Grade 10 Telangana Mathematics 2023

Instructions:

1 Answer all the questions under Part-A on a separate answer book.

2 Write the answers to the questions under Part-B on the Question Paper itself and attach them to Part-A's answer book.

Part-A

SECTION -I

Note:

1. Answer ALL the following questions.
2. Each question carries 2 marks.

Q1. Find the centroid of the triangle whose vertices are $(2,3),(-4,7)$ and $(2,-4)$.

Solution:

The coordinates of the centring of triangle having vertices $A(2,3), B(4,7)$ and $C(2,-4)$

is given by $G(x, y)$

where $x=\left(\frac{2+(-4)+2}{3}\right)=0, y=\left(\frac{3+7-4}{3}\right)=2$

So, the centroid of the triangle is $G(0,2)$.

Q2. Find the probability of getting a 'vowel' if a letter is chosen randomly from the word "INNOVATION".

Solution:

Probability of getting a Vowel from the Word INNOVATION $=\frac{5}{11}$.

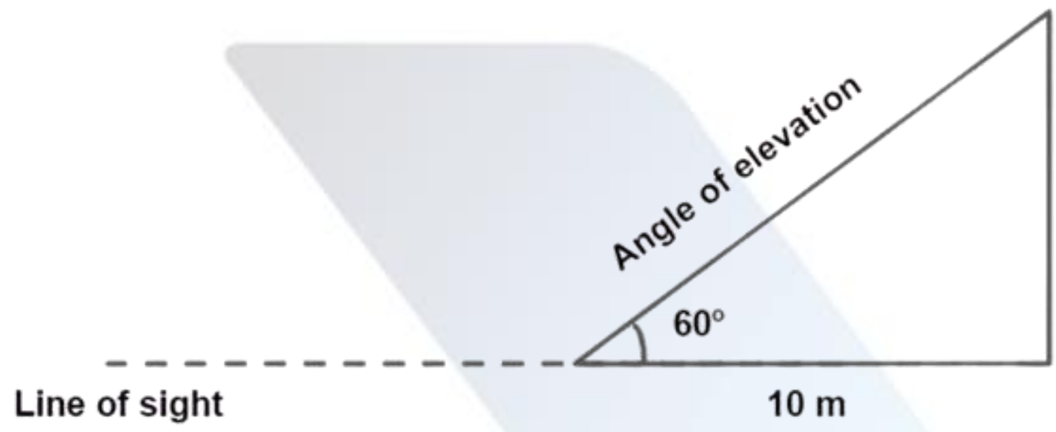
Q3. Express ' $\tan \theta$ ' in terms of ' $\sin \theta$ '.

Solution:

$\tan \theta=\frac{\sin \theta}{\cos \theta}=\frac{\sin \theta}{\sqrt{1-\sin ^2 \theta}}$

Q4. An observer standing at a distance of 10 m from the foot of a tower, observes its top with an angle of elevation of 60°. Draw a suitable diagram for this situation.

Solution:



Q5. The sides of a triangle measure $2\sqrt{2}, 4$ and $2\sqrt{6}$ units. Is it a right-angled triangle? Justify.

Solution:

To check if the triangle is a right-angled triangle, we need to apply the Pythagorean theorem. According to the Pythagoras theorem, in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides.

Let us assume that the side of length $2\sqrt{6}$ is the hypotenuse of the triangle. Then, the sum of the squares of the other two sides must be equal to the square of the length of the hypotenuse.

So, $(2\sqrt{2})^2 + (4)^2 = (2\sqrt{6})^2$

$8 + 16 = 24$

Hence, the given triangle is a right-angled triangle.

Q6. $2\sin^2\theta - 3\sin\theta + 1 = 0$, where $0° < \theta \leq 90°$

Solution:

Let $t = \sin\theta$ hence we have that

$2t^2 - 3t + 1 = 0$

$\Rightarrow 2t^2 - 2t - t + 1 = 0$

$\Rightarrow 2t(t-1) - (t-1) = 0$

$\Rightarrow (t-1)(2t-1) = 0$

$\Rightarrow t = 1$ and $t = \frac{1}{2}$

Hence, we have that $\sin\ \theta = 1$ and $\sin\ \theta = \frac{1}{2}$. Solving these, we get

Remember that θ belongs to $[0, 2\pi]$.

