Grade 10 Kerala Mathematics 2015

Answer ALL 4 questions from 1 to 4. Each question carries 2 scores.

Q1. The first term of an arithmetic sequence is 10 and its common difference is 3. Write the first three terms of the sequence. Verify whether 100 is a term of this sequence.

Solution:

The first term of the sequence is 10 and the common difference is 3 .

$a_1 = 10$ and $d = 3$

Next term $= a_2 = a_1 + d = 10 + 3 = 13$

$a_3 = a_2 + d = 13 + 3 = 16$

Thus, first three terms of the sequence are 10,13 and 16.

Let 100 be the n^{th} term of the sequence.

$a_n = a_1 + (n-1)d$

$100 = 10 + (n-1)3$

$90 = (n-1)3$

$n - 1 = 30$

$n = 31$, which is a whole number.

Therefore, 100 is the 31^{st} term of the sequence.

Q2. Which number added to the polynomial $3x^2 - 4x - 1$ gives a polynomial with $(x-1)$ as a factor.

Solution:

$P(1) = 3(1)^2 - 4(1) - 1 = -2$

Add 2 to the given polynomial.

Q3. If the equation $x^2 + kx + k = 0$ has only one solution, find the possible value of k.

Solution:

$x^2 + kx + k = 0$ has only one solution

The discriminant $b^2 - 4ac = 0$

$k^2 - 4k = 0$

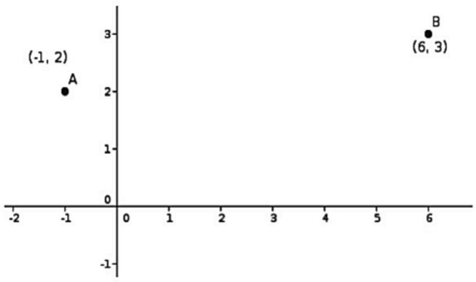
$k^2 = 4k$

$\mathrm{k} = 0$ or 4

So for $k = 0$ or $k = 4$ the equation $x^2 + 4x + 4 = 0$ has only one solution.

Q4. Draw x and y axes and mark the points $\mathrm{A}(-1,2), \mathrm{B}(6,3)$.

Solution:



Answer ALL 4 questions from 5 to 8. Each question carries 3 scores.

Q5. The scores obtained by 50 students in an examination is tabulated as shown below:

Score	Number of students
below 10	3
below 20	7
below 30	13
below 40	22
below 50	32
below 60	40
below 70	46
below 80	50

Find the median score.

Solution:

Score	10	20	30	40	50	60	70	80
Number of Students	3	7	13	22	32	40	46	50

$y = \left(\frac{50}{2}\right) = 25$

$\frac{(x-4)}{(50-40)} = \frac{(25-22)}{(32-22)}$

The median is 43.

Q6. Sum of first n terms of an arithmetic sequence is $3n^2 + n$. Find the first term and common difference of this sequence.

Solution:

Sum of 1 term $= 3 \times 1^2 + 1 = 4$

So, the first term = 4

Sum of two terms $= 3 \times 2^2 + 2 = 14$

2^{nd} term $= 14 - 4 = 10$

For finding $d = 10 - 4 = 6$

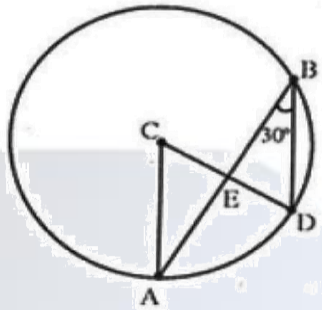
Or

Sum of coefficients of n^2 and $n = 3 + 1 = 4$

The first term $= 4$

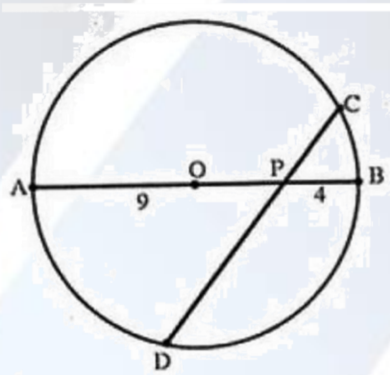
Double the coefficient of n^2 is a common difference.
So, $d = 6$.

Q7.



