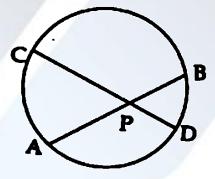


Grade 10 Kerala Mathematics 2020

Answer any 3 questions from 1 to 4. Each question carries 2 scores. Q1. (a) Write the 6th term of the arithmetic sequence 1, 25, 49, 73, 97 (b) How many perfect square terms are there in the arithmetic sequence 97, 73, 49...? **Solution:** (a) Given an arithmetic sequence is 1, 25, 49, 73, 97, First term (a) = 1d = 25 - 1 = 24. $T_6 = a + (n-1)d$ 6^{th} term = a + 5 d \Rightarrow 1 + 5 × 24 $\Rightarrow 1 + 120 = 121$ (b) Given an arithmetic sequence is 97, 73, 49, The perfect square numbers are 1, 4, 9, 16,.... Hence the given sequence is 97, 73, 49, 25, 1. From the above sequence, the perfect square numbers are 49,25, and 1. : The number of perfect square terms are 3.

Q2. Chords AB and CD are intersecting at P.AB = 10 cm, PB = 4 cm, PD = 3 cm.



(a) What is the length of PA?
(b) Find the length of the PC.
Solution:
Given:
AB = 10 cm
PB = 4 cm
PD = 3 cm
(a) PA = AB - PB



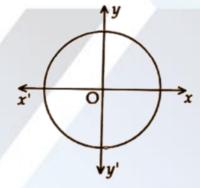
= 10 - 4= 6 cm (b) PC × PD = PA × PB PC = $\frac{PA \times PB}{PD}$ = $\frac{6 \times 4}{3}$ = 8 cm

Q3. Write the polynomial $p(x) = x^2 - 4$ as the product of two first degree polynomials.

Solution:

Given polynomial = $p(x) = x^2 - 4$ First-degree polynomial $x^2 - 4$ = (x + 2)(x - 2)

Q4. In the figure, O is the centre of the circle and $x^2 + y^2 = 25$ is the equation of the circle.



(a) What is the radius of the circle?

(b) Write the equation of the circle whose centre is at origin and the radius is 3.

Solution:

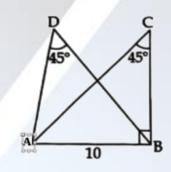
Given circle is $x^2 + y^2 = 25$ (a) $x^2 + y^2 = r^2$ $r^2 = 25$ $r = \sqrt{25} = 5$ $\therefore r = 5$ The radius of the circle is 5 units. (b) Given radius = 3 Hence the equation of the circle = $x^2 + y^2 = r^2$. $\Rightarrow x^2 + y^2 = 3^2$



 $\Rightarrow x^2 + y^2 = 9$

Answer any 5 questions from 5 to 11. Each question carries 3 scores.

- Q5. (a) Write the first term and the common difference of the arithmetic sequence whose algebraic expression is 3n + 5. (b) The first term of an arithmetic sequence is 8 and the common difference is 5. Write its algebraic form. **Solution:** Given $x_n = 3n + 5$ (a) If n = 1, then the first term can be obtained. First term $= 3 \times 1 + 5 = 3 + 8 = 8$ Common difference $= 3[\because \text{ coefficient of } n \text{ be the } d]$ (b) Given a = 8d = 5 $x_n = dn + (a - d)$ = 5n + (8 - 5)= 5n + 3
- Q6. In the figure, $\angle ABC = 90^\circ$, $\angle C = \angle D = 45^\circ$, AB = 10 cm.



(a) What is the length of AC?

(b) What is the radius of the circumcircle of triangle ABC?

(c) What is the radius of the circumcircle of triangle ABD?

Solution:

In right \bigtriangleup ABC, the angles are 45°, 45°, 90°

- $\Rightarrow 1:1:\sqrt{2}$
- \Rightarrow AB: BC: AC
- $\Rightarrow x: x: x\sqrt{2}$
- $\Rightarrow 10:10:10\sqrt{2}$
- (a) The length of AC = $10\sqrt{2}$ cm.



