqrt{7}$ is rational.
Thus, our assumption is false, and we conclude that:
$\sqrt{5}+\sqrt{7}$ is an irrational number.

Q23. Show that the distance of the points (5,12), (7,24) and (35,12) from the origin are arranged in ascending order forms an arithmetic progression. Find the common difference of the progression.

Solution:

Step 1: Find the distances from the origin
Using the distance formula:
$d=\sqrt{x^2+y^2}$
Distance of (5,12) from (0,0)
$d_1=\sqrt{5^2+12^2}=\sqrt{25+144}=\sqrt{169}=13$
Distance of (7,24) from (0,0)
$d_2=\sqrt{7^2+24^2}=\sqrt{49+576}=\sqrt{625}=25$
Distance of (35,12) from (0,0)
$d_3=\sqrt{35^2+12^2}=\sqrt{1225+144}=\sqrt{1369}=37$
Step 2: Check if the distances form an AP
An arithmetic progression (AP) has a common difference, given by:
$d_2-d_1=25-13=12$
$d_3-d_2=37-25=12$
Since the common difference $d=12$ is the same for all terms, the distances form an AP.

Q24. From the given Venn diagram, write the sets $A \cup B, A \cap B, A - B$ and $B - A$.



Solution:

Union of Sets $(A \cup B)$

The union of two sets is the set of all distinct elements present in either A or B :

$A \cup B = \{1,2,3,4,5,6,8\}$

Intersection of Sets $(A \cap B)$

The intersection of two sets consists of the common elements in both sets:

$A \cap B = \{2,4\}$

Difference of Sets $(A - B)$

The difference of two sets consists of the elements in A that are not in B :

$A - B = \{1,3,5\}$

Difference of Sets $(B - A)$

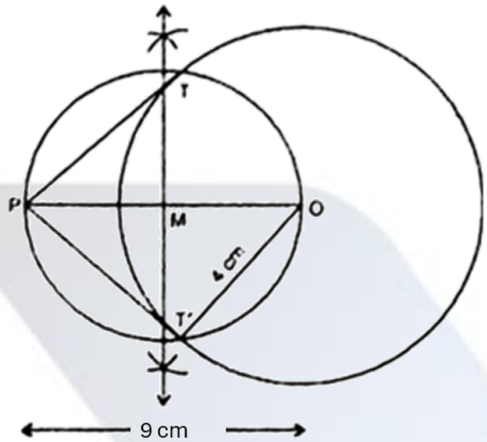
The difference of two sets consists of the elements in B that are not in A :

$B - A = \{6,8\}$

GROUP - B

Q25. Draw a circle of radius 4 cm . From a point 9 cm away from it's centre, construct a pair of tangents to the circle.

Solution:



Steps of Construction:
(i) Take a point O in the plane of the paper and draw a circle of radius 4 cm.
(ii) Mark a point P at a distance of 6 cm from the centre O and join OP.
(iii)Bisect the line segment OP. Let the point of bisection be M.
(iv)Taking M as centre and OM as radius, draw a circle to intersect the given circle at the points T and T'.
(v) Join PT and PT' to gel the required tangents.

Q26. If $\sec\theta + \tan\theta = \mathrm{P}$, then prove that $\sin\theta = \frac{\mathrm{P}^2-1}{\mathrm{P}^2+1}$.

Solution:

Step 1: Express $\sec\theta$ and $\tan\theta$ in terms of P

Using the identity:

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta}$$

$$\sec\theta - \tan\theta = \frac{1}{P}$$

Now, we have two equations:

$$\sec\theta + \tan\theta = P$$

$$\sec\theta - \tan\theta = \frac{1}{P}$$

Step 2: Find $\sec\theta$ and $\tan\theta$

Adding both equations:

$$2\sec\theta = P + \frac{1}{P}$$

$$\sec\theta = \frac{P^2 + 1}{2P}$$

Subtracting the second equation from the first:

$$2\tan\theta = P - \frac{1}{P}$$

$$\tan\theta = \frac{P^2 - 1}{2P}$$

Step 3: Find $\sin\theta$

Using the identity:

$$\sin\theta = \frac{\tan\theta}{\sqrt{1 + \tan^2\theta}}$$

Substituting $\tan\theta = \frac{P^2-1}{2P}$:

$$\sin^2\theta = \frac{\left(\frac{P^2 - 1}{2P}\right)^2}{1 + \left(\frac{P^2 - 1}{2P}\right)^2}$$

Simplifying,

$$\sin^2\ \theta = \frac{(P^2 - 1)^2}{4P^2 + (P^2 - 1)^2}$$

$$\sin^2\ \theta = \frac{(P^4 - 2P^2 + 1)}{P^4 + 2P^2 + 1}$$

$$\sin\ \theta = \frac{P^2 - 1}{P^2 + 1}$$

Q27. Find the median for the following data.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	10	12	8	8

Solution:

Step 1: Compute Cumulative Frequency (CF)

Class Interval	Frequency (f)	Cumulative Frequency (CF)
0 – 10	6	6
10 – 20	10	16
20 – 30	12	28
30 – 40	8	36
40 – 50	8	44

Total number of observations:

$$n = \sum f = 44$$

The median class is the class whose cumulative frequency first exceeds $\frac{n}{2} = \frac{44}{2} =$ 22.
From the cumulative frequency column, the median class is $20 - 30$.
Step 2: Apply the Median Formula

$$\text{Median} = l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$$

where:
$l = 20$ (lower boundary of median class)
$n = 44$ (total number of observations)
$cf = 16$ (cumulative frequency of the class preceding the median class)
$f = 12$ (frequency of the median class)
$h = 10$ (class width)
Step 3: Compute Median

$$\text{Median} = 20 + \left(\frac{22 - 16}{12}\right) \times 10$$

$$= 20 + \left(\frac{6}{12} \times 10\right)$$

$$= 20 + (0.5 \times 10)$$

$$= 20 + 5$$

$$= 25$$

Q28. The sum of the radius of base and height of a solid right circular cylinder is 37 cm . If it's total surface area is 1628 square centimetres (cm^2), then find the volume of the cylinder (use $\pi = \frac{22}{7}$.)

Solution:
Step 1: Define Given Information
Let:
$r =$ Radius of the base of the cylinder
$h =$ Height of the cylinder
Given:
$r + h = 37$
Total Surface Area $= 1628$
We use the Total Surface Area formula:
$\text{TSA} = 2\pi r(r + h)$
Substituting the given values:

$$2 \times \frac{22}{7} \times r \times 37 = 1628$$

Step 2: Solve for r

$$\frac{44}{7} \times r \times 37 = 1628$$
$$\frac{1628 \times 7}{44 \times 37} = r$$
$$r = \frac{11396}{1628} = 7 \text{ cm}$$
Using $r + h = 37$:
$$7 + h = 37$$
$$h = 30 \text{ cm}$$
Step 3: Find the Volume
Volume of a cylinder is given by:
$$V = \pi r^2 h$$
Substituting values:
$$V = \frac{22}{7} \times 7^2 \times 30$$
$$V = \frac{22}{7} \times 49 \times 30$$
$$V = \frac{32340}{7}$$
$$V = 4620 \text{ cm}^3$$