Hence, we have that $\sin\theta = 1$

$\Rightarrow \sin\theta = \sin 90°$

$\Rightarrow \theta = 90°$

And $\sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin 30°$

$\Rightarrow \theta = 30°$

SECTION - II

Note:

1. Answer ALL the following questions.
2. Each question carries 4 marks.

Q7. Write the formula for Median of a grouped data and explain each term of it.

Solution:

For grouped data-

Median $= l + \frac{\frac{N}{2} - c}{f} \times h$

Whereas

$l =$ Lower limit of the median class

$N =$ Total frequency

$c =$ cumulative frequency of class preceding the median class

$f =$ Frequency of median class

$h =$ Width of the median class

Q8. If $x^2 + y^2 = 10xy$, then prove that $2\log\ (x + y) = \log\ x + \log\ y + 2\log\ 2 + \log\ 3$.

Solution:

Adding $2xy$ on both sides,

$x^2 + y^2 + 2xy = 10xy + 2xy$

$(x + y)^2 = 12xy$ $[\because (a + b)^2 = a^2 + b^2 + 2ab]$

Applying logarithm on both sides,

$log\ (x + y)^2 = log\ 12xy$

$2log\ (x + y) = log\ 12xy[\because log_a\ x^n = nlog_a\ x]$

$= log\ 12 + log\ x + log\ y\ [\because log\ abc = log\ a + log\ b + log\ c]$

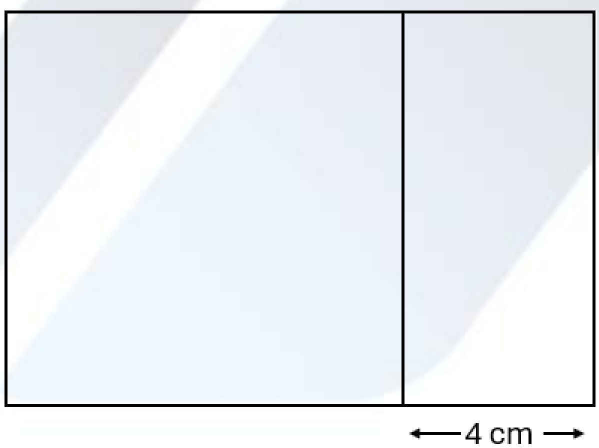
$= log\ (4 \times 3) + log\ x + log\ y$

$= log\ 4 + log\ 3 + log\ x + log\ y$

$= [log\ 2^2 + log\ 3 + log\ x + log\ y]$

$= 2log\ 2 + log\ 3 + log\ x + log\ y\ = 2log\ (x + y)$

Q9. A strip of width 4 cm is attached to one side of a square to form a rectangle, The area of the rectangle formed is 77 cm^2, then find the length of the side of the square.



Solution:

Given that, to create a rectangle, a 4 cm wide strip is fastened to one side of a square. The new rectangle is 77 cm^2 in size.

