Grade 10 Telangana Maths 2024 - PART A

Time: 2 Hours 30 Minutes **Maximum Marks : 60**

SECTION – I

Note:

1. Answer all the following questions.
2. Each question carries 2 marks.

Q1. Express 360 as a product of prime factors.

Solution:

To express 360 as a product of prime factors, we use the Fundamental Theorem of Arithmetic.

Prime factorizing 360 :

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

or in exponent form,

$360 = 2^3 \times 3^2 \times 5$

Q2. Is the pair of linear equations $3x - 5y = 7$ and $6x - 10y = 13$ are inconsistent? Justify your answer.

Solution:

The given equations are:

$3x - 5y = 7$

$6x - 10y = 13$

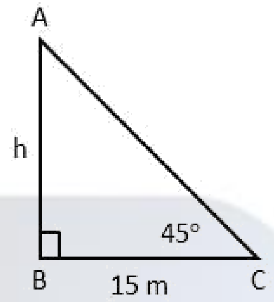
Comparing coefficients:

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{7}{13}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system is inconsistent. Hence, the given equations represent parallel lines with no solution.

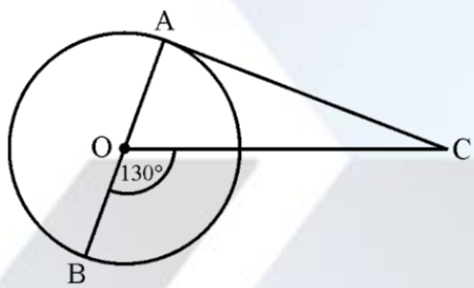
Q3. A flag pole stands vertically on the ground. From a point which is 15 metres away from the foot of the tower, the angle of elevation of the top of the tower is 45°. Draw a suitable diagram for the given data.

Solution:



AB- Height of the Tower, BC- Distance between tower and flagpole, C- Foot of the flagpole

Q4. AOB is the diameter of a circle with centre ' O ' and AC is a tangent to the circle at A. If $\angle BOC = 130^\circ$, then find $\angle ACO$.



Solution:

$\angle BOC + \angle AOC = 180^\circ (Linear\ Pair)$
$\Rightarrow \angle AOC = 180^\circ - 130^\circ = 50^\circ$
$\angle CAO = 90^\circ$ (Because the tangent at any point of a circle is perpendicular to the radius through the point of contact)
In $\triangle AOC$,
$\angle AOC + \angle CAO + \angle ACO = 180^\circ$ (Angle sum property of a triangle)
$\angle ACO = 180^\circ - 90^\circ - 50^\circ$
$\angle ACO = 40^\circ$

Q5. Express ' $\sin\theta$ ' in terms of ' $\tan\theta$ '.
Solution:
We know that:
$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
Rearranging for $\sin\ \theta$:
$\sin\theta = \tan\theta \times \cos\theta$

Using the identity:

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$$

Substituting this in the equation:

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

Thus, sin θ in terms of tan θ is:

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$$

Q6. Construct a Quadratic equation having the roots $\log_2 8$ and $\log_{10} 100$.

Solution:

The given roots are $\log_2 8$ and $\log_{10} 100$.

Calculating the values:

$\log_2 8 = \log_2(2^3) = 3$

$\log_{10} 100 = \log_{10}(10^2) = 2$

A quadratic equation with roots α and β is given by:

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

Substituting $\alpha = 3$ and $\beta = 2$:

$x^2 - (3+2)x + (3\times 2) = 0$

$x^2 - 5x + 6 = 0$

Thus, the required quadratic equation is:

$x^2 - 5x + 6 = 0$

SECTION – II

Note:

1. Answer all the following questions.

2. Each question carries 4 marks.

Q7. Write the formula for Mode of a grouped data and explain each term.

Solution:

$$\text{Mode} = L + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

Where:

L = Lower boundary of the modal class

f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding the modal class

f_2 = Frequency of the class succeeding the modal class

h = Class width (class size).

Q8. Prove that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

Solution:

The given expression is:

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$$

Taking LHS:

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$$

Taking the LCM $(1+\cos\theta)\sin\theta$:

$$\frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

Expanding $(1+\cos\theta)^2$:

$$=\frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{(1+\cos\theta)\sin\theta}$$

Since $\sin^2\theta+\cos^2\theta=1$, we simplify:

$$=\frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$=\frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

Canceling $(1+\cos\theta)$:

$$=\frac{2}{\sin\theta}$$

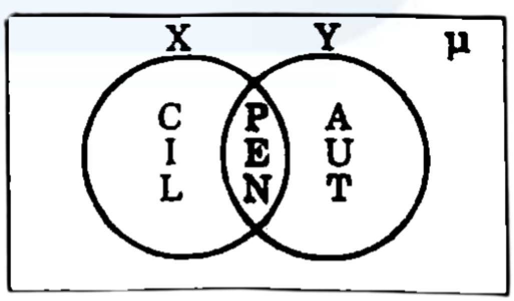
$$=2\operatorname{cosec}\theta$$

Thus,

$$\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$$

Hence proved.