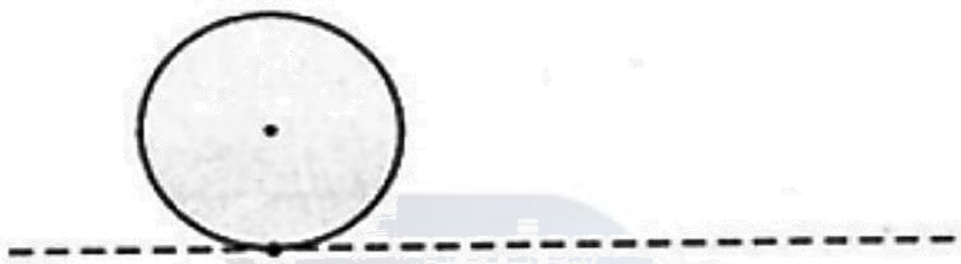
In the figure, O is the centre of the circle and $\angle ABD = 30°$.
(a) What is the measure of $\angle ACD$?
(b) If $\angle ABD = \angle CAB$ and $AB = 6$ cm, find the radius of the circle.
Solution:
(a) $\angle ACD = 60°$
(b) CE bisects AB or $\angle AEC = 90°$
Using the property of $30°, 60°, 90°$ triangle,
$AC = 2\sqrt{3}$
The radius of the circle $= 2\sqrt{3}$ cm

OR



In the figure, O is the centre of the circle. CD is the chord which is not perpendicular to the diameter AB. $PA = 9$ cm and $PB = 4$ cm.
(a) What is $PC \times PD$?
(b) Show that the length of PC and PD cannot be natural numbers at a time.
Solution:
(a) $PC \times PD = 9 \times 4 = 36$
(b) If $PC \times PD = 9 \times 4 = 36$ then natural number values of PC and PD are (1,36), (2,18), (3,12), (4,9), (6,6). Here the longest chord or diameter is 13 cm.
So, (1, 36), (2,18), (3,12) are not acceptable values for PC and PD. CD is not a diameter hence it cannot be 13 cm. So, (4,9) is also not acceptable. CD is not perpendicular to AB, hence (6,6) is also not acceptable. So, the length of PC and PD cannot be natural numbers

Q8.



There is a mark on the outermost part of the wheel of radius 30 cm . Now the mark is close to the ground as shown in the figure. If the wheel rolls 31.4 cm on a straight line, then

(a) Find the angle by which the wheel rotates.

(b) What will be the height of the mark from the ground?

Solution:

(a) For finding the circumference of the wheel $= 2\pi r$

$= 2 \times 3.14 \times 30$

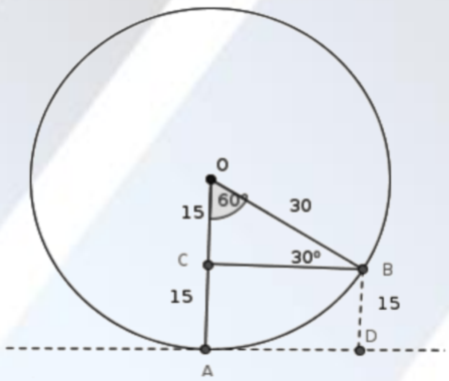
$= 188.4$ cm

The wheel rolls 31.4 cm it rotates $\frac{1}{6}$ part of its circumference.

Angle $= \left(\frac{1}{6}\right) \times 360^0$

$= 60^0$

(b)



$\angle O = 60°$

$\angle OBC = 30°, OB = 30$ cm

OC = 15 cm

$\Rightarrow AC = 15$ cm

$\Rightarrow BD = 15$ cm

The height of the mark from the ground $= 15$ cm.

Answer ALL 10 questions from 9 to 18. Each question carries 4 scores.

Q9. A box contains 8 black beads and 12 white beads. Another box contains 9 black beads and 6 white beads. One bead from each box is taken.

(a) What is the probability that both the beads are black?

(b) What is the probability of getting one black and one white bead?

