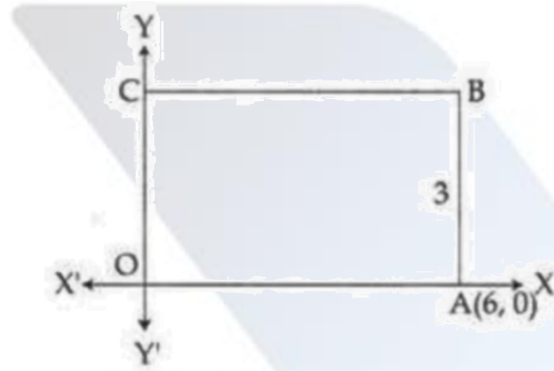# Grade 10 Kerala Mathematics 2018

Answer ANY 3 questions from 1 to 4. Each question carries 2 scores.

Q1.

In the figure, OABC is a rectangle and its breadth is 3. Write the coordinates of the vertices B and C.

Solution:

Given OABC is a rectangle.

Breadth = 3 cm

A (6,0)

From the figure AB = 3 cm (breadth)

Hence, B = (6,3) and C = (0,3)[OC = 3 cm breadth]

Q2. The letters of the word MALAYALAM are written in paper slips and put into a box. A child is asked to take one slip from the box without looking.

(a) What is the probability of getting the letter A ?

(b) What is the probability of not getting the letter A?

Solution:

MALAYALAM

Total letters = 9

$n(S) = 9$

Number of times A occurs = 4

$n(E) = 4$

(a) Probability of getting letter A = $\dfrac{\text{Favourable events}}{\text{Total number of outcomes}}$

$= \left(\dfrac{n(E)}{n(S)}\right)$

$= \left(\dfrac{4}{9}\right)$

(b) Probability of not getting A $= 1 - P(E) = 1 - \dfrac{4}{9} = \dfrac{5}{9}$

Q3.   The algebraic form of an arithmetic sequence is 5n + 3.
       (a) What is the first term of the sequence?
       (b) What will be the remainder if the terms of the sequence are divided by 5?
       Solution:
       $a_n = 5n + 3$
       (a) To obtain the first term of the sequence, put n = 1,
       $a_1 = 5 \times 1 + 3 = 8$
       (b) d = 5 (coefficient of n be a common difference)
       The remainder obtained when divided by 5 is 3 $\left\{\left(\frac{8}{5}\right) = 3, \left(\frac{13}{5}\right) = 3, \left(\frac{18}{5}\right) = 3\right\}$,.etc.

Q4.   The weights of 11 children in a school cricket club are 35,39,32, 36, 40, 30, 34, 37,
       38, 33, 31 (kgs). Find the median weight.
       Solution:
       Weight of 11 children in ascending order (in kgs)
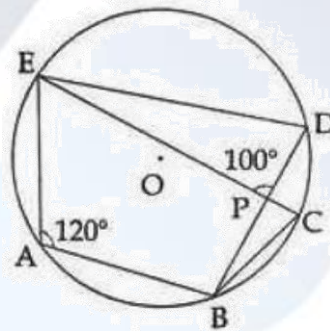   30,31,32,33,34,35,36,37,38,39,40
       Here n = 11, which is odd.
       So, median $= \left\{\frac{[n+1]^{th}}{2}\right\}$ term $= \frac{[11+1]^{th}}{2} = 6^{th}$ term
       The median weight of children = 35 kg.

       Answer ANY 5 questions from 5 to 11. Each question carries 3 scores.
Q5.   In the figure, O is the centre of the circle and A, B, C, D, E are the points on it.

       ∠EAB = 120°, ∠EPD = 100°. Write the measures of ∠EDB, ∠ECB and ∠DBC.
       Solution:
       ABCDE is a cyclic quadrilateral.
       ∠EDB + ∠EAB = 180°
       120° + ∠EDB = 180°
       ∠EDB = 60°
       ABCE is a cyclic quadrilateral.
       ∠EAB + ∠ECB = 180°
       120° + ∠ECB = 180°

∠ECB = 60°
∠BPE = ∠DPE [vertically opposite angles]
∠BPE = 100°
In triangle BPC,
∠BPC + ∠BCP + ∠PBC = 180°
100° + 60° + ∠PBC = 180°
∠PBC = 180° − 160°
∠PBC = 20°
∠DBC = 20° [same arc angles]

Q6. The algebraic form for the sum of first n terms of an arithmetic sequence is $2n^2 + 8n$. How many consecutive terms of the sequence, starting from the first, are to be added to get 330?