- (b) The radius of the circumcircle of \triangle ABC
- = Half of the hypotenuses AC

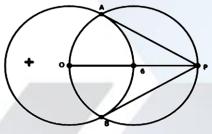
$$=\frac{10\sqrt{2}}{2}$$

$$=5\sqrt{2}$$
 cm

- (c) The radius of the circumcircle of \triangle ABD
- = Half of the hypotenuses AC

$$=\frac{10\sqrt{2}}{2}$$
$$=5\sqrt{2} \text{ cm}$$

Q7. Draw a circle of radius 3 cm . Mark a point P at a distance of 6 cm from the centre of the circle. Draw tangents from P to the circle. Solution:



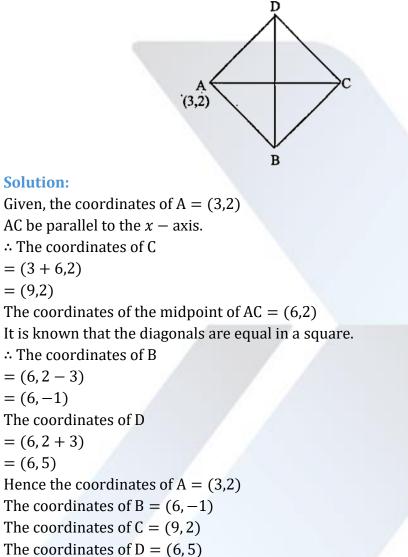
Q8. (a) What is the common difference of the arithmetic sequence x-1, x, x + 1,
(b) If x - 1 is an even number, what is the next even number?
(c) Prove that the product of 2 consecutive even numbers added to 1 gives a perfect square.

Solution:

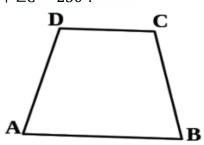
(a) Given sequence x - 1, x, x + 1, d = x - (x - 1) = x - x + 1 d = 1(b) Given even number = x - 1Next even number = x - 1 + 2 = x + 1(c) Let consecutive two even number be (x - 1) and (x + 1)By question, the product of two consecutive numbers is +1, $(x - 1)(x + 1) + 1 = x^2 - 1 + 1 = x^2$ Here x^2 being a perfect square.



Q9. In the figure, ABCD is a square. Its diagonals are parallel to the coordinate axes. AC = 6 and the coordinates of A is (3,2). Write the coordinates of the vertices of C, B and D.



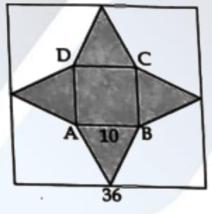
Q10. In the figure, ABCD is a cyclic quadrilateral. Also, $\angle A + \angle D = 210^\circ$, $\angle D + \angle C = 250^\circ$.





(a) What is $\angle A + \angle C$? (b) Find the measures of $\angle A$ and $\angle C$. **Solution:** Given that $\angle A + \angle D = 210^\circ$, $\angle D + \angle C = 250^\circ$, (a) $\angle A + C = 180^{\circ}$ (cyclic quadrilateral) $\angle A + \angle D = 210^{\circ} \rightarrow (1)$ $\angle D + \angle C = 250^{\circ} \rightarrow (2)$ Adding (1) + (2) $\Rightarrow \angle A + \angle C + 2 \angle D = 460^{\circ}$ \Rightarrow 180 + 2 \angle D = 460° $\Rightarrow 2 \angle D = 28^{\circ}$ $\angle D = 140^{\circ}$ (b) $\angle A = 210^{\circ} - 140^{\circ}$ $\angle A = 70^{\circ}$ $\angle C = 250^{\circ} - 140^{\circ}$ $\angle C = 110^{\circ}$

Q11. The figure of a square sheet paper is shown below. Length of one side of the paper sheet is 36 cm and AB = 10 cm. The shaded portion is cut out and folded into a square pyramid.