Solution:

Total number of pairs $= 20 \times 15 = 300$

The probability that both beads are black $= \left(\frac{72}{300}\right)$

$= \left(\frac{6}{25}\right)$

The probability that both beads are white $= \left(\frac{72}{300}\right)$

$= \left(\frac{6}{25}\right)$

The probability of getting one black and one white bead $= \left(\frac{156}{300}\right)$

$= \left(\frac{13}{25}\right)$

Q10. Write the polynomial $3x^2 - 5x - 2$ as a product of two first degree polynomials.

Solution:

$3x^2 - 5x - 2$

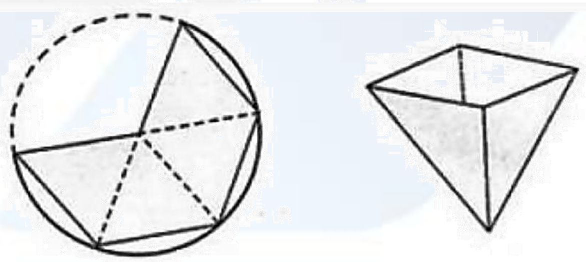
$= 3x^2 - 6x + x - 2$

$= 3x(x - 2) + (x - 2)$

$= (3x + 1)(x - 2)$

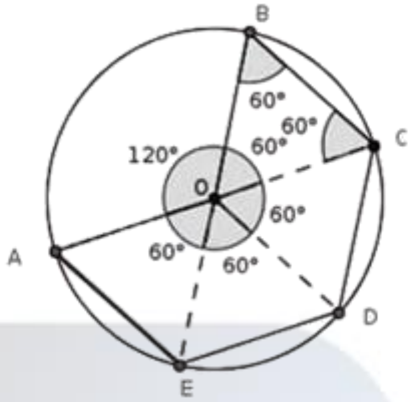
$x = 2, \qquad z\frac{-1}{3}$

Q11.



From a tin sheet, a sector of radius 20 cm and central angle 240° is divided into 4 equal parts as shown in the figure. The shaded portion is cut off. Using this, a vessel is in the shape of a square pyramid is made. What is the capacity of this vessel?

Solution:



In $\triangle$ OBC,

$\angle O = \angle B = \angle C = 60^\circ$

$OB = BC = OC = 20 \text{ cm}$

$e = 20 \text{ cm}$

$a = 20\ cm \Rightarrow\ d = 20\sqrt{2}$

$h^2 = e^2 - \left(\frac{d}{2}\right)^2 = 20^2 - (10\sqrt{2})^2$

$= 400 - 200$

$= 200$

$h = \sqrt{200} = 10\sqrt{2} \text{ cm}$

Volume $= \frac{1}{3} a^2\ h$

$= \frac{1}{3} \times 20^2 \times 10\sqrt{2}$

$= \frac{4000\sqrt{2}}{3} \text{ cm}^3$

Q12. The table below shows the classification of students participated in a camp, according to their height. Calculate the mean height of the students.

Height (cm) ·	Number of students
130 – 135 ·	8
135 – 140	12
140 – 145	20
145 – 150	28
150 – 155	32
155 – 160	22
160 – 165	16
165 – 170	12

Solution:

Height	Number of students (f)	Mid Value (x)	Σfx
130 – 135	8	132.5	1060
135 – 140	12	137.5	1650
140 – 145	20	142.5	2850
145 – 150	28	147.5	4130
150 – 155	32	152.5	4880
155 – 160	22	157.5	3465
160 – 165	16	162.5	2600
165 – 170	12	167.5	2010
	150		22645

$\text{Mean} = \frac{22645}{150} = 150.97$

Q13. (a) What is the volume of a solid metal cylinder of height 4 cm and radius 5 cm?
(b) The solid is melted and recast into 5 cones of equal height and radius 2 cm. Find the height of such a cone.

Solution:

(a) Volume of cylinder $= \pi r^2 h$

$= \pi \times 5 \times 5 \times 4$

$= 100\pi$ cubic cm

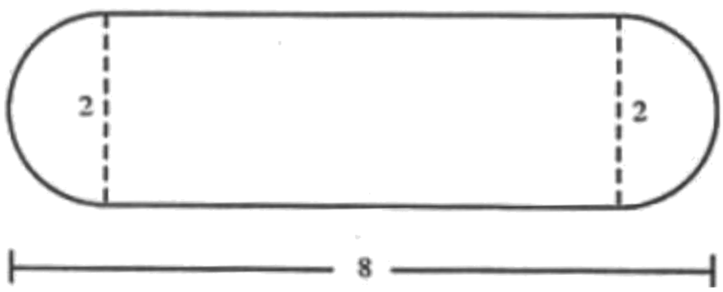
(b) Volume of 5 cones $= 5 \times 13\pi r^2 h$

Volume of cylinder = volume of 5 cones

To find the height of one cone $= 15$ cm

OR

A tank is in the shape of a cylinder with two hemispheres attached to both ends as shown in the figure.