Solution:
$S_n = 2n^2 + 8n$
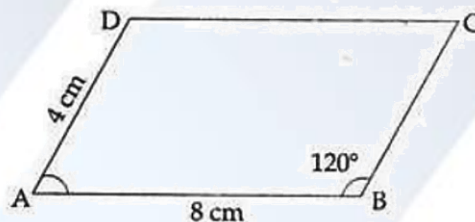$330 = 2n^2 + 8n$
$2n^2 + 8n − 330 = 0$
$n^2 − 4n − 165 = 0$
$n(n + 15) − 11(n + 15) = 0$
$(n + 15)(n − 11) = 0$
$n = 11, −15$
Since the number of terms cannot be negative, n = 11.

Q7.

ABCD is a parallelogram. AB = 8 cm, AD = 4 cm, ∠ B = 120°
(a) What is ∠A ?
(b) What is the perpendicular distance from D to AB ?
(c) What is the area of ABCD?

Solution:
Given ABCD is a parallelogram.
AB = 8 cm, AD = 4 cm, ∠B = 120°
(a) ∠B = ∠D = 120° (opposite angles equal)
∠A + ∠C = 360° − (120° + 120°)
= 360° − 240°

$= 120°$

$\therefore \angle A = \angle C = \left(\dfrac{120°}{2}\right) = 60°$

$\therefore \angle A = 60°$

(b) Draw DE as perpendicular from D to AB .

In triangle APD, $\angle P = 90°$,

$\dfrac{DP}{AD} = sinA$

$\dfrac{DP}{4} = sin60°$

$\dfrac{DP}{4} = \dfrac{\sqrt{3}}{2}$
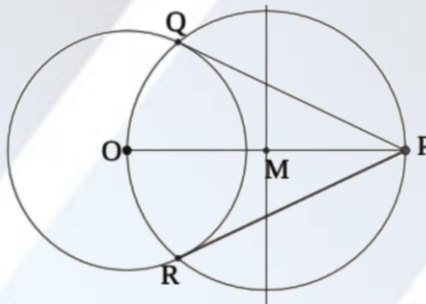
$DP = \dfrac{\sqrt{3}}{2} \times 4$

$= 2\sqrt{3}$ cm

(c) The area of parallelogram ABCD = base × height

$= 8 \times 2\sqrt{3}$

$= 16\sqrt{3}$ cm²

Q8.   Draw a circle of radius 3 cm . Mark a point 7 cm away from the centre. Draw the tangents to the circle from this point.
Solution:

Steps of construction:
1. Draw a circle of 3 cm radius and O as its centre.
2. From the centre, mark OP = 7 cm.
3. Draw a perpendicular bisector of the line OP such that it meets at M.
4. Draw a circle with radius OM and cut the circle at Q and R .
5. Joint PR and PQ.
    Hence, PR and PQ are tangents.

Q9.   The perimeter of the base of a square pyramid is 96 cm and its height is 16 cm.
(a) What is the length of a base edge?

(b) What is the slant height?

(c) Find the lateral surface area.

Solution:

The perimeter of the base of a square pyramid = 96 cm

Height = 16 cm

(a) 4 × base edge = 96

4a = 96

$a = \left(\frac{96}{4}\right)$

= 24 cm

∴ Base edge = 24 cm.

(b) Slant height (l) = $l^2 = \sqrt{\left(\frac{base}{2}\right)^2 + (\text{height})^2}$

$= \sqrt{\left(\frac{24}{2}\right)^2 + 16^2}$

$= \sqrt{12^2 + 16^2}$

$= \sqrt{144 + 256}$

$= \sqrt{400}$

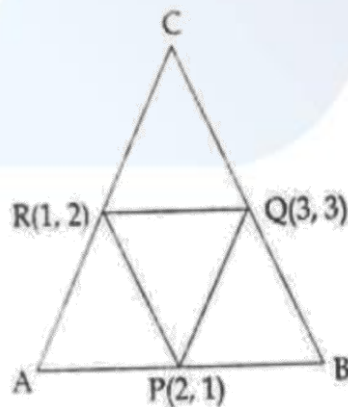= 20 cm

(c) Lateral surface area (LSA) = $\left(\frac{1}{2}\right)$ × perimeter × slant height

$= \left(\frac{1}{2}\right) \times 96 \times 20$

= 960 cm²

Q10.

P, Q, R are the midpoints of the sides of triangle ABC.

(a) What type of quadrilateral is PQCR?

(b) Write the coordinates of the vertices A and C.

Solution:

(a) Q & R are midpoints of sides BC and AC respectively

$\therefore$ QR| | AB

Similarly PQ || AC

$\therefore$ PQCR is a parallelogram.

(b) Let the vertex of A, B and C be $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) respectively R is the midpoint of AC.