Q9. From the Venn diagram, find the following sets.



(i) X ∪ Y

(ii) X ∩ Y

(iii) X − Y

(iv) Y − X

Solution:

(i) Union $(X \cup Y)$:
The union includes all unique elements from both sets:
$X \cup Y = \{P, E, N, C, I, L, A, U, T\}$
(ii) Intersection $(X \cap Y)$:
The intersection includes only the elements common to both sets:
$X \cap Y = \{P, E, N\}$
(iii) Difference $(X - Y)$:
Elements in X but not in Y :
$X - Y = \{C, I, L\}$
(iv) Difference $(Y - X)$:
Elements in Y but not in X :
$Y - X = \{A, U, T\}$

Q10. In an arithmetic progression, if 4 times of fourth term is equal to 8 times of the eigth term, then prove that twelfth term of the progression is zero.

Solution:

Let the first term of the arithmetic progression be a and common difference be d.
The general term of an AP is given by:
$T_n = a + (n-1)d$
The fourth term:
$T_4 = a + 3d$
The eighth term:
$T_8 = a + 7d$
Given that:
$4T_4 = 8T_8$
Substituting the values:
$4(a + 3d) = 8(a + 7d)$
Expanding both sides:
$4a + 12d = 8a + 56d$
Rearrange the equation:
$4a - 8a = 56d - 12d$
$-4a = 44d$

$a=-11d$
Now, we find the twelfth term (T_{12}) :
$T_{12}=a+11d$
Substituting $a=-11d$:
$T_{12}=-11d+11d=0$
Thus, the twelfth term of the given arithmetic progression is zero. Hence, proved.

Q11. In a bag, there are 5 Red balls, 2 Black balls and 3 White balls. If one ball is selected randomly from the bag, then find the probability of -
(i) getting a Red ball.
(ii) getting not a Red ball.
Solution:
Total number of balls in the bag:
$5(\text{ Red })+2(\text{ Black })+3(\text{ White })=10$
(i) Probability of getting a Red ball:
$$P(\text{ Red })=\frac{\text{Number of Red balls}}{\text{Total number of balls}}=\frac{5}{10}=\frac{1}{2}$$
(ii) Probability of getting not a Red ball:
$$P(\text{ Not Red })=1-P(\text{ Red })=1-\frac{1}{2}=\frac{1}{2}$$

Q12. In a rectangle ABCD, AB $=2x-y$, BC $=15$, CD $=2$ and DA $=x+3y$, then find the values of x and y.
Solution:
$AB=CD$
$2x-y=2$
$BC=DA$
$15=x+3y$
Solving the Equations:
$2x-y=2..(1)$
$x+3y=15..(2)$
Multiply equation (2) by 2:
$2x+6y=30$
Now subtract equation (1):
$(2x+6y)-(2x-y)=30-2$
$7y=28$
$y=4$
Substituting $y=4$ in equation (2):
$x+3(4)=15$

$x + 12 = 15$
$x = 3$
Conclusion:
Thus, $x = 3$ and $y = 4$

SECTION – III

Note:

1. Answer any 4 questions from the given six questions.

2. Each question carries 6 marks.

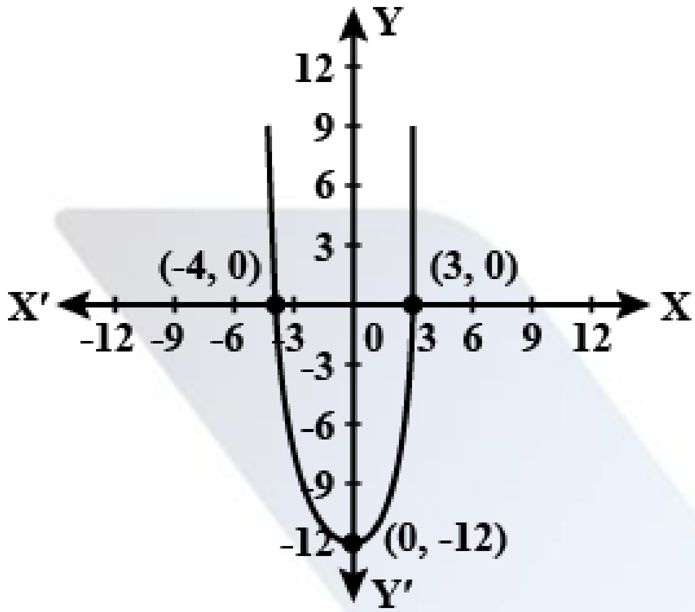
Q13. Prove that $3\sqrt{5} + \sqrt{7}$ is an irrational number.

Solution:

We need to prove that $3\sqrt{5} + \sqrt{7}$ is an irrational number.
Step 1: Assume the given number is rational
Let us assume, for contradiction, that $3\sqrt{5} + \sqrt{7}$ is a rational number.
That means, we can write:
$3\sqrt{5} + \sqrt{7} = r$
where r is a rational number.
Step 2: Express one term in terms of the other
Rearrange the equation:
$\sqrt{7} = r - 3\sqrt{5}$
Since r and $3\sqrt{5}$ are rational and $\sqrt{5}$ is irrational, it follows that $3\sqrt{5}$ is also irrational. Thus, the right-hand side of the equation $(r - 3\sqrt{5})$ is of the form:
(Rational) - (Irrational)
which is irrational.
Step 3: Contradiction
We know that $\sqrt{7}$ is irrational, but the right-hand side of our equation is also irrational.
This contradicts the fact that an irrational number cannot be equal to a rational number.
Thus, our assumption that $3\sqrt{5} + \sqrt{7}$ is rational is wrong.
Conclusion:
Hence, $3\sqrt{5} + \sqrt{7}$ is irrational. Proved.

Q14. Draw the graph of the Quadratic polynomial $p(x) = x^2 + x - 12$ and find the zeroes of the polynomial from the graph.

Solution:



The given quadratic equation is $x^2 + x - 12 = 0$.
Let $y = x^2 + x - 12$, when we substitute different values of x in the equation $y = x^2 + x - 12$ then value of y changes accordingly.
When $x = 0$ then
$y = (0)^2 + 0 - 12 = 0 + 0 - 12 = -12$
The point is $(0, -12)$
When $x = 3$ then
$y = (3)^2 + 3 - 12 = 9 + 3 - 12 = 0$
The point is $(3,0)$
When $x = -4$ then
$y = (-4)^2 - 4 - 12 = 16 - 4 - 12 = 0$
The point is $(-4,0)$
Therefore, the coordinates are $(0, -12)$, $(3,0)$ and $(-4,0)$ and the graph of the quadratic equation $y = x^2 + x - 12$ is as shown above.
Hence, from above graph, we get that $x = -4$ or $x = 3$.

Q15. Find the Arithmetic mean of the following data.

Class Interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	11	14	15	20	15	13	12

Solution:
We will use the direct method to find the arithmetic mean.
Step 1: Prepare the table

Class Interval	Frequency (f)	Midpoint (x)	$f \times x$
$0-10$	11	5	55
$10-20$	14	15	210
$20-30$	15	25	375
$30-40$	20	35	700
$40-50$	15	45	675
$50-60$	13	55	715
$60-70$	12	65	780

Step 2: Apply the formula for Arithmetic Mean

The formula for the arithmetic mean is:

$$\bar{x} = \frac{\sum fx}{\sum f}$$

Summing up the values:

$$\sum f = 11 + 14 + 15 + 20 + 15 + 13 + 12 = 100$$

$$\sum fx = 55 + 210 + 375 + 700 + 675 + 715 + 780 = 3510$$

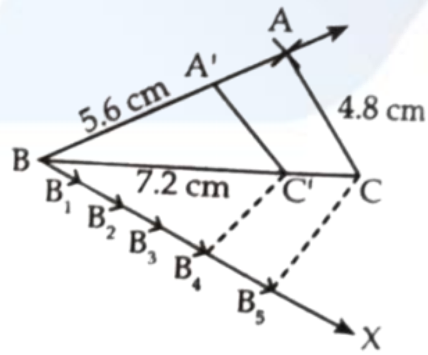
$$\bar{x} = \frac{3510}{100} = 35.1$$

Conclusion:

The Arithmetic Mean of the given data is 35.1

Q16. Construct a triangle ABC with AB $= 5.6$ cm, BC $= 7.2$ cm and CA $= 4.8$ cm. Construct another triangle similar to $\triangle$ ABC, whose sides are $\frac{3}{5}$ times of the corresponding sides of $\triangle ABC$.