Its common diameter is 2m and total length is 8 m. Find the total cost of painting the outer surface of this tank at the rate of 60 rupees per square meter.

Solution:

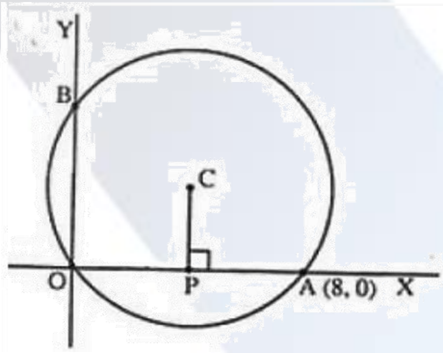
Curved surface area of 2 hemispheres $= 2 \times 2\pi r^2 = 4\pi$ sq.m

Curved surface area of cylinder $= 2\pi rh = 2 \times \pi \times 1 \times 6 = 12\pi$ sq.m

Total area to be painted $= 4\pi + 12\pi = 16\pi$ sq. m

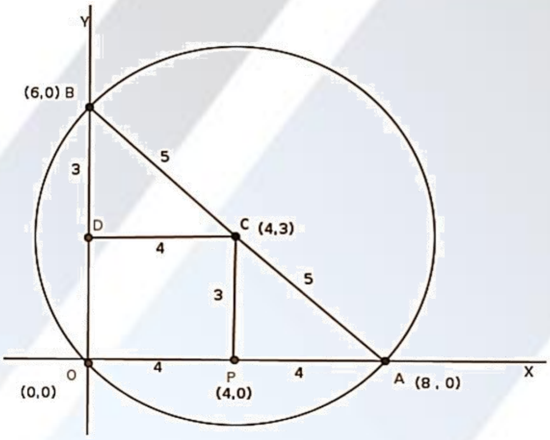
Cost of painting $= 16 \times 3.14 \times 60 =$ Rs. 3014.40

Q14.



In the figure, the radius of the circle centred at C is 5. The circle passes through point A(8,0). If PC is perpendicular to the x-axis, find the coordinates of the points P, B and C.

Solution:



$OA = 8$

$OP = 4$

$\Rightarrow P(4,0)$

$AC = 5, PA = 4$

$\Rightarrow PC = 3$

$\Rightarrow C(4,3)$

$CD = 4, BC = 5$

$\Rightarrow BD = 3$

$\Rightarrow B(6,0)$

Q15. The terms of an arithmetic sequence with common difference 4 are natural numbers.

(a) If x is a term in this sequence, what is the next term?

(b) If the sum of the reciprocals of two consecutive terms of this sequence is $\frac{4}{15}$, find those terms.

Solution:

(a) The next term $= x + 4$

(b) $\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right) + 4 = \left(\frac{4}{15}\right)$

$$\frac{(x+4+x)}{x(x+4)} = \left(\frac{4}{15}\right)$$

$$\frac{(2x+4)}{(x^2+4x)} = \left(\frac{4}{15}\right)$$

$$4(x^2+4x) = 15(2x+4)$$

$$4x^2 + 16x = 30x + 60$$

$$4x^2 + 16x - 30x - 60 = 0$$

$$4x^2 - 14x - 60 = 0$$

$$2x^2 - 7x - 30 = 0$$

$$x = \frac{\left(-b \pm \sqrt{b^2 - 4ac}\right)}{2a}$$

$$= \frac{(7 \pm \sqrt{49} + 240)}{4}$$

$$= \frac{(7 \pm 17)}{4}$$

$x = 6$ and $x = \frac{-5}{2}$

OR

(a) Lengths of sides of a right-angled triangle are in arithmetic sequence with common difference d. If the length of the smallest side of the triangle is x − d, write the length of its other two sides.