- (a) What is the length of the base of the pyramid?
- (b) What is the slant height of the pyramid?

(c) Find the lateral surface area of the pyramid.

Solution:

Given, Side of the paper sheet = 36 cm

AB = 10 cm

(a) Base edge of the pyramid



AB = 10 cm (b) Slant height of the pyramid $= \frac{(36 - 10)}{2}$ $= \frac{26}{2}$ = 13 cm [:: a + 2l = 36, side of the larger square] (c) Lateral surface area = 2al= $2 \times 10 \times 13$ = 260 cm^2

Answer any 7 questions from 12 to 21. Each question carries 4 scores.

Q12. (a) What is the sum of the first 5 terms of the arithmetic sequence 1,3,5,7

....?

(b) What is the sum of the first *n* terms of the arithmetic sequence 1,3,5,7

....?

(c) Find the sum of the first *n* terms of the arithmetic sequence $\left(\frac{1}{n}\right), \left(\frac{3}{n}\right), \left(\frac{5}{n}\right), \left(\frac{7}{n}\right)$

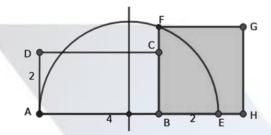
(d) What is the sum of the first 2020 terms of the arithmetic sequence $\left(\frac{1}{2020}\right)$,

```
\left(\frac{3}{2020}\right), \left(\frac{5}{2020}\right), \left(\frac{7}{2020}\right), \dots
Solution:
(a) Given sequence = 1,3,5,7,...
Sum = n^2
Here n = 5
∴ The sum of the first 5 terms
= 5^{2}
= 25
(b) The sum of the first n terms = n^2
(c) Given sequence = \left(\frac{1}{n}\right), \left(\frac{3}{n}\right), \left(\frac{5}{n}\right), \left(\frac{7}{n}\right).....
The sum of the first n terms =\frac{n^2}{n}=n
(d) Given sequence (\frac{1}{2020}), (\frac{3}{2020}), (\frac{5}{2020}), (\frac{7}{2020}), \dots
The sum of the first n terms = \frac{n^2}{n} = n
The sum of the first 2020 terms
= n
= 2020
```



Q13. Draw a rectangle of length 4 cm and breadth 2 cm . Draw a square having the same area of the rectangle.

Solution:



Step 1 Construct a rectangle ABCD with length 4 cm and breadth 2 cm.

Step 2 Extend the line AB to BE, such that BE = BC.

Step 3 Draw a semi-circle with AE as diameter.

Step 4 Draw a parallel line BF through B and then BF as length, draw the square BFGH.

Q14. In a school, the total number of students in 10A division is equal to the number of students in 10B. One student is to be selected from each division. The number of

boys in 10A is 20. The probability of selecting a boy from 10A is $\left(\frac{2}{5}\right)$ and that of class B is $\left(\frac{3}{5}\right)$

class B is $\left(\frac{3}{5}\right)$.

(a) How many students are there in 10A?

(b) What is the probability of selecting a girl from 10A?

(c) How many boys are there in 10B?

(d) What is the probability of both the selected students being boys? **Solution:**

| Class | XA | XB |
|-------|----|----|
| Boys | 20 | 30 |
| Girls | 30 | 20 |
| Total | 50 | 50 |

Given the probability of boys in $XA = \frac{2}{5}$

Given the probability of boys in XB = $\frac{3}{5}$

(a) Number of boys in XA

$$= 20 \times \frac{5}{2}$$
$$= 50$$



(b) Probability of girl from XA = $1 - \frac{2}{5}$ = $\frac{3}{5}$ (c) Number of boys in XB = $50 \times \frac{3}{5}$ = 10×3 = 30(d) Both being boys = $\frac{2}{5} \times \frac{3}{5}$ = $\frac{6}{25}$