Solution:



Q17. The three vertices of a parallelogram ABCD are $A(-1,-2), B(4,-1)$ and $C(6,3)$. Find the coordinates of vertex D and find the area of parallelogram ABCD .

Solution:

Given that $ABCD$ is a parallelogram, the diagonals bisect each other.

Midpoint of diagonal AC should be equal to the midpoint of diagonal BD.

Coordinates of $A(-1,-2), C(6,3)$:

Midpoint of $AC =$

$$\left(\frac{-1+6}{2}, \frac{-2+3}{2}\right)=\left(\frac{5}{2}, \frac{1}{2}\right)$$

Coordinates of $B(4,-1)$ and $D(x, y)$:

Midpoint of $BD =$

$$\left(\frac{4+x}{2}, \frac{-1+y}{2}\right)$$

Equating midpoints:

$$\frac{4+x}{2}=\frac{5}{2}, \frac{-1+y}{2}=\frac{1}{2}$$

Solving for x :

$4+x=5 \Rightarrow x=1$

Solving for y :

$-1+y=1 \Rightarrow y=2$

Thus, the coordinates of D are $(1,2)$.

The area of a parallelogram given its vertices $(x_1, y_1),(x_2, y_2),(x_3, y_3),(x_4, y_4)$ is:

$$\text{Area }=\left|\frac{1}{2}\left(x_1 y_2+x_2 y_3+x_3 y_4+x_4 y_1-\left(y_1 x_2+y_2 x_3+y_3 x_4+y_4 x_1\right)\right)\right|$$

Substituting $A(-1,-2), B(4,-1), C(6,3), D(1,2)$:

$$\text{Area }=\left|\frac{1}{2}((-1)(-1)+4(3)+6(2)+1(-2)-(-2)(4)-(-1)(6)-3(1)-2(-1))\right|$$

$$=\left|\frac{1}{2}(1+12+12-2-(-8)-(-6)-3-(-2))\right|$$

$$=\left|\frac{1}{2}(1+12+12-2+8+6-3+2)\right|$$

$$=\left|\frac{1}{2}(36)\right|$$

$$=18$$

Thus, the area of parallelogram $ABCD$ is 18 square units .

Q18. Due to heavy floods in the state thousands were rendered homeless. The State Government decided to provide canvas for 1500 tents. The lower part of each tent is cylindrical of base radius 2.8 meters and height 3.5 meters with conical upper part of same base radius but of height 2.1 meters. If the canvas used to make the tent costs Rs 100 per square meter, find the total cost of canvas to construct the tents.

Solution:

The given tent consists of:

A cylindrical lower part with radius $r = 2.8$ m and height $h_1 = 3.5$ m.

A conical upper part with same radius $r = 2.8$ m and height $h_2 = 2.1$ m.

We need to find the total canvas required and then calculate the cost, taking $\pi = \frac{22}{7}$.

Step 1: Find the Curved Surface Area (CSA)

The total canvas required covers only the curved surface area of the cylinder and cone, excluding the base.

Curved Surface Area of Cylinder:

$\text{CSA}_{\text{cyl}} = 2\pi r h_1$

Substituting the values:

$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5$$

$$= \frac{44 \times 2.8 \times 3.5}{7}$$

$$= \frac{431.2}{7} = 61.6 \text{ m}^2$$

Curved Surface Area of Cone:

First, we find the slant height l using the formula:

$$l = \sqrt{r^2 + h_2^2}$$

$$= \sqrt{(2.8)^2 + (2.1)^2}$$

$$= \sqrt{7.84 + 4.41} = \sqrt{12.25} = 3.5 \text{ m}$$

Now, the CSA of the cone is:

$\text{CSA}_{\text{cone}} = \pi r l$

$$= \frac{22}{7} \times 2.8 \times 3.5$$

$$= \frac{22 \times 2.8 \times 3.5}{7}$$

$$= \frac{215.6}{7} = 30.8 \text{ m}^2$$

Step 2: Find Total Canvas Required

Total Canvas Area $= \text{CSA}_{\text{cyl}} + \text{CSA}_{\text{cone}}$

$= 61.6 + 30.8 = 92.4 \text{ m}^2$

For 1500 tents:

Total Canvas $= 1500 \times 92.4$

$= 1{,}38{,}600 \text{ m}^2$

Step 3: Calculate the Total Cost
Given that the cost per square meter is Rs. 100:
Total Cost $= 1{,}38{,}600 \times 100$
$= \text{Rs}1{,}38{,}60{,}000$
Final Answer:
The total cost of canvas to construct 1500 tents is Rs. 1,38,60,000