$1 = \dfrac{x_1 + x_3}{2}$ and $2 = \dfrac{y_1 + y_3}{2}$

$x_1 + x_3 = 2$ …. (i)

$y_1 + y_3 = 4$ …. (ii)

P is the midpoint of side AB

$\therefore \ 2 = \dfrac{x_1 + x_2}{2}$ and $1 = \dfrac{y_1 + y_2}{2}$

$x_1 + x_2 = 4$ …. (iii)

$y_1 + y_2 = 2$ … (iv)

Q is the midpoint of side BC

$\therefore \qquad \dfrac{x_2 + x_3}{2} = 3$ and $\dfrac{y_2 + y_3}{2} = 3$

$\therefore \qquad x_2 + x_3 = 6 \qquad$ …. (v)

and $\quad y_2 + y_3 = 6 \qquad$ ….. (vi)

Adding equations (i), (iii) & (v)

$2(x_1 + x_2 + x_3) = 2 + 4 + 6$

$x_1 + x_2 + x_3 = 6$ …. (vii)

Adding equations (ii), (iv) & (vi)

$2(y_1 + y_2 + y_3) = 4 + 2 + 6$

$y_1 + y_2 + y_3 = 6$ …. (viii)

from equ. (vii) & (v)

$6 + x_1 = 6$

$x_1 = 0$

from equ. (viii) & (vi)

$6 + y_1 = 6$

$y_1 = 0$

$\therefore$ Coordinates of the vertex A(0,0)

from equ. (vii) & (iii)

$4 + x_3 = 6$

$x_3 = 2$

from equ. (viii) & (iv)

$2 + y_3 = 6$

$y_3 = 4$

$\therefore$ Coordinates of the vertex C(2,4)

Q11. P(x) is a second degree polynomial with $P(1) = 0$ and $P(-2) = 0$.

(a) Find two first degree factors of P(x).

(b) Find the polynomial P(x).

Solution:

Given: $P(1) = 0$ and $P(-2) = 0$.

(a) First degree factors $= (x - 1)(x + 2)$ [using factor theorem]

(b) $P(x) = (x - 1)(x + 2)$

$= x^2 + x - 2$

Answer ANY 7 questions from 12 to 21. Each question carries 4 scores.

Q12. There are 20 terms in an arithmetic sequence. Sum of the first and last terms is 88.

(a) What is the sum of 2nd and 19th terms?

(b) If the 10th term is 42 , what is the 11th term?

(c) What is the common difference of the sequence?

(d) What is the first term?

Solution:

Given n = 20.

Sum of the first and last term = 88.

(a) The sum of the 2nd and 19th terms

ie., $x_2 + x_{19} = x_1 + d + x_{20} - d$

$= x_1 + x_{20}$

$= 88$

(b) Given 10th term 42.

$x_{10} + x_{11} = (x_1 + 9\,d) + (x_{20} - 9\,d)$

$42 + x_{11} = x_1 + x_{20}$

$x_{11} = x_1 + x_{20} - 42$

$= 88 - 42$

$= 46$

Hence 11th term $= 46$

(c) Common difference (d) $= x_{11} - x_{10}$

$= 46 - 42$

$= 4$
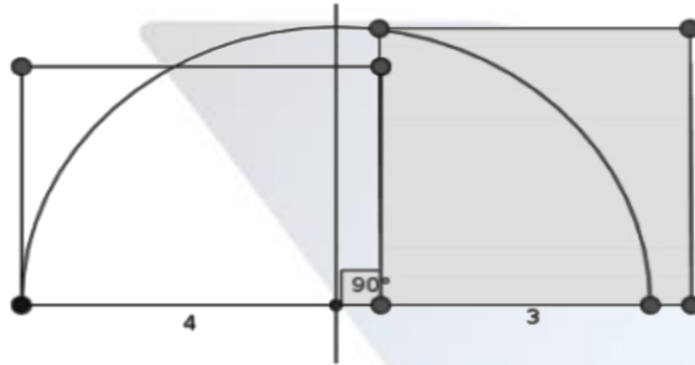
(d) $x_{10} = x_1 + 9\,d = 42$ (given 10th term as 42)

$= 42 - 9\,d$

$= 12 - 9 \times 4$

a = 6

Hence the first term = 6.

Q13. Draw a rectangle of length 4 cm , breadth 3 cm and draw a square of the same area.

Solution:

Steps of construction:

1. Draw a rectangle PQRS in the given measurement.
2. PQ produced up to T as QT = 3 cm.
3. Draw a perpendicular bisector of the line PT, meet Q on PT.
4. Produce QR to meet the semicircle at M.
5. Draw the square QMNO as QM = MN = ON = QO.
6. Hence QMNO is the required square.

Q14. There are 30 scouts and 20 guides in a school. In another school, there are 20 scouts and 15 guides. From each school, one student among them is to be selected for participation in a seminar.
(a) What is the total number of possible outcomes?
(b) What is the probability of both being scouts?
(c) What is the probability of both being guides?
(d) What is the probability of one scout and one guide?