(b) Show that any right-angled triangle with sides in an arithmetic sequence is similar to the right-angled triangle with sides 3,4 and 5.

Solution:

(a) The lengths of the other two sides are $x, x + d.$

(b) By Pythagoras theorem,

$$(x-d)^2 + x^2 = (x+d)^2$$

$$x^2 - 2xd + d^2 + x^2$$

$$\Rightarrow x^2 + 2xd + d^2$$

$$\Rightarrow x^2 - 4xd = 0$$

$$x(x-4d) = 0$$

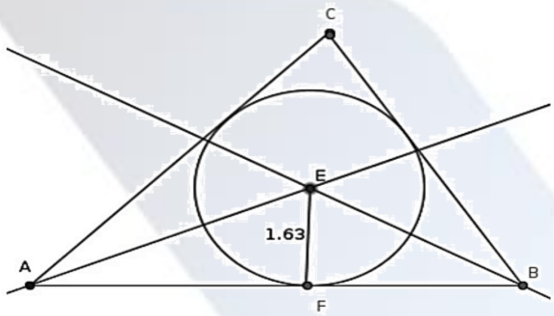
$x = 0$ or $x = 4d$

Length of the sides are,

$x - d = 4d - d = 3d$
$x = 4d$
$x + d = 4\,d + d = 5d$
Hence the sides $3d$, $4d$, $5d$ are similar to pythagorean triplet sides 3, 4, 5.

Q16. Draw a triangle of sides 5 cm, 6 cm and 7 cm . Draw its incircle. Measure and read the radius of the incircle.

Solution:



Radius of the incircle $= 1.6$ cm.

Q17. A line of slope 2 passes through the point A(1,3).

(a) Check whether the B(3,7) is a point on this line.
(b) Write down the equation of this line.
(c) Find the coordinates of a point C on the line such that BC $=$ 2AB.

Solution:

The slope of the line is 2 and passes through A(1,3)

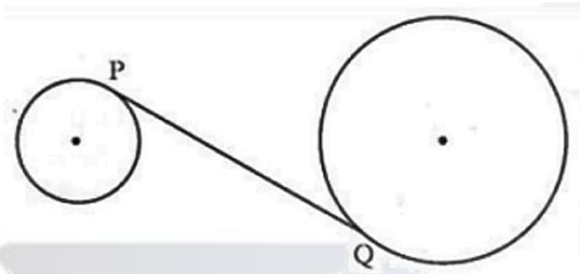
$$\frac{[y-3]}{[x-1]} = 2$$

$y - 3 = 2x - 2$
$y = 2x + 1$
(a) For point B(3,7)
$2(3) + 1 = 7$
Hence the point B(3,7) lies on the line $y = 2x + 1$.
(b) Equation of the line *is* $y = 2x + 1$.
[c] Let (x_1, y_1) be the coordinates of point C on line.
Therefore it satisfies the equation of line a $y = 2x + 1$.
BC $=$ 2AB

$$\Rightarrow \sqrt{(x_1-3)^2 + (y_1-7)^2} = 2\sqrt{(3-1)^2 + (7-3)^2}$$

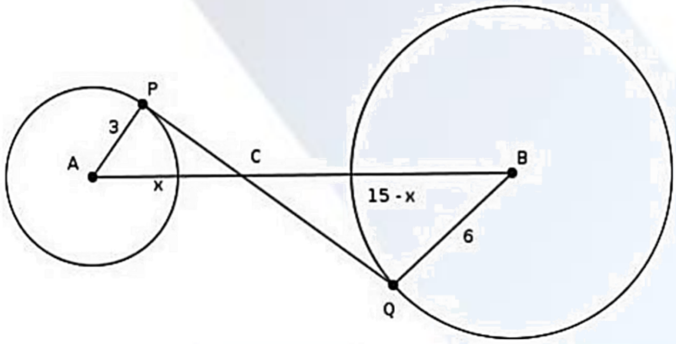
On squaring both sides,
$(x_1 - 3)^2 + (y_1 - 7)^2 = 4[(3-1)^2 + (7-3)^2]$
$5(x_1 - 3)^2 = 80$
$(x_1 - 3)^2 = 16$
$x_1 - 3 = \pm 4$
$x_1 = 7 \; or \; -1$
$y_1 = 2x_1 + 1 = 2(7) + 1 = 15$
$y_1 = 2x_1 + 1 = 2(-1) + 1 = -1$
The coordinates of point C are (7,15) or $(-1, -1)$.