Let, the width of new rectangle is x and length is $x + 4$

Area of rectangle = length $\times$ breadth

So, $77=(x+4)\times(x)$

$\Rightarrow 77=x^2+4x$

$\Rightarrow x^2+4x-77=0$

$\Rightarrow x^2+11x-7x-77=0$

$\Rightarrow x(x+11)-7(x+11)=0$

$\Rightarrow (x+11)(x-7)=0$

So, $x=-11$ and 7

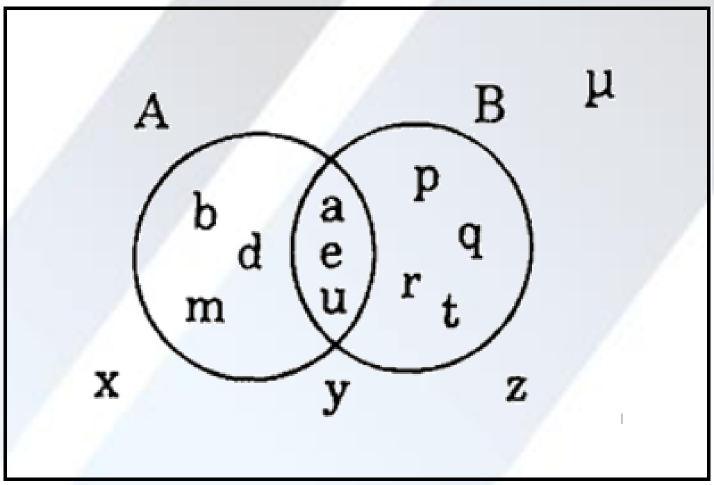
So, the length cannot be negative so $x=7$ which is the side of the square.

Length $=x+4=7+4=11$

Therefore, The width of the new rectangle is 7 cm, length is 11 cm and the length of the side of the square is 7 cm.

Q10. From the given Venn diagram show that

$n(A\cup B)=n(A)+n(B)-n(A\cap B)$



Solution:

From the given figure,

$A=\{b,d,m,a,e,u\}$ i.e. $n(A)=6$

$B=\{p,q,r,t,a,e,u\}$ i.e. $n(B)=7$

$A\cup B=\{b,d,m,a,e,u,p,q,r,t\}$ i.e. $n(A\cup B)=10$

$A\cap B=\{a,e,u\}$ i.e. $n(A\cap B)=3$

Now,

L.H.S. $= n(A \cup B) = 10 \ldots$ (i)

R.H.S. $= n(A) + n(B) - n(A \cap B)$

$= 6 + 7 - 3$

$= 13 - 3$

$= 10 \ldots$ (ii)

From (i) and (ii),

L.H.S. = R.H.S. Hence Proved.

Q11. A box contains four slips numbered 1,2,3,4 and another box contains five slips numbered $5, 6, 7, 8, 9$. If one slip is taken randomly from each box,

(i) How many number pairs are possible?

(ii) What is the probability of both being odd?

(iii) What is the probability of getting the sum of the numbers 10?

Solution:

(i) Possible pair

$1-5, 2-5, 3-5, 4-5, 1-6, 2-6, 3-6, 4-6, 1-7, 2-7,$

$3-7,\ 4-7,\ 1-8,\ 2-8,\ 3-8,\ 4-8,\ 1-9,\ 2-9,\ 3-9,\ 4-9$

Possible pair $= 20$

(ii) Probability of both being odd $= \frac{6}{20} = \frac{3}{10}$

(iii) Probability of getting the sum of the numbers $10 = \frac{4}{20} = \frac{1}{5}$

Q12. Which term of the A.P. $21, 18, 15, \ldots m$ is -81 ? Also find the term which becomes zero.

Solution:

Here, $a = 21, d = 18 - 21 = -3$

and $a_n = -81$, and we have to find n.

As $a_n = a + (n-1)d$

we have $-81 = 21 + (n-1)(-3)$

$-81 = 24 - 3n$

$-105 = -3n$

$n=35$

Therefore, the 35 th term of the given AP is -81 .

Next, we want to know if there is any n for which $a_n=0$. If such an n is there, then

$21+(n-1)(-3)=0$, i.e., $3(n-1)=21$ i.e., $n=8$

So, the eighth term is 0.

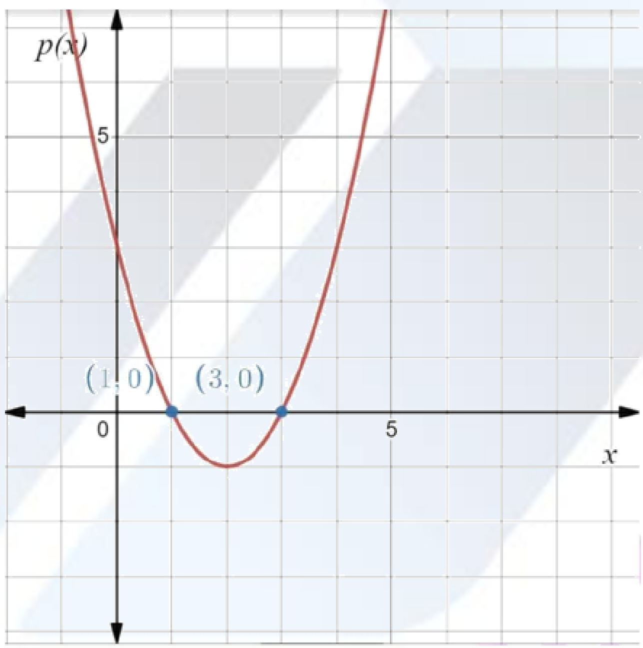
SECTION - III

Note:

1. Answer any 4 questions from the given six questions,
2. Each question carries 6 marks.

Q13. Draw the graph of the quadratic polynomial $p(x)=x^2-4x+3$ and find the zeroes of the polynomial from the graph.

Solution:



We need to recall the following definition.

- The zeroes of a polynomial are the value of a variable that makes the polynomial true.

Given:

$p(x)=x^2-4x+3$

Simplify the polynomial.

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-3)(x-1) = 0$

$x - 3 = 0$ OR $x - 1 = 0$

$x = 3$ OR $x = 1$

Hence, the zeros of the polynomial are $x = 1$ and $x = 3$.

Q14. In an acute angled triangle ABC, if $sin\ (A + B - C) = \frac{1}{2}$ and $cos\ (B + C - A) = \frac{1}{2}$ then find $\angle A, \angle B$ and $\angle C$.

Solution:

$si\,n(A + B - C) = \frac{1}{2} = si\,n\left(\frac{\pi}{6}\right)$

$\Rightarrow A + B - C = 30^\circ \ldots\ldots (1)$

$cos\ (B + C - A) = \frac{1}{2} = cos\ 60^\circ$

$\Rightarrow B + C - A = 60^\circ \ldots\ldots. (2)$

By (1) & (2)

$2B = 90^\circ$

$\Rightarrow B = \left(\frac{90}{2}\right)^\circ = 45^\circ$

$\because A + B + C = 180^\circ$ so by this and (1)

$2A + 2B = 180^\circ + 30^\circ = 210^\circ$

$A + B = 105^\circ$

$A = 105^\circ - 45^\circ$

$A = 60^\circ$

Now $C = 180^\circ - (60^\circ + 45^\circ)$

$C = 75^\circ$

Hence, $\angle A = 60^\circ, \angle B = 45^\circ$ and $\angle C = 75^\circ$

Q15. Find the mode for the following data.

Class Interval	Frequency
$1000 - 1500$	24
$1500 - 2000$	40
$2000 - 2500$	33
$2500 - 3000$	28
$3000 - 3500$	30
$3500 - 4000$	22
$4000 - 4500$	16
$4500 - 5000$	7

Solution:

Firstly, we know the formula of the mode:

$$\text{Mode } = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

Where as;

Modal class = Interval with highest frequency $= 1500 - 2000$

Lower limit of modal class, $(I) = 1500$

Height in class Interval, $(h) = 500$

Frequency of the modal class, $(f_1) = 40$

Frequency of class before modal class, $(f_0) = 24$

Frequency of class after modal class, $(f_2) = 33$

$\therefore$ Substitute the given values : Mode $= 1500 + \frac{40-24}{2(40)-24-33} \times 500$

$$\text{Mode} = 1500 + \frac{16}{80-57} \times 500$$

$$\text{Mode} = 1500 + \frac{16}{23} \times 500$$

$$\text{Mode} = 1500 + \frac{8000}{23}$$

$$\text{Mode} = 1500 + 347.82$$

Mode = 1847.82

Thus;

The mode will be 1847.82

Q16. If $A(-2, 2)$, $B(a, 6)$, $C(4, b)$ and $D(2, -2)$ are the vertices of a parallelogram ABCD, then find the values of a and b. Also find the lengths of its sides.

Solution:

Given that, $A(-2,2)$, $B(a, 6)$, $C(4, b)$ and $D(2, -2)$ are the vertices of a parallelogram $ABCD$.

We know that diagonals of Parallelogram bisect each other.

Mid-point let say O of diagonal AC is given by

$x = \left(\frac{x_1+x_2}{2}\right)$ and $y = \left(\frac{y_1+y_2}{2}\right)$

$O = \left(\frac{-2+4}{2}, \frac{2+b}{2}\right) = \left(1, \frac{2+b}{2}\right)$.

Mid-point let say P of diagonal BD is given by

$P = \left(\frac{a+2}{2}, \frac{6+(-2)}{2}\right) = \left(\frac{a+2}{2}, 2\right) \ldots (2)$

Points O and P are same

Equating the corresponding co-ordinates of both midpoints, we get

$$1 = \frac{a+2}{2} \text{ and } \frac{2+b}{2} = 2$$

$$2 = a + 2 \text{ and } \frac{b}{2} = 1$$

$a = 0$ and $b = 2$

Thus vertices of the parallelogram ABCD are $A(-2,2)$, $B(0,6)$, $C(4,2)$ and $D(2, -2)$.

Using distance formula,

$AB = \sqrt{(-2-0)^2 + (2-6)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units

$BC = \sqrt{(0-4)^2 + (6-2)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

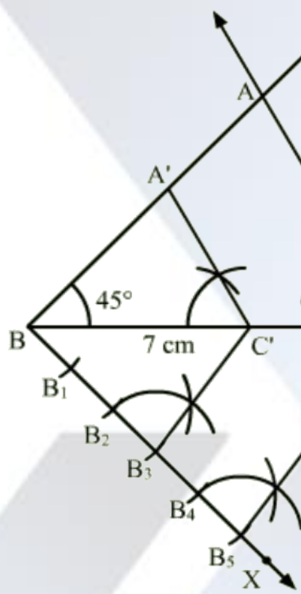
$CD = \sqrt{(4-2)^2 + (2-(-2))^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ units

$\text{DA} = \sqrt{(2-(-2))^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

Hence, lengths of parallelogram $AB = 2\sqrt{5}$ units, $BC = 4\sqrt{2}$ units, $CD = 2\sqrt{5}$ units and $DA = 4\sqrt{2}$ units.

Q17. Construct triangle ABC with $BC = 7$ cm, $\angle B = 45^\circ$ and $\angle C = 60^\circ$. Then construct another triangle similar to $\triangle$ ABC, whose sides are $\frac{3}{5}$ times of the corresponding sides of $\triangle$ ABC.

Solution:



Steps of Construction:

Step I: $BC = 7$ cm is drawn.

Step II: At B, a ray is drawn making an angle of 45° with BC.

Step III: At C, a ray making an angle of 60° with BC is drawn intersecting the previous ray at A.

Step IV: A ray BX is drawn making an acute angle with BC opposite to vertex A.

Step V : Five points B_1, B_2, B_3, B_4 and B_5 at equal distance is marked on BX.

Step VI: B_5C is joined and B_3C is made parallel to B_5C.

Step VII: CA is made parallel CA.

Thus, ABC is the required triangle.

Q18. Prove that $2\sqrt{3} + \sqrt{5}$ is an irrational number.

Solution:

Let us assume that $2\sqrt{3}+\sqrt{5}$ is a rational number.

Let $\mathrm{P}=2\sqrt{3}+\sqrt{5}$ is rational

On squaring both sides we get

$\mathrm{P}^2=(2\sqrt{3}+\sqrt{5})^2=(2\sqrt{3})^2+(\sqrt{5})^2+2\times 2\sqrt{3}\times\sqrt{5}$

$\mathrm{P}^2=12+5+4\sqrt{15}$

$\mathrm{P}^2=17+4\sqrt{15}$

$$\frac{\mathrm{P}^2-17}{4}=\sqrt{15}$$

Since P is rational number, therefore P^2 is also rational & $\frac{\mathrm{P}^2-17}{4}$ is also rational.

But $\sqrt{15}$ is irrational from equation(1)

$$\frac{\mathrm{P}^2-17}{4}=\sqrt{15}$$

Rational $\neq$ irrational

Hence our assumption is incorrect & $2\sqrt{3}+\sqrt{5}$ is an irrational number.

Hence, it's proved.