Solution:

Name of school	Scouts	Guides	Total
A	30	20	50
B	20	15	35

Total number of possible selection = m × n (fundamental counting theorem)

(a) m × n = 50 × 35 = 1750

(b) Probability of both being scouts

$$P(S) = \frac{\text{Favourable outcomes}}{\text{total number of outcomes}}$$

F = 30 × 20 = 600

N = 1750.

$$P(S) = \left(\frac{600}{1750}\right) = \left(\frac{12}{35}\right)$$

(c) Probability both being guide = m × n = 20 × 15 = 300 (f)

$$P(G) = \frac{\text{Favourable outcomes}}{\text{total number of outcomes}}$$

$$= \left(\frac{300}{1750}\right)$$
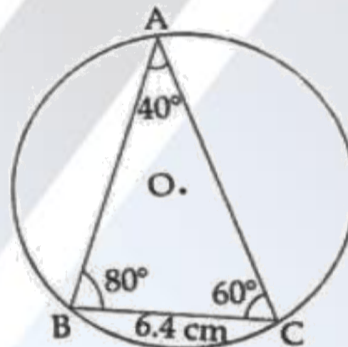
$$= \left(\frac{6}{35}\right)$$

(d) Probability of one scout and one guide = 1 − P(S) + P(G)

$$= 1 - \left(\frac{12}{35} + \frac{6}{35}\right)$$

$$= 1 - \left[\frac{18}{35}\right]$$

$$= \left(\frac{17}{35}\right)$$

Q15.   O is the centre of the circumcircle inscribed in the triangle ABC.

∠A = 40°, ∠B = 80°, ∠C = 60°, BC = 6.4 cm.

(a) What is the diameter of the circle?

(b) What is the length of the other two sides?

	40°	60°	80°
sin	0.64	0.87	0.98
cos	0.77	0.50	0.17
tan	0.84	1.73	5.67

Solution:

In triangle ABC ,

$AB = c, BC = a = 6.4$ cm

$AC = b$

$\angle A = 40°, \angle B = 80°, \angle C = 60°, BC = 6.4$ cm

(a) Diameter $(d) = \left(\dfrac{BC}{\sin A}\right)$

$= \left(\dfrac{6.4}{\sin 40°}\right)$

$= \left(\dfrac{6.4}{0.64}\right)$

$= 10$ cm

(b) Side $AC = d \times \sin B$

$= 10 \times \sin 80°$

$= 10 \times 0.98$

$= 9.8$ cm

Side $AB = d \times \sin C$

$= 10 \times \sin 60°$

$= 10 \times 0.87$

$= 8.7$ cm

Q16. A circle with centre (3, 2) passes through the point (6,3).

(a) What is the radius of the circle?

(b) Check whether each of the points with coordinates (0,2), (3,6), (0,3) is inside, outside, or on the circle.

Solution:

(a) Given points (3,2) and (6,3)

Using the distance formula,

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6-3)^2 + (3-2)^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{10}$

Radius of the circle > 3 units.

(b) (3,2) and (0,2)

Using the distance formula,

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-3)^2 + (2-2)^2}$

$= \sqrt{9}$

$= 3$

Here 3 is less than the radius, so the point is inside the circle.

(3,2) and (3,6)

Using the distance formula,

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3 - 3)^2 + (6 - 2)^2}$$
$$= \sqrt{0^2 + 4^2}$$
$$= \sqrt{16}$$
$$= 4$$

Here 4 is greater than the radius, so the point is outside the circle.

(3,2) and (0,3)

Using the distance formula,
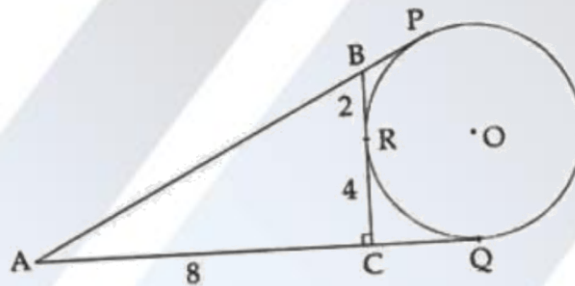
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(0 - 3)^2 + (3 - 2)^2}$$
$$= \sqrt{9 + 1}$$
$$= \sqrt{10}$$

Here $\sqrt{10}$ is equal to the radius, so the point is on the circle.

Q17.

In the figure, the circle with centre ' O ' is the excircle of the right-angle triangle ACB and P, Q, R is the point where the circle touches the sides of a triangle. AC = 8 cm, CR = 4 cm, BR = 2 cm.

(a) What is the length of AQ ?

(b) What is the perimeter of triangle ACB?

(c) What is the area of the triangle ACB?

(d) What is the radius of the incircle of triangle ACB?

Solution:

Given, AC = 8 cm, CR = 4 cm, BR = 2 cm.

(a) From the figure,

AQ = AC + CQ = AC + (CR = CQ)(tangents are equal)

= 8 + 4

= 12 cm

∴ AQ = 12 cm

(b) Perimeter of the △ ACB,

△ ACB is a right-angled triangle, by Pythagoras theorem,

$AB^2 = AC^2 + BC^2$

$AC = 8 \text{ cm}, BC = CR + RB = 4 + 2 = 6$

$AB^2 = 8^2 + 6^2$

$= 64 + 36$

$= 100$

$AB = \sqrt{100}$

$= 10 \text{ cm}$

Hence the perimeter $= 8 + 6 + 10 = 24 \text{ cm}.$

(c) Area of the △ ACB $= \left(\frac{1}{2}\right) \times \text{base} \times \text{height}$

$= 12 \times 8 \times 6$

$= 24 \text{ cm}^2$

(d) The radius of the in-circle of △ ACB

$r = \dfrac{\text{Area}}{\text{Semiperimeter}}$

$= \left(\dfrac{24}{12}\right)$

$= 2 \text{ cm}$

Q18. A circular sheet with radius 36 cm is divided into 4 equal sectors and one of them bent into a cone.

(a) What is the slant height of the cone?

(b) What is the radius of the cone?

(c) What is the curved surface area?

Solution:

The radius of the circular sheet $= 36 \text{ cm (l)}$

(a) Slant height of the cone $= 36 \text{ cm}$

(b) Radius of the cone , r

$\dfrac{r}{l} = \dfrac{x}{360°}$

Centre angle $(x) = \left(\dfrac{360°}{4}\right) = 90°$

$\dfrac{r}{36} = \dfrac{90°}{360°}$

$r = \left(36 \times \dfrac{90}{360}\right)$

$= 9 \text{ cm}$

Hence, the radius of the cone = 9 cm.

(c) Curved surface area (CSA) = πrl

= π × 9 × 36

= 324π cm²

= 324 × 3.14

= 1017.36 cm²

Q19. The coordinates of the vertices of a triangle are A (1, 1), B(5, 5), C (2,5).

(a) Write the coordinates of the midpoint D of AB.

(b) What is the length of the CD?

(c) What are the coordinates of the point dividing the line CD in the ratio 2:1?

Solution:

Given, the vertices of the △ ABC,

A (1,1), B(5,5), C(2,5)

(a) Midpoint of D of AB

$$\left[\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)\right]$$

Points A (1,1), B(5,5)

$$D = \left[\left(\frac{1 + 5}{2}\right), \left(\frac{1 + 5}{2}\right)\right]$$

= (3,3)

(b) Points C (2,5), D(3,3)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - 2)^2 + (3 - 5)^2}$$

$$= \sqrt{1 + 4}$$

$$= \sqrt{5}$$

(c) Given, CD = 2: 1. (ie., m: n)

$$\left(\frac{(mx_2 + nx_1)}{(m + n)}, \frac{(my_2 + ny_1)}{(m + n)}\right)$$

$$= \left(\frac{2 \times 3 + 1 \times 2}{2 + 1}, \frac{2 \times 3 + 1 \times 5}{2 + 1}\right)$$

$$= \left(\frac{6 + 2}{3}, \frac{6 + 5}{3}\right)$$

$$= \left(\frac{8}{3}, \frac{11}{3}\right)$$

Q20. $P(x) = x^3 + ax^2 - x + b$ and
(a) Find the relation between a and b for $x - 1$ to be a factor of $P(x)$.
(b) Find the relation between a and b for $x - 2$ to be a factor of $P(x)$.
(c) Find a and b such that both $x - 1$ and $x - 2$ are factors of $P(x)$.
Solution:
Given $P(x) = x^3 + ax^2 - x + b$
(a) $x - 2$ is a factor of $P(x)$
$P(1) = 0$
$P(1) = (1)^3 + a \times (1)^2 - 1 + b = 0$
$= 1 + a - 1 + b = 0$
$a + b = 0$
(b) $x - 2$ is a factor of $P(x)$
$P(2) = 0$
$P(2) = 2^3 + a \times 2^2 - 2 + b = 0$
$= 8 + 4a - 2 + b = 0$
$= 4a + b = -6$
(c) We have, $a + b = 0$ and $4a + b = -6$.
Solve these two equations, $a = -2$ and $b = 2$.

Q21.   The table below shows the members in "Stree-Shakti Kudumbashree" sorted
according to their ages.

Age group	Number of members
20 − 30	4
30 − 40	8
40 − 50	10
50 − 60	7
60 − 70	4
70 − 80	2
Total	35

(a) If the members are arranged in increasing order of their ages, the age of the
member at what position is taken as the median?
(b) What is the assumed age of the member at the 13th position?
(c) Find the median of the ages.

Solution:

(a) Median $= [(\frac{N}{2})]^{th}$ term $= (\frac{35}{2}) = 17.5^{th}$ term

Median class $= 40 - 50$

Age of the member which is taken as the median position $= 40 - 50$

(b) Age of the member at the 13th position $= 40$ years

(c) Median $= l_1 + \{(\frac{(\frac{N}{2})-cf}{f})\} \times h$

Age group	Number of members	Cumulative Frequency
$20 - 30$	4	4
$30 - 40$	8	12
$40 - 50$	10	22
$50 - 60$	7	29
$60 - 70$	4	33
$70 - 80$	2	35
Total	35	

$(\frac{N}{2}) = (\frac{35}{2}) = 17.5$

Median class $= 40 - 50$

$l_1 = 40, c = 12, f = 10$

Hence, median

$= 40 + \{(\frac{(17.5 - 12)}{10})\} \times 10$

$= 40 + 5.5$

$= 45.5$

Answer ANY 5 questions from 22 to 28 . Each question carries 5 scores.

Q22. Consider the numbers between 100 and 300 which leave the remainder 2 on division by 3.

(a) What is the first number in this sequence?

(b) What is the last number in this sequence?

(c) How many such numbers are there in this sequence?

(d) Find the sum of all numbers in the sequence.

Solution:

First term = $99 + 2 = 101.$, $d = 3$

So, the sequence is $101, 104, 107, 110 \ldots \ldots 299.$

Last term = $300 - 1 = 299$

(a) The first term = 101

(b) Last term = 299

(c) $n = \left[\left(\frac{(X_n - X_1)}{d}\right)\right] + 1$

$= \left[\left(\frac{(299 - 101)}{3}\right)\right] + 1$

$= 67$

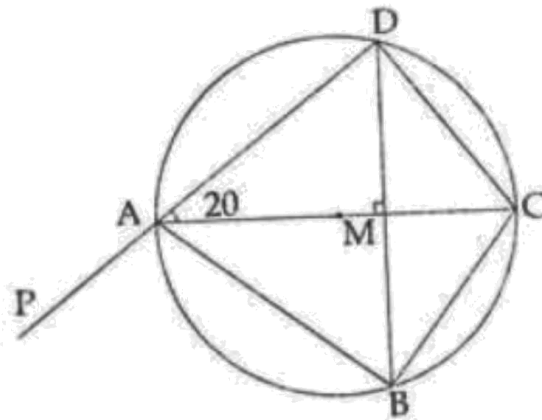(d) $S_n = \left(\frac{n}{2}\right)(a + a_n)$

$= \left(\frac{67}{2}\right)(101 + 299)$

$= 67 \times 200$

$= 13400$

Q23. In the figure, the chord BD is perpendicular to the diameter AC. Find the measures of the following angles.

(a) ∠BAC

(b) ∠BCD

(c) ∠ADC

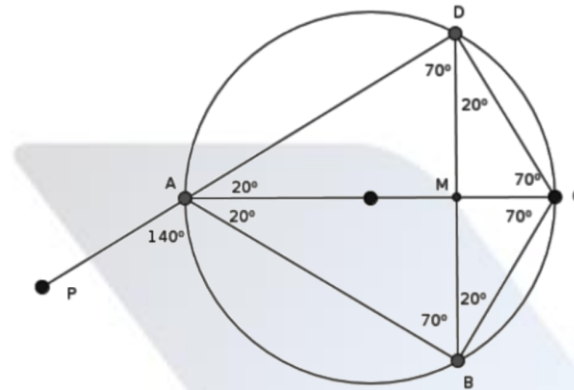(d) ∠CDM

(e) ∠BAP

Solution:

(a) ∠BAC = ∠CAD = 20°

(b) ∠BCD = 140°(180° − 40° = 140°, cyclic quadriateral)

(c) ∠ADC = 90° (angle of the semicircle)

(d) ∠CDM = ∠BAC = 20° (angle in the same arc)

(e) ∠BAP = 140°(180° − 40° = 140°, linear pair)

Q24. A rope of length 40 m is cut into two pieces and two squares are made on the floor with them. The sum of the areas enclosed is 58 sq.m.

(a) If the length of one piece is taken as x, what is the length of the other piece?

(b) What are the lengths of the sides of the squares?

(c) Write the given fact about the area as an algebraic equation.

(d) What is the length of each piece?

Solution:

Length of the rope = 40 cm

Let the length of the one-piece = x

(a) Length of the other piece = 40 − x

(b) Length of the sides of the square = $\left(\frac{x}{4}\right)$ and $\left(\frac{[40-x]}{4}\right)$.

(c) The algebraic form, given, the sum of the area = 58 cm²,

$a^2 + a^2 = 58$ cm²

$$\Rightarrow \left(\frac{x}{4}\right)^2 + \left[\frac{(40-x)}{4}\right]^2 = 58$$

$$\Rightarrow \left[\frac{x^2}{16}\right] + \frac{[1600 - 80x + x^2]}{16} = 58$$

$$\Rightarrow \frac{[x^2 + 1600 - 80x + x^2]}{16} = 58$$

$$\Rightarrow 2x^2 - 80x + 1600 = 58 \times 16$$

$\Rightarrow 2x^2 - 80x + 1600 = 928$

Dividing through out by 2 ,

$x^2 - 40x + 800 = 464$

$\Rightarrow x^2 - 40x + 800 - 464 = 0$

$\Rightarrow x^2 - 40x + 336 = 0$

(d) Length of each piece,

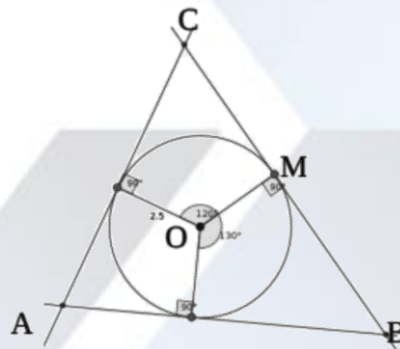$x^2 - 40x + 336 = 0$

$\Rightarrow (x - 28)(x - 12) = 0$

$\Rightarrow x - 28 = 0$ or $x - 12 = 0$

$x = 28$ or $x = 12$.

The length of each piece $= 28$ m or 12 m .

Q25. Draw a circle of radius 2.5 cm. Draw a triangle of two angles 500 , 60° with all its sides touching the circle.

Solution:

Steps of construction:

Draw a circle of radius 2.5 cm with centre O .

Draw OM as the radius.

Construct ∠MON $= 120°$ and ∠MOP $= 130°$.

Draw the tangents through M, N and P respectively and meet at A, B and C.

△ ABC is the required triangle.

Q26. A boy saw the top of a building under construction at an elevation of 30°. The completed building was 12 m higher and the boy saw its top at an elevation of 60° from the same spot.

(a) Draw a rough figure based on the given details.

(b) What is the height of the building?

(c) What is the distance between the building and the boy?

Solution:

(a)
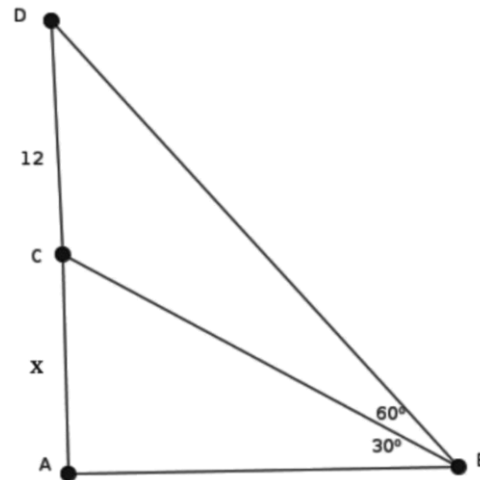
(b) Consider AC = x , AD = x +12 .

In triangle ABD, $\left(\dfrac{AD}{AB}\right)$ = tan 60°

$AB = \left(\dfrac{AD}{\tan 60°}\right)$

$\Rightarrow AB = \left(\dfrac{x+12}{\sqrt{3}}\right) \dots (1)$

In triangle ABC ,

$\left(\dfrac{AC}{AB}\right)$ = tan 30°

$\Rightarrow AB = \left(\dfrac{AC}{\tan 30°}\right)$

$\Rightarrow AB = \left(\dfrac{x}{\left(\dfrac{1}{\sqrt{3}}\right)}\right)$

$= x\sqrt{3} \dots \dots \dots (2)$

Comparing equations (1) and (2),

$\left(\dfrac{x+12}{\sqrt{3}}\right) = x\sqrt{3}$

$\Rightarrow 3x = x + 12$

$\Rightarrow 3x - x = 12$

$\Rightarrow 2x = 12$

$x = 6$

Hence, the height of the building = 6 + 12 = 18 m.

(c) Consider the equation (2), $AB = \sqrt{3}x$

$= \sqrt{3} \times 6$

$= 1.73 \times 6$

$= 10.38$ m

Distance between the building and the boy $= 10.38$ m.

Q27. The picture shows the shape of a boiler. The total height of the boiler is 12 m and the diameter is 6m, the height of the cylindrical part is 6m.

(a) What is the height of the cone?

(b) How many litres can the boiler hold?

Solution:

Given, total height $= 12$ m

Radius $= \left(\frac{6}{2}\right) = 3$ m

Height of the cylinder part (h) $= 6$ m

(a) Height of the cone $= 12 - (6 + 3) = 3$ m.

(b) Volume of the boiler = Volume of the cone + volume of the cylinder + volume of the hemisphere.

$\left(\frac{1}{3}\right)\pi r^2 h + \pi r^2 h + \left(\frac{2}{3}\right)\pi r^3$

$= \left(\frac{1}{3}\right) \times \pi \times 3 \times 3 \times 3 + \pi \times 3 \times 3 \times 6 + \left(\frac{2}{3}\right) \times 3 \times 3 \times 3$

$= 9\pi + 54\pi + 18\pi$

$= 81\pi$ m³

$= 81 \times 3.14$

$= 254.34 \text{ m}^3$

Hence, the volume in liters $= 254.34 \times 1000 = 254340$ liters.
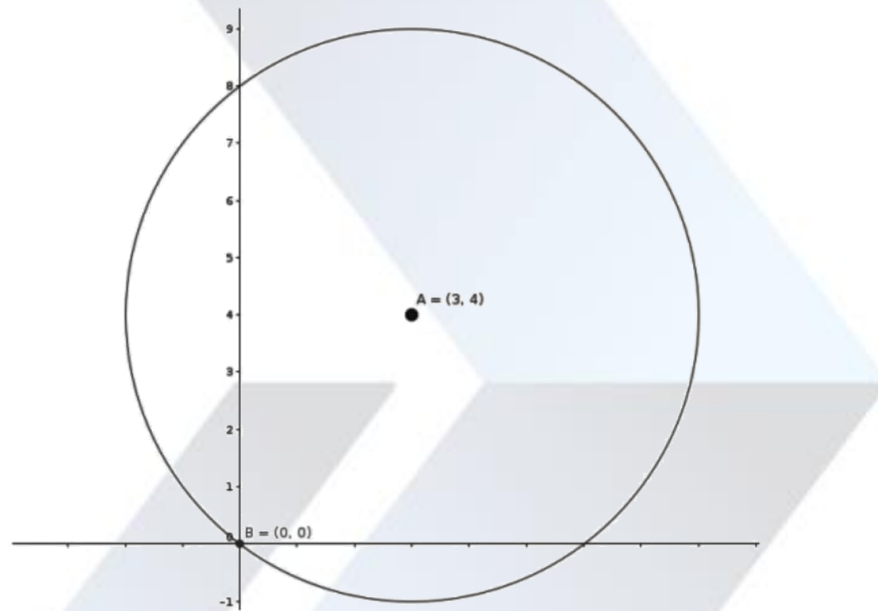
Q28. A circle with centre (3,4) passes through the origin.
(a) What is the radius of the circle?
(b) If a point on the circle is (x, y), write the relation between x, y.
(c) Check whether the point $(-2,1)$ lies on this circle.
Solution:

Given , center (3, 4) ; Origin (0,0).
(a) Radius (r) $= \sqrt{x^2 + y^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

(b) Equation of the circle $= (x - a)^2 + (y - b)^2 = r^2$

$(x - 3)^2 + (y - 4)^2 = 5^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = 25$

$\Rightarrow x^2 + y^2 - 6x - 8y + 25 - 25 = 0$

$\Rightarrow x^2 + y^2 - 6x - 8y = 0$ is the equation of the circle.

(c) $(-2,1)$, substitute this value in the equation of the circle, we get,

$(-2)^2 + (1)^2 - 6 \times -2 - 8 \times 1 = 0$

$4 + 1 + 12 - 8 = 0$

$17 - 8 = 0$

$9 \neq 0$

AC > radius of the circle.

Hence, the point $(-2,1)$ lies outside the circle.

Q29. Read the following, understand the mathematical idea expressed in it and answer the questions that follow.

1,4,9,16 … … are the squares of the counting numbers. The remainders got by dividing the square numbers with natural numbers have a cyclic property. For example, the remainder on dividing these numbers by 4 are tabulated here.

Number	1	4	9	16	25	-	-	-
Remainder	1	0	1	0	1	-	-	-

On dividing by 4 perfect squares leave only 0 and 1 as remainders. From this, we can conclude that an arithmetic sequence whose terms leaves remainder 2 on dividing by 4 does not have a perfect square.

(a) Which are the possible remainders on dividing any number with 4 ?

(b) Which are the numbers we would not get on dividing a perfect square by 4 ?

(c) What is the remainder that leaves on dividing the terms of the arithmetic sequence 2,5,8,11, by 4 ?

(d) Does the arithmetic sequence 3,7,11,. .contain perfect squares?

(e) Write a sequence with common difference 4 which contains many perfect squares?

Solution:

(a) 0, 1, 2 and 3

(b) 2 and 3

(c) 2, 1, 0 and 3

(d) No, the sequence doesn't contain perfect square numbers.

3,7,11, … ….. is divided by 4 we get the remainder 3. When divided by the perfect square numbers by 4, we get the remainder 0 and 1.

(e) 1, 5, 9, 13