Q18.



In the figure, the radius of the smaller circle is 3 cm, that of the bigger circle is 6 cm and the distance between the centres of the circles is 15 cm. PQ is a tangent to both circles. Find its length.

Solution:



In $\triangle$ APC and $\triangle$ BQC

$\angle P = \angle Q = 90^0$

$\angle ACP = \angle BCQ$

$\triangle APC \sim \triangle BQC$

$\left(\frac{AP}{AC}\right) = \left(\frac{BQ}{BC}\right)$

$\left(\frac{3}{x}\right) = \frac{6}{15 - x}$

$6x = 3(15 - x)$

$= 45 - 3x$

$9x = 45$

$x = 5$

$AP = 3 \text{ cm}, AC = 5 \text{ cm}$

$\Rightarrow PC = 4 \text{ cm}$

$BQ = 6 \text{ cm}, BC = 10 \text{ cm}$

$\Rightarrow QC = 8 \text{ cm}$

$PQ = PC + QC$

$= 4 + 8$

$PQ = 12 \text{ cm}$

Q19. Consider the arithmetic sequence 9, 15, 21,............

(a) Write the algebraic form of this sequence.

(b) Find the 25^{th} term of this sequence.

(c) Find the sum of terms from 25^{th} to 15^{th} of this sequence.
(d) Can the sum of some terms of this sequence be 2015? Why?

Solution:

The sequence is 9,15,21, ...
First term $= a = 9$
Common difference $= d = 15 - 9 = 6$
(a) For natural number n
nth term $= [(n-1) \times 6] + 9 = 6n + 3$ where $n = 1,2,3, \ldots.$
Hence the algebraic form of sequence 9,15,21, ... is $x_n = 6n + 3$.
(b) nth term $= 6n + 3$
25^{th} term $= 6(25) + 3$
$= 150 + 3$
= 153
(c) Sum of first n terms is given by,

$$S_n = \left(\frac{1}{2}\right)$$

$n(x_1 + x_n)$
Sum of 24 terms is

$$S_{24} = \left(\frac{1}{2}\right) n[x_1 + x_{24}]$$

$$= \left(\frac{24}{2}\right) [9 + 147]$$

$= 1872$
Sum of 50 terms is

$$S_{50} = \left(\frac{1}{2}\right) n[x_1 + x_{50}]$$

$$= \left(\frac{50}{2}\right) [9 + 6(50) + 3]$$

$= 7800$
Sum of terms from twenty fifth to fiftieth
$= S_{50} - S_{24}$
= $7800 - 1872$
= 5928
(d) Let the sum of first n terms is 2015,

$$\mathrm{S_n} = \left(\frac{1}{2}\right) n[x_1 + x_n]$$

$$2015 = \frac{1}{2} n[9 + 6n + 3]$$

$$2015 = 3n^2 + 6n$$

$$n = -1 \pm \sqrt{\frac{(6054)}{3}}$$

$n = -26.94, 24.94$
The values of n are not natural numbers.
Hence 2015 cannot be the sum of some terms of this sequence.

Q20. In triangle ABC, AB = 5 cm, $\angle A = 80°$, $\angle B = 70°$. Calculate the radius of the circumcircle and length of the other 2 sides.

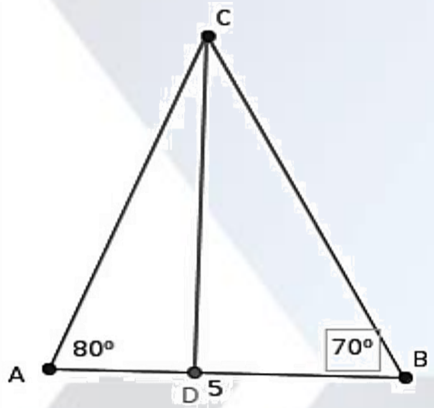
OR

Gopi and Gautham stand on opposite sides of a tower. The children and the tower are on a straight line. Gopi sees the top of the tower at an angle of elevation of 36° and Gautham sees it at an angle of elevation of 52°. The distance between the children is 60m .