Q15. Perimeter of the rectangle in the figure is 36 cm. AC = $\sqrt{164}$ cm.



```
(a) What is AB + BC ?

(b) Find the length of AB.

Solution:

Given the perimeter = 36 cm

AC = \sqrt{164} cm

(a) 2(l + b) = 36

\therefore AB + BC = \frac{36}{2}

= 18 cm

(b) Let AB = x, BC = 18 - x

Since \triangle ABC is right-angled, according to Pythagoras theorem,

AC<sup>2</sup> = AB<sup>2</sup> + BC<sup>2</sup>

164 = x^2 + (18 - x)^2

x^2 + 324 - 36x + x^2 = 164

2x^2 - 36x = 164 - 324 = -160

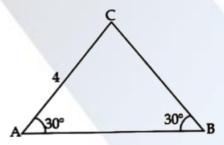
Dividing by 2

x^2 - 18x = -80 [ square completion method]
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 $x^{2} - 18x + 81 = -80 + 81$ $(x - 9)^{2} = 1$ $x - 9 = \pm 1$ x - 9 = 1 or x - 9 = -1 x = 10 or = 8So, AB = 10 cm.

Q16. In triangle ABC, $\angle A = \angle B = 30^\circ$, AC = 4 cm.

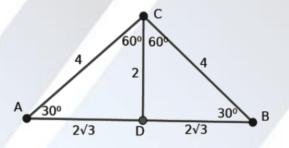


(a) What is the length of BC?

(b) Find the length of AB.

(c) In triangle PQR, PQ = $4\sqrt{3}$ cm, $\angle P = \angle Q = 30^{\circ}$. Draw the triangle.

Solution:

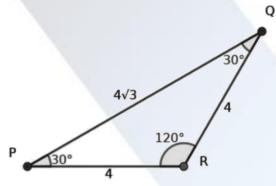


Given , $\angle A = \angle B = 30^{\circ}$ AC = 4 cmDraw CD \perp AB. In right \triangle ADC, $30^{\circ}: 60^{\circ}: 90^{\circ}$ $1: \sqrt{3}: 2$ DC: AD:AC $x: x\sqrt{3}: 2x$ $2: 2\sqrt{3}: 4$ DC = 2 AD = $2\sqrt{3}$



AC = 4 (a) Length of BC = AC = 4 cm (b) Length of AB = AD + DB = $2\sqrt{3} + 2\sqrt{3}$ = $4\sqrt{3}$ cm

(c) Draw PR = 4 cm and make R be 120° and join PQ. \triangle PRQ is the required triangle.



Q17. (a) If $p(x) = x^2 - 7x + 13$, what is p(3)?

(b) Write the polynomial p(x) - p(3) as the product of two first degree polynomials.
(c) Find the solutions of the equation p(x) - p(3) = 0.

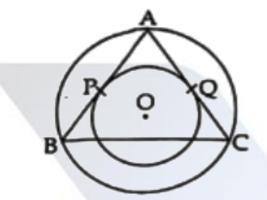
(c) i mu the solutions of the equ

Solution:

(a) Given polynomial $p(x) = x^2 - 7x + 13$ $p(3) = 3^2 - 7 \times 3 + 13$ = 9 - 21 + 13 = 1(b) $p(x) - p(3) = x^2 - 7x + 13 - 1$ $= x^2 - 7x + 12$ = (x - 3)(x - 4)Hence the product two first degree polynomial = (x - 3)(x - 4)(c) p(x) - p(3) = 0 $x^2 - 7x + 12 = 0$ $\Rightarrow (x - 3)(x - 4) = 0$ $\Rightarrow (x - 3) = 0$ or (x - 4) = 0ie., x = 3 or x = 4. Hence the solution is x = 3 and 4.



Q18. In the figure, O is the centre of both circles. AB and AC touch the small circle at P and Q. A, B and C are points on the larger circle.



- (a) If AP = 5 cm, then what is the length of AQ?
- (b) Prove that AB = AC.

(c) If AP = 5 cm and $\angle A = 90^\circ$, then what is the radius of the small circle? **Solution:**

(a) Given AP = 5 cm

Hence the length of AQ = 5 cm. [: Same tangents from A]

(b) AB and AC are tangents

OP \perp AB and OQ \perp AC [:: Chord bisector theorem]

AP = BP and AQ = QC

$$AB = AC$$

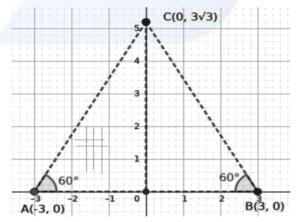
(c) Given $\angle A = 90^{\circ}$

So it can be seen that APOQ is a square.

[:: OP and OQ be radii $\therefore \angle APO = \angle AQO = 90^{\circ}$. ie., $\angle POQ = 90^{\circ}$]

Hence the radius of the small circle = 5 cm.

Q19. Draw the coordinate axes and mark the points A (-3,0), B(3,0), C $(0,3\sqrt{3})$. **Solution:**





Q20. A sector of radius 12 cm and the central angle 120° is rolled up to a cone.

(a) What is the slant height of the cone?

(b) Find the radius and height of the cone.

(c) What is the central angle of the sector to be used to make a cone of base radius

 $\sqrt{2}$ cm and height 4 cm ?

Solution:

(a) The radius of the sector = 12 cm [The radius of the sector to be the slant height of the cone]

(b)
$$\frac{r}{l} = \frac{x^{\circ}}{360^{\circ}}$$

 $\Rightarrow \frac{r}{12} = \frac{120^{\circ}}{360^{\circ}}$
 $\Rightarrow 360r = 12 \times 120$
 $\Rightarrow r = \frac{[12 \times 120]}{360}$
 $\therefore r = 4 \text{ cm}$
Radius = 4 cm
 $h = \sqrt{l^2 - r^2}$
 $= \sqrt{12^2 - 4^2}$
 $= \sqrt{124} - 16$
 $= \sqrt{128}$
 $= 8\sqrt{2} \text{ cm}$
(c) $\frac{r}{l} = \frac{x^{\circ}}{360^{\circ}}$
Central angle $(x^{\circ}) = \frac{(360 \times r)}{l}$
To find 'l'
 $l = \sqrt{h^2 + r^2}$
 $r = \sqrt{2}, h = 4 \text{ cm}$
 $\therefore l = \sqrt{4^2 + \sqrt{2^2}}$
 $= \sqrt{16 + 2}$
 $= \sqrt{18}$
 $= 3\sqrt{2}$
 \therefore Center angle $(x^{\circ}) = 360 \times rl$
 $= \frac{(360 \times \sqrt{2})}{3\sqrt{2}}$
 $= 120^{\circ}$



Q21. (a) What is the slope of the line passing through the points (5,0) and (3,2)? Write the equation of the line.

(b) The *x* coordinate of a point on the line x - y = 5 is 5. What is the *y* coordinate of that point?

(c) Write the coordinates of the point of intersection of the lines x + y = 5 and x - y = 5.

Solution:

(a) Given points are (5,0) and (3,2).

Slope =
$$\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

= $\frac{(2 - 0)}{(3 - 5)}$
= $\frac{2}{-2}$
= -1
Equation of the line is
 $y - y_1 = m(x - x_1)$
= $y - 0$
= $-1(x - 5)$
 $y = -x + 5$
 $x + y - 5 = 0$ is the equation.
(b) If $x = 5$, then
 $5 - y = 5$
 $-y = 5 - 5$
= 0
The y coordinate of that point is 0
(c) Given $x + y = 5$ and $x - y = 0$
 $x + y = 5 \rightarrow (1)$
 $x - y = 0 \rightarrow (2)$
Solve (1) and (2),
 x and $y = 5$, and 0
So the coordinates = (5,0).

Answer any 5 questions from 22 to 28. Each question carries 5 scores.

Q22. Sum of the first four terms of an arithmetic sequence is 72. Sum of first *n* terms is also 72.

(a) What is the 5th term of the sequence?

- (b) Find the sum of the first five terms.
- (c) Write the sequence.

Solution:

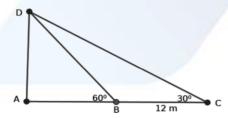


Given sum of the first 4 terms = 72Sum of the first 9 terms = 72(a) 5^{th} term $x_5 = \frac{(S_9)}{9}$ $=\frac{72}{9}$ = 8 (b) Sum of the first 5 terms S_5 $= S_4 + x_5$ = 72 + 8= 80(c) $x_3 = \frac{(S_5)}{5}$ $=\frac{80}{5}$ = 16 $x_3 + 2d = x_5$ 16 + 2d = 82d = -8d = -4 $x_1 = x_3 - 2d$ $= 16 - [2 \times -4]$ = 24

The sequence = 24,20,16,12,8,...

Q23. A boy standing at the edge of a canal sees the top of a tree on the other edge at an elevation of 60°. Stepping 12 m back, he sees it at an elevation of 30°. Find the height of the tree.

Solution:



Let AB be the height of the tree.

B be the first position of the boy.

C be the second position of the boy.

BC = 12

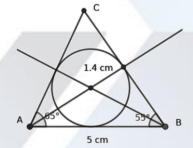
 $\angle C = 30^{\circ}$

 $\angle BPA = 60^{\circ}$



 $\angle PBA = 30^{\circ}$ $\angle A = 90^{\circ}$ $\triangle CBD$ is an isosceles triangle. $\therefore BC = BD = 12$ From right $\triangle BAP$, 30° ; 60° ; 90° $1:\sqrt{3}: 2$ $\Rightarrow AD: AB: BD$ $\Rightarrow x : x\sqrt{3}: 2x$ $\Rightarrow 6: 6\sqrt{3}: 12$ $\therefore AB = 6\sqrt{3}$ Hence the height of the tree = $6\sqrt{3}$ m.

Q24. In \triangle ABC, AB = 5 cm, \angle A = 65°, \angle B = 55°. Draw the triangle ABC and the incircle. Measure the radius of the incircle. Solution:



- Q25. A circle is drawn with (5,3) as a centre. (5,6) is a point on the circle.
 - (a) What is the radius of the circle?
 - (b) Write the equation of the circle.
 - (c) What is the distance from the centre of the circle to the x-axis?

(d) What is the length of the tangents from the origin to the circle? **Solution:**

(a) Radius of the circle = 6 - 3 = 3 units

(b) Equation of the circle

$$\Rightarrow (x-a)^2 + (y-b)^2 = r^2$$

$$\Rightarrow (x-5)^2 + (y-3)^2 = 3^2$$

$$\Rightarrow x^2 - 10x + 25 + y^2 - 6y + 9 = 9$$

 $\Rightarrow x^{2} + y^{2} - 10x - 6y + 25 = 0$

(c) The distance from the centre of the circle to the x -axis is the radius of the circle = 3 units.

(d) The length of the tangents from the origin to the circle = 5 units. (The x –axis being itself a tangent.)



Q26. (a) The radius of a solid sphere is 6cm. Find its volume and surface area.(b) It is cut into two equal halves. What is the total surface area of each hemisphere? What is the volume of a hemisphere?Solution:

(a) Given radius = 6 cm; Volume = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi 6^3$ = 288 π cm³ Total Surface Area = $4\pi r^2$ = $4\pi \times 6^2$ = 144 π cm² (b) Total Surface Area of hemisphere = $3\pi r^2$ = $3\pi \times 6^2$ = 108 π cm² The volume of hemisphere = $\frac{2}{3}\pi r^3$ = $\frac{2}{3}\pi 6^3$ = 144 π cm³

Q27. The table below shows children of a class sorted according to their marks in the examination.

| | Marks | Number of Children |
|---|---------|-----------------------|
| | 0 - 10 | 4 |
| | 10 - 20 | 7 |
| 4 | 20 - 30 | 10 |
| | 30 - 40 | 12 |
| | 40 - 50 | 8 |
| | | 41 |



(a) If we arrange the children from the one with the least mark to the one with the greatest, then what will be the assumed mark of the 12^{th} student?

(b) Compute the median mark.

Solution:

| Class | Frequency | Marks | Cumulative Frequency |
|---------|-----------|-------|-------------------------|
| 0 - 10 | 4 | < 10 | 4 |
| 10 – 20 | 7 | < 20 | 11 |
| 20 - 30 | 10 | < 30 | 21 |
| 30 - 40 | 12 | < 40 | 33 |
| 40 – 50 | 8 | < 50 | 41 |
| Total | 41 | | |

(a) Assumed mark of the 12th student

$$= 20 + \frac{[30 - 20]}{[10 \times 2]}$$

= 20 + $\frac{1}{2}$
= 20.5
(b) $\frac{N}{2} = \frac{41}{2} = 20.5$
Median class = 20 - 30
 $l = 20$
 $F = 11$
 $f = 10$
Median = $l + [\frac{(\frac{N}{2} - F)}{f}] \times c$
= 20 + $\frac{(20.5 - 11)}{10} \times 10$
= 20 + $\frac{9.5}{10} \times 10$
= 20 + 9.5



= 29.5 ∴ Median mark = 29.5

Q28. In the figure, O is the centre of the large circle. The centre of the small circle is C. OP is a tangent to the small circle. $\angle BOQ = 50^{\circ}$.

```
(a) \angle OAQ = ?
(b) \angle OCP = ?
(c) \angle APO = ?
(d) \angle POQ = ?
Solution:
Given \angle BOQ = 50^{\circ}
(a) Since \triangle AOQ is isosceles, their base angles are equal.
\angle AOQ = 180^{\circ} - 50^{\circ} = 130^{\circ}
\angle A = \angle Q = \frac{(180^\circ - 130^\circ)}{2}
=\frac{50^{\circ}}{2}
= 25^{\circ}
\therefore \angle 0AQ = 25^{\circ}
(b) \angle 0CP = 25° × 2 = 50°
(c) \angle APO = 25^{\circ} + 90^{\circ} = 115^{\circ}
(d) \angle POQ = 180^{\circ} - (50^{\circ} + \angle AOP)
= 180^{\circ} - 50^{\circ} - 40^{\circ}
= 90^{\circ}
```

Q29. Read the following passage. Understand the mathematical concept in it and answer the questions that follow. Each question carries 1 score. The common difference of the arithmetic sequence 15, 14, 13, 12, 11, 10 is 14 – 15 = -1. The first term of the sequence is 15 and the 15th term is 15 + $14 \times -1 = 15 - 14 = 1$. Similarly, the 4th term is 12 and the 12th term is 4. Its 16th term is $x_{16} = 15 + 15 \times -1 = 15 - 15 = 0$. So, the sum of the first 31 terms



is also 0. That is if the n^{th} term of an arithmetic sequence with common difference -1 is m, then the m^{th} term is n and the $(m + n)^{th}$ term is 0.

(a) 7^{th} term of an arithmetic sequence is 10 and the 10^{th} term is 7. What is the common difference?

(b) What is the 21st term of the arithmetic sequence 21,20,19

(c) 5^{th} term of an arithmetic sequence is 17 and the 17^{th} term is 5. Which term of the sequence is 0?

(d) 5^{th} term of an arithmetic sequence is 17 and the 17^{th} term is 5. What is the 44^{th} term?

(e) 1^{st} term of an arithmetic sequence is *n* and the n^{th} term is 1. What is the $(n + 1)^{th}$ term?

(f) The 1^{st} term of an arithmetic sequence is n and the n^{th} term is 1. Sum of how many terms, starting from the first term of this sequence is 0?

Solution:

(a) The common difference d = -1

- (b) 21st term becomes 1.
- (c) 22nd term becomes 0.
- (d) 44^{th} term = 0 22 = -22
- [e] The $(n + 1)^{th}$ term is 0.
- [f] 0 is the sum of $(2n + 1)^{th}$ term.