(a) Draw a rough figure according to the given information.

(b) Find the height of the tower.

Solution:



$$CD = \frac{5 \times \tan 80^0 \times \tan 70°}{\tan 80° + \tan 70°}$$

$$= \frac{(5 \times 2.75 \times 5.67)}{(2.75 + 5.67)}$$

$$= \frac{77.9625}{8.42}$$

$= 9.26$ cm

$$\sin 70° = \left(\frac{CD}{BC}\right)$$

$$BC = \left(\frac{CD}{\sin 70°}\right)$$

$$= \left(\frac{9.26}{0.94}\right)$$

$= 9.85$ cm

$$AC = \left(\frac{CD}{\sin 80°}\right)$$

$$= \left(\frac{9.26}{0.98}\right)$$

$= 9.45$ cm

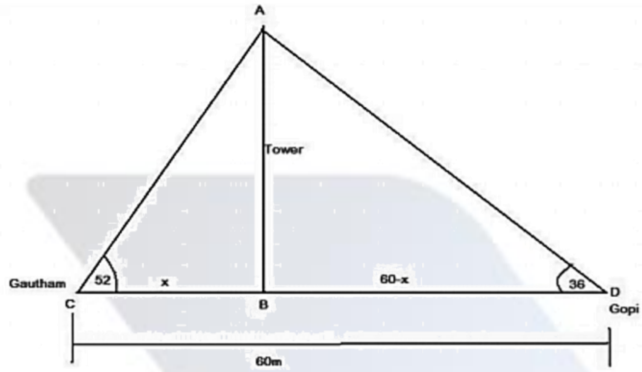
$$\text{Diameter} = \left(\frac{AB}{\sin 30°}\right)$$

$$= \left(\frac{5}{0.5}\right)$$

$= 10$ cm

Circumradius $= \left(\frac{10}{2}\right) = 5$ cm

OR



From $\triangle$ ABC, AB $= x\tan 52^0 = 1.28x$ ––– (1)

From $\triangle$ ABD, AB $= (60 - x)\tan 360^0$

$= (60 - x)0.72$

$= 43.2 - 0.72x$ – – – – – (2)

From (1) and (2)

$1.28x = 43.2 - 0.72x$

$2x = 43.2$

$x = 21.6$

Height of the tower

$= 1.28 \times 21.6$

$= 27.648$ m or 27.65 m

Q21. Equation of a line is $y = 2x$.

(a) A is a point on the line. If the coordinate of A is -2, find its y coordinate.

(b) Verify whether a circle of radius 5 centred at A passes through point B(5, 5).

(c) Radius of the circle passing through B is 5 and its centre is on the above mentioned line. Find the coordinates of its centre.

Solution:

Equation of a line is $y = 2x$.

(a) $x = -2$

$\Rightarrow y = 2 \times -2$

$\Rightarrow -4$

(b) A (-2, - 4)

AB = 5

$(5 - [-2])^2 + (5 - [-4])^2$

$= 49 + 81$

$= 25$

$= 5^2$

Hence a circle of radius 5 centred at point A$(-2, -4)$ does not pass through the point B(5,5).

(c) Let (x_1, y_1) be the coordinates of centre C on line.

$(x_1 - 5)^2 + (y_1 - 5)^2 = 52$
$x_1{}^2 - 10x_1 + 25 + y_1{}^2 - 10y_1 + 25 = 25$
$y_1 = 2x_1$
$x_1{}^2 - 10x_1 + 4x_1{}^2 - 20x_1 = -25$
$5x_1{}^2 - 30x_1 = -5$
$x_1{}^2 - 6x_1 = -5$
$x_1{}^2 - 6x_1 + 9 = -5 + 9$
$(x_1 - 3)^2 = 4$
$x_1 - 3 = \pm 2$
$x_1 = 2 + 3 = 5$ or $x_1 = -2 + 3 = 1$
$y_1 = 2 \times 5 = 10$ or $y_1 = 2 \times 1 = 2$
The coordinates of point of C are (5,10) or (1,2).

Q22. Draw a triangle with sides 5 cm, 6 cm and 6 cm. Draw a square having the same area of the triangle.

Solution: