Grade 10 Kerala Mathematics 2019

Answer any 3 questions from 1 to 4. Each question carries 2 scores.

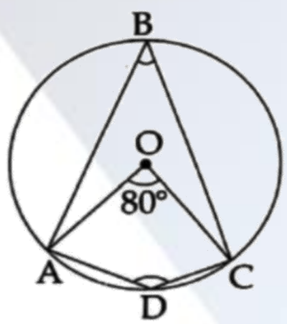
Q1. In the figure, O is the centre of the circle.

$\angle AOC = 80°$

[i] What is the measure of $\angle ABC$?

[ii] What is the measure of $\angle ADC$?

Solution:



Given $\angle AOC = 80°$

[i] The measurement $\angle ABC = \left(\frac{1}{2}\right) \times \angle AOC = \frac{1}{2} \times 80 = 40°$.

[ii] $\angle ABC + \angle ADC = 180°$

$40° + \angle ADC = 180°$

$\angle ADC = 180° - 40°$

$\angle ADC = 140°$

Q2. [i] Write the first integer term of the arithmetic sequence $\left(\frac{1}{7}\right), \left(\frac{2}{7}\right), \left(\frac{3}{7}\right)$ ....

[ii] What is the sum of the first 7 terms of the above sequence?

Solution:

[i] Given arithmetic sequence $= \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \ldots \ldots \ldots.$

Common difference $d = \frac{2}{7} - \frac{1}{7} = \frac{1}{7}$.

Hence the first integer term $= \frac{7}{7} = 1$

[ii] $a = \left(\frac{1}{7}\right)$

$d = \frac{2}{7} - \frac{1}{7}$

$= \frac{1}{7}$

$n = 7$

$S_n = \left(\frac{n}{2}\right)(2a + [n-1]d)$

$S_7 = \left(\frac{7}{2}\right)(2 \times \left[\frac{1}{7}\right] + [7-1] \times \left[\frac{1}{7}\right])$

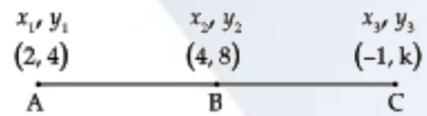
$$= \left(\frac{7}{2}\right)\left[\left(\frac{2}{7}\right) + 6 \times \left(\frac{1}{7}\right)\right]$$
$$= \left(\frac{7}{2}\right)\left[\left(\frac{2}{7}\right) + \left(\frac{6}{7}\right)\right]$$
$$= \left(\frac{7}{2}\right)\left(\frac{8}{7}\right)$$
$$= 4$$

Q3. [i] If $C(-1, k)$ is a point on the line passing through the points A (2,4) and B(4,8) which number is k ?

[ii] What is the relation between the x coordinate and the y coordinate of any point on this line?

Solution:

[i]

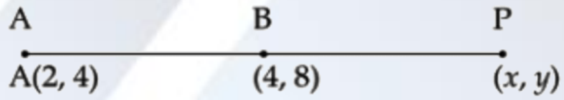


Points A, B and C are collinear.

Area of triangle ABC = 0

$$\left(\frac{1}{2}\right)(x_1[y_2 - y_3] + x_2[y_3 - y_1] + x_3[y_1 - y_2])$$
$$|2(8 - k) + 4(k - 4) + (-1)(4 - 8)| = 0$$
$$16 - 2k + 4k - 16 - 4 + 8 = 0$$
$$2k = -4$$
$$k = -2$$

[ii]



Area of triangle ABP = 0

$$\left(\frac{1}{2}\right)(x_1[y_2 - y_3] + x_2[y_3 - y_1] + x_3[y_1 - y_2])$$
$$|2(8 - y) + 4(y - 4) + (x)(4 - 8)| = 0$$
$$16 - 2y + 4y - 16 - 4x = 0$$
$$2y - 4x = 0$$
$$2y = 4x$$
$$y = 2x$$
$$2x - y = 0$$

Q4. [i] Find $P(1)$ if $P(x) = x^2 + 2x + 5$

[ii] If $(x - 1)$ is a factor of $x^2 + 2x + k$, what is the value of k ?

Solution:

[i] $P(x) = x^2 + 2x + 5$

$P(1) = 1^2 + 2 \times 1 + 5$

$= 1 + 2 + 5$
$P(1) = 8$
[ii] Since $(x - 1)$ is the factor of $x^2 + 2x + k$, then
$x - 1 = 0$
$x = 1$
$(1)^2 + 2(1) + k = 0$
$1 + 2 + k = 0$
$k = -3$

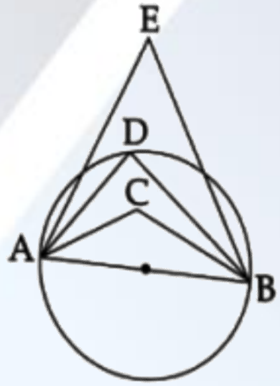
Answer any 5 questions from 5 to 11. Each question carries 3 scores.

Q5. [i] What is the remainder on dividing the terms of the arithmetic sequence 100,107,114 ..... by 7?
[ii] Write the sequence of all three-digit numbers. Which leaves the remainder 3 on division by 7? Which is the last term of this sequence?

Solution:

[i] Given sequence be 100,107,114, … … ....
d = 7
Remainder $= \frac{100}{7} = 2$
[ii] 101, 108, 115
Hence the last three-digit term = 997.

Q6. AB is the diameter of the circle. D is the point on the circle.



$\angle ACB + \angle ADB + \angle AEB = 270^\circ$. The measure of one among $\angle ACB$, $\angle ADB$ and $\angle AEB$ is 110°. Write the measures of $\angle ACB$, $\angle ADB$, $\angle AEB$.

Solution:

$\angle ADB = 90^\circ$ (Measurement of semi circle angle)
$\angle ACB + \angle ADB + \angle AEB = 270^\circ$ (given)
$\angle ACB + 90^\circ + \angle AEB = 270^\circ$
$\angle ACB + \angle AEB = 270^\circ - 90^\circ = 180^\circ$
The given condition is that any one of the angles $\angle ACB$,
$\angle AEB$ be 110°.

Take $\angle ACB = 110°$

Hence $\angle AEB = 180° - 110° = 70°$

So the angles, $\angle ADB = 90°, \angle ACB = 110°, \angle AEB = 70°$.

Q7. If x is a natural number,

[a] What number is to be added to $x^2 + 6x$ to get a perfect square?

[b] If $x^2 + ax + 16$ is a perfect square number, then which number is a?

[c] If $x^2 + ax + b$ is a perfect square, prove that $a^2 = 4b$.

Solution:

Given $x^2 + 6x$

[a] $6x = 2ab$

$a = x$

$b = ?$

$b = \frac{6x}{2x} = 3$

Perfect square $= b^2 = 3^2 = 9$.

Hence 9 is to be added to them.

[b] Given, $x^2 + ax + 16$ is perfect square

This is the form of $a^2 + 2ab + b^2 = (a+b)^2$

$2ab = ax$

$a = x$

$b^2 = 16$

$b = \sqrt{16} = 4$

So, $(x+4)^2 = x^2 + ax + 16$

Hence $a = 2ab = 2 \times 4 = 8$.

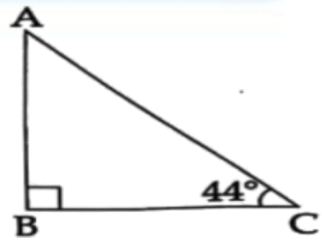
[c] Here b = the square of the half of a

$b = \left(\frac{a}{2}\right)^2$

$b = \frac{a^2}{4}$

$a^2 = 4b$

Q8. In the figure, $\angle B = 90°, \angle C = 44°$.



[a] What is the measure of $\angle A$?

[b] Which among the following is $\tan 44°$:

$\left(\frac{AB}{BC}\right), \left(\frac{AB}{AC}\right), \left(\frac{BC}{AB}\right), \left(\frac{BC}{AC}\right)$

[c] Prove that $\tan 44^\circ \times \tan 46^\circ = 1$

Solution:

[a] $\angle A + \angle B + \angle C = 180^\circ$

$\angle A + 90^\circ + 44^\circ = 180^\circ$

$\angle A = 180^\circ - 90^\circ - 44^\circ$

$\angle A = 46^\circ$

[b] In ΔABC,

$\tan 44^\circ = \frac{\text{opposite}}{\text{adjacent}}$

$= \frac{AB}{BC}$ {from the figure }

[c] Take LHS $= \tan 44^\circ \times \tan 46^\circ$

$= \tan 44^\circ \times \cot(90^\circ - 46^\circ)[\tan \theta = \cot(90 - \theta)]$

$= \tan 44^\circ \times \cot 44^\circ$

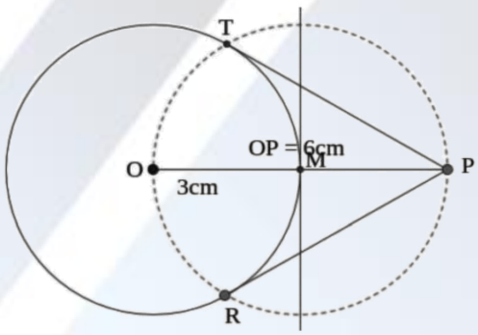
$= \tan 44^\circ \times \left[\frac{1}{\tan 44^\circ}\right]$

$= 1$

= RHS

Q9. Draw a circle of radius 3 centimetres. Mark a point P at a distance of **6cm** from the centre of the circle. Draw tangents from P to the circle.

Solution:



Steps of construction:

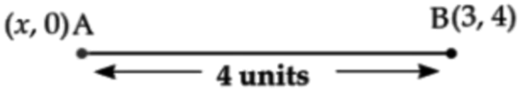
1. Draw a circle of radius 3 cm with O as the centre.
2. From the centre O, draw OP = 6 cm and perpendicular to OP marking it as M.
3. Draw another circle with centre M cutting T and R respectively.
4. Join PT and PR which are the required tangents.

Q10. [i] Find the coordinates of the point on the x-axis, which is at a distance of 4 units from (3, 4).

[ii] Find the coordinates of the point on the x-axis at a distance of 5 units from (3,4).

Solution:

[i]



$4 = \sqrt{(x-3)^2 + (0-4)^2}$

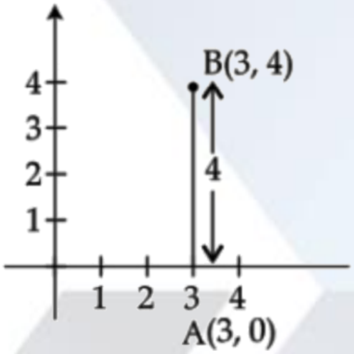
On squaring both sides,

$4^2 = (x-3)^2 + 16$

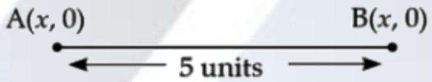
$x - 3 = 0$

$x = 3$

A(3,0) is the required point.



[ii]



$AB = 5$

$\sqrt{(x-3)^2 + (0-4)^2 = 5}$

On squaring both sides,

$(x-3)^2 + 16 = 5^2$

$(x-3)^2 + 16 = 25$

$(x-3)^2 = 25 - 16$

$(x-3)^2 = 9$

$(x-3) = \pm 3$

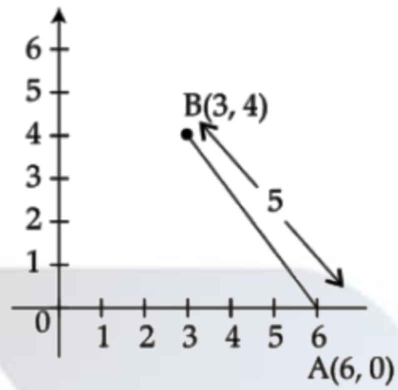
$x - 3 = 6$

$x = 6$

$x - 3 = -3$

$x = 0$

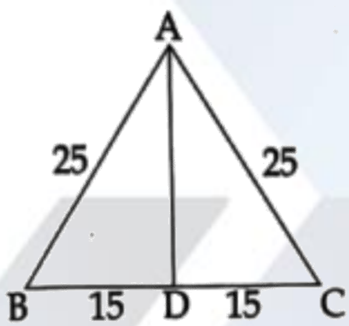
Hence, (6,0) and (0,0) is the required point.



Q11. The given figure is the lateral face of a square pyramid. AB = AC = 25 centimeters and BD = DC = 15 centimeters.

[i] What is the length of its base edge?

[ii] Find the lateral surface area of the pyramid.



Solution:

Side of the base $= \frac{\text{diagonal}}{\sqrt{2}}$

$= \left(\frac{30}{\sqrt{2}}\right) \times \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$= 15\sqrt{2}$ cm

= 15 × 1.414

= 17.210

Side of the base = 17.210 cm

Lateral surface area $= \left(\frac{1}{2}\right) \times$ perimeter of the base × slant height

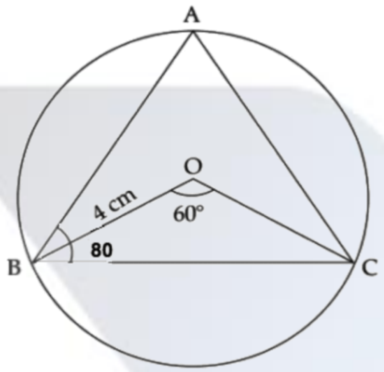
$= \left(\frac{1}{2}\right) \times (17.21) \times 4 \times 25$

$= 860.50 \text{ cm}^2$

Answer any 7 questions from 12 to 21. Each question carries 4 scores.

Q12. In triangle ABC, $\angle A = 30^\circ, \angle B = 80^\circ$, the circumradius of the triangle is 4 centimetres. Draw the triangle. Measure the length of its smallest side.

Solution:



Steps of construction:

1. Draw a circle of radius 4 cm having a centre at O .
2. Make an angle $\angle BOC = 60^\circ$.
3. Construct an angle $\angle CBA = 80^\circ$.
4. Join AC.
5. $\triangle$ ABC is the required triangle.

$\angle A + \angle B + \angle C = 180^\circ$

$30^\circ + 80^\circ + \angle C = 180^\circ$

$\angle C = 180^\circ - 110^\circ$

$\angle C = 70^\circ$

$30^\circ < 70^\circ < 80^\circ$

$\angle A < \angle C < \angle B$

The smallest angle is $\angle A$.

BC is the smallest side of $\triangle$ ABC.

Q13. Find the following sums:

[i] $1 + 2 + 3 + 100$

[ii] $1 + 3 + 5 + \cdots \ldots 99$

[iii] $2 + 4 + 6 + \cdots ..100$

[iv] $3 + 7 + 11 + \cdots \ldots ..199$

Solution:

[i] $1 + 2 + 3 + \ldots\ldots\ldots..100$

$a = 1$

$d = 2 - 1 = 1$

Last term $= 100 = l$

$l = a + (n - 1)d$

$100 = 1 + (n - 1)1$

$100 = 1 + n - 1$

$n = 100$

$S_n = \left(\frac{n}{2}\right)(n + l)$

$S_{100} = \left(\frac{100}{2}\right)(100 + 1)$

$= (50) \times (101)$

$= 5050$

[ii] $1 + 3 + 5 + \cdots \ldots 99$

$a = 1$

$d = 3 - 1 = 2$

Last term = 99

$l = a + (n - 1)d$

$99 = 1 + (n - 1)2$

$99 = 1 + 2n - 2$

$99 = 2n - 1$

$100 = 2n$

$\frac{100}{2} = n$

$50 = n$

$S_n = \left(\frac{n}{2}\right)(a + a_n)$

$S_{50} = \left(\frac{50}{2}\right)(1 + 99)$

$= (25) \times (100)$

$= 2500$

[iii] $2 + 4 + 6 + \cdots \ldots 100$

$a = 2$

$d = 4 - 2 = 2$

Last term $= 100 = l$

$l = a + (n - 1)d$

$100 = 2 + (n - 1)2$

$100 = 2 + 2n - 2$

$100 = 2n$

$n = \frac{100}{2}$

$n = 50$

$S_n = \left(\frac{n}{2}\right)(a + a_n)$

$S_{50} = \left(\frac{50}{2}\right)(2 + 100)$

$= (25) \times (102)$

$= 2550$

[iv] $3 + 7 + 11 + \cdots \ldots .199$

$a = 3$

$d = 7 - 3 = 4$

Last term $= 199 = l$

$l = a + (n - 1)d$
$199 = 3 + (n - 1)4$
$199 = 3 + 4n - 4$
$199 = 4n - 1$
$\frac{200}{4} = n$
$n = 50$
$S_n = \left(\frac{n}{2}\right)(a + a_n)$
$S_{50} = \left(\frac{50}{2}\right)(3 + 199)$
$= (25) \times (202)$
$= 5050$

Q14. A box contains some green and blue balls. 7 red balls are put into it. Now the probability of getting a red ball from the box is $\frac{7}{24}$ and that of the blue ball is $\frac{1}{6}$.
[i] How many balls are there in the box?
[ii] How many of them are blue?
[iii] What is the probability of getting a green ball from the box?

Solution:

Let the number of green balls be x.
The number of blue balls is y.
Number of red balls = 7
Total number of balls $= x + y + 7$
P (red ball) $= \frac{7}{24}$
P(blue ball) $= \frac{1}{3}$
[i] Since P(red ball) $= \frac{7}{24}$,
$\frac{7}{(x + y + 7)} = \frac{7}{24}$
$24 = x + y + 7$
$24 - 7 = x + y$
$17 = x + y$ ----(1)
P(blue ball) $= \frac{1}{3}$
$\frac{y}{x + y + 7} = \frac{1}{3}$
$3y = x + y + 7$
$2y = x + 7$
$-x + 2y = 7$---- (2)
On adding equation (1) and (2),
$17 = x + y$
$-x + 2y = 7$
$3y = 24$

$y = \frac{24}{3}$

$y = 8$

Put $y = 8$ in equation (1),

$17 = x + 8$

$17 - 8 = x$

$x = 9$

Total number of balls $= 8 + 9 + 7 = 24$

[ii] Number of blue balls

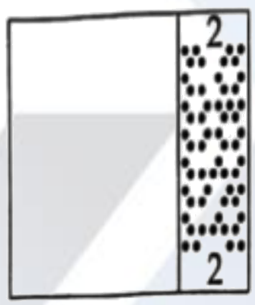
$\frac{y}{24} = \frac{1}{3}$

$3y = 24$

$y = 8$

[iii] P (green ball) $= \frac{x}{24} = \frac{9}{24} = \frac{3}{8}$

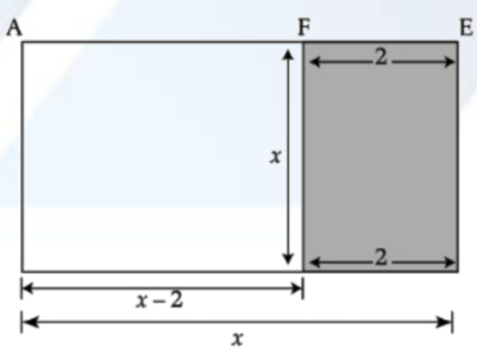
Q15. Land is acquired for road widening from a square ground, as shown in the figure. The width of the acquired land is 2 meters. Area of the remaining ground is 440 square meters.



[i] What is the shape of the remaining ground?

[ii] What is the length of the remaining ground?

Solution:



[i] The shape of the remaining ground is rectangular.

[ii] Let the length be x and breadth be $x - 2$.

Given,

Area $= 440\ m^2$

$L \times B = 440$

$x \times (x - 2) = 440$

$x^2 - 2x = 440$

$x^2 - 2x - 440 = 0$

$(x - 22)(x + 20) = 0$

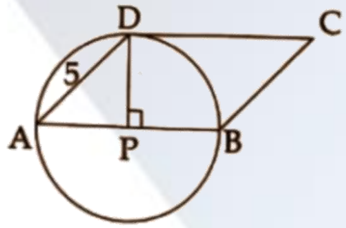
$x = 22, -20$

Since the values must be positive, $x = 22$ is taken.

Length $= 22$ m

Breadth $= 22 - 2 = 20$ m

Q16. In the figure, P is the centre of the circle. A, B and D are points on the circle. $\angle P = 90^\circ$, AD $=$ 5 cm.



(a) What is the measure of $\angle A$?

(b) What is the area of the ΔAPD?

(c) Find the area of the parallelogram ABCD.

Solution:

[a] In ΔAPD, $\angle P = 90^\circ$

$\angle A = \angle D$ [angle opposite to equal side are equal]

$\angle A + \angle ADP + \angle APD = 180^\circ$ [angle sum property of a triangle]

$\angle A + \angle A + 90^\circ = 180^\circ$

$2\angle A = 90^\circ$

$\angle A = 45^\circ$

[b] In $\triangle APD$,

$\sin\ 45^\circ = \frac{PD}{AD}$

$\frac{1}{\sqrt{2}} = \frac{PD}{5}$

$\frac{5}{\sqrt{2}} = PD = AP$

Area of $\triangle$ ADP $= \left(\frac{1}{2}\right) \times AP \times PD$

$= \left(\frac{1}{2}\right) \times \left(\frac{5}{\sqrt{2}}\right) \times \left(\frac{5}{\sqrt{2}}\right)$

$= \frac{25}{4}\ cm^2$

[c] Area of a parallelogram $=$ base $\times$ height

$= AB \times PD$

$= 2AP \times PD$

$= 2 \times \left(\frac{5}{\sqrt{2}}\right) \times \left(\frac{5}{\sqrt{2}}\right)$

$= 25\ \text{cm}^2$

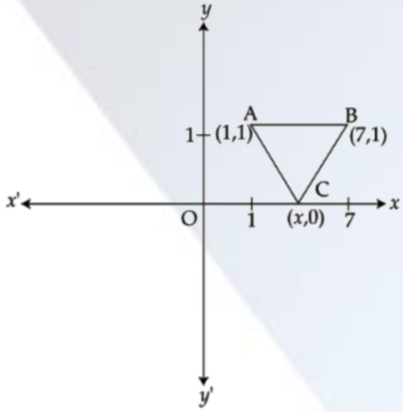
Q17. [a] Draw the coordinates and mark the points A (1, 1), B (7, 1).

[b] Draw an isosceles triangle ABC with AB as the hypotenuse.

[c] Write the coordinates of C.

Solution:

[a]



AC = BC

$\sqrt{(x-1)^2+1} = \sqrt{(x-7)^2+1}$

On squaring both sides,

$(x-1)^2 + 1 = (x-7)^2 + 1$

$x^2 + 1 - 2x + 1 = x^2 + 49 - 14x + 1$

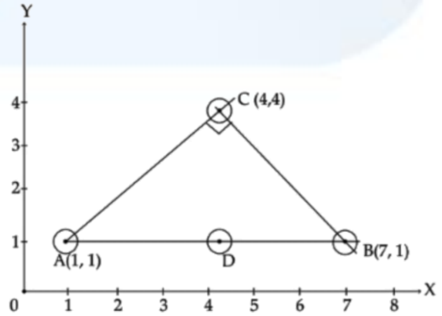
$-2x + 14x = 49 - 1$

$12x = 48$

$x = \frac{48}{12}$

$x = 4$

[b]

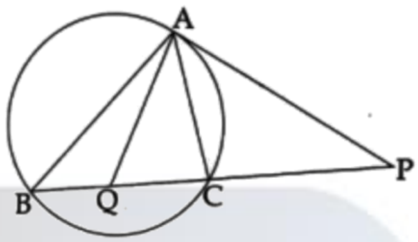


C (4,0) is the required point.

Coordinate of C(4,4)[AD = BC = CD]

The midpoint of the hypotenuse is equal distance from the vertex of the triangle.

Q18. In the figure, chord BC is extended to P. Tangent from P to the circle is PA. AQ is the bisector of ∠BAC.

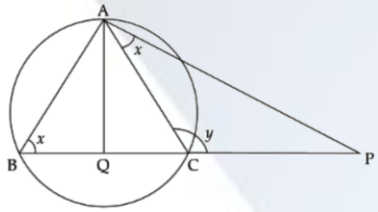


[a] Write one pair of equal angles from the figure.

[b] If $\angle PAC = x$ and $\angle PCA = y$, then prove that $\angle BAC = y - x$.

[c] Prove that $\angle PAQ = \frac{[y+x]}{2}$

Solution:



[a]$\angle BAC = \angle PAC$

[b]$\angle PAC = \angle ABC$

$\angle ACP = \angle BAC + \angle ABC$ [exterior angle property]

$y = \angle BAC + x$

$\angle BAC = y - x$

[c]$\angle PAQ = \angle PAC + \angle CAQ$

$= x + (\frac{1}{2}) \times \angle BAC$

$= x + (\frac{1}{2}) \times (y - x)$

$= x + (\frac{1}{2})y - (\frac{1}{2})x$

$\angle PAQ = (\frac{1}{2})(x + y)$

Q19. If $(x - 1)$ is a factor of the second-degree polynomial $P(x) = ax^2 + bx + c$ and $P(0) = -5$.

[a] What is the value of c ?

[b] Prove that $a + b = 5$.

[c] Write a second-degree polynomial whose one factor is $x - 1$.

Solution:

[a] Given that $x - 1$ is a factor of the polynomial $ax^2 + bx + c$

$x - 1 = 0$

$x = 1$

$P(1) = 0$

$a(1)^2 + b \times 1 + c = 0$

$a + b + c = 0$

Now, at $x = 0, P(0) = -5$

$a \times 0 + b \times 0 + c = -5$

$c = -5$

[b] $a + b + c = 0$

$a + b - 5 = 0$

$a + b = 5$

[c] Second-degree polynomial $= 3x^2 + 2x - 5$ or $2x^2 - 3x + 5$ or $4x^2 + x - 5$ [any of them]

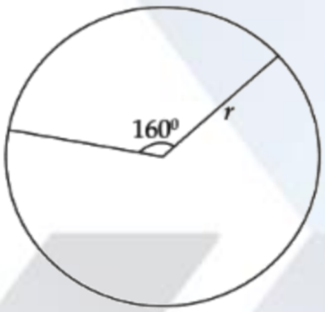
Q20. A circular sheet of paper is divided into two sectors. The central angle of one of them is 160°.

[a] What is the central angle of the remaining sector?

[b] These sectors are bent into cones of maximum volume. If the radius of the small cone is 8 centimetres, what is the radius of the other?

[c] What is the slant height of the cone?

Solution:



[a] Central angle of the remaining sector $= 360° - 160° = 200°$

[b] R_1 is the radius of the small cone = 8 cm

$$2\pi R_1 = \frac{2\pi r(\theta_1)}{360°}$$

$$8 = r \times \left(\frac{160°}{360°}\right)$$

$$r = \frac{(360° \times 8)}{160°}$$

$r = 18$ cm

$$2\pi R_2 = \frac{2\pi r(\theta_2)}{360°}$$

$$R_2 = \frac{(18 \times 200°)}{360°}$$

$R_2 = 10$ cm

[c] Slant height $(l_1) = 18$ cm

Slant height $(l_2) = 18$ cm

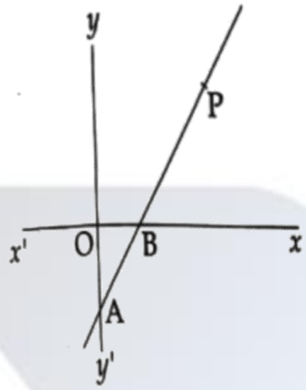
Q21. Equation of the line AB is $3x - 2y = 6$. P is a point on the line. The line intersects the y-axis at A and the x-axis at B.

[a] What is the x coordinate of A ?

[b] What is the length of OA?

[c] What is the length of OB?

[d] The x coordinate and the y coordinate of P are the same. Find the coordinates of P.



Solution:

Given, the equation of line AB is $3x - 2y = 6$

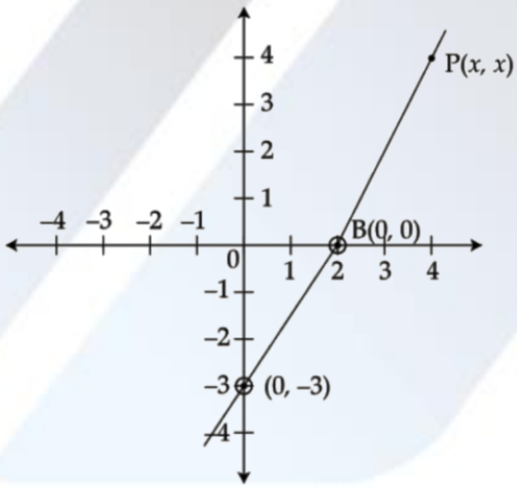
x	0	2
y	-3	0

[a] x coordinate of $A = 0$

[b] $OA = 3$ units

[c] $OB = 2$ units

[d]



A, B, P are collinear.

Area of $\triangle$ ABP $= 0$

$\left(\frac{1}{2}\right)[0(0 - x) + 2(x + 3) + x(-3 - 0)] = 0$

$2x + 6 - 3x = 0$

$x = 6$

Hence, the coordinates of P are (6,6).

Answer any 5 questions from 22 to 28. Each question carries 5 scores.

Q22. If the terms of the arithmetic sequence $\left(\frac{2}{9}\right)$, $\left(\frac{3}{9}\right)$, $\left(\frac{4}{9}\right)$, $\left(\frac{5}{9}\right)$... Are represented as $x_1, x_2, \ldots$ then

[a] $x_1 + x_2 + x_3 =$

[b] $x_4 + x_5 + x_6 =$

[c] Find the sum of the first 9 terms.

[d] What is the sum of the first 300 terms?

Solution:

[a] $x_1 + x_2 + x_3$

$= \left(\frac{2}{9}\right) + \left(\frac{3}{9}\right) + \left(\frac{4}{9}\right)$

$= \left(\frac{9}{9}\right)$

$= 1$

[b] $x_4 + x_5 + x_6$

$= \left(\frac{5}{9}\right) + \left(\frac{6}{9}\right) + \left(\frac{7}{9}\right)$

$= \left(\frac{18}{9}\right)$

$= 2$

[c] $n = 9$

$a = \left(\frac{2}{9}\right)$

$d = \left(\frac{3}{9}\right) - \left(\frac{2}{9}\right) = \left(\frac{1}{9}\right)$

$S_n = \left(\frac{n}{2}\right)(2a + [n-1]d)$

$S_9 = \left(\frac{9}{2}\right)(2 \times \left[\frac{2}{9}\right] + [9-1] \times \left(\frac{1}{9}\right))$

$= \left(\frac{9}{2}\right)[\left(\frac{4}{9}\right) + \left(\frac{8}{9}\right)]$

$= \left(\frac{9}{2}\right)\left(\frac{12}{9}\right)$

$= 6$

[d] $n = 300$

$a = \left(\frac{2}{9}\right)$

$d = \left(\frac{3}{9}\right) - \left(\frac{2}{9}\right) = \left(\frac{1}{9}\right)$

$S_n = \left(\frac{n}{2}\right)(2a + [n-1]d)$

$S_{300} = \left(\frac{300}{2}\right)(2 \times \left[\frac{2}{9}\right] + [300-1] \times \left(\frac{1}{9}\right))$

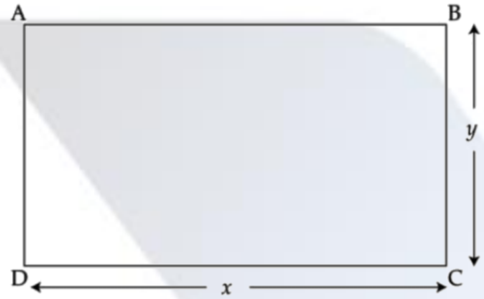
$= \left(\frac{300}{2}\right)[\left(\frac{4}{9}\right) + \left(\frac{299}{9}\right)]$

$= (150)\left(\frac{303}{9}\right)$

$= 5050$

Q23. Draw a rectangle of area 12 square centimetres. Draw a square having the same area.

Solution:



For the given rectangle,

Area $= 12\ cm^2$

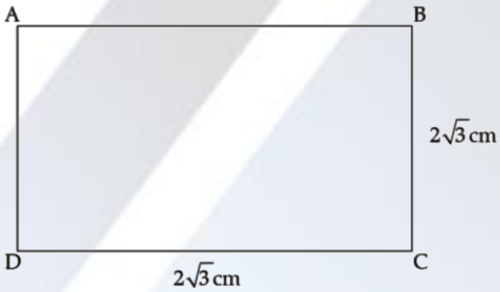
$x \times y = 12\ cm^2$

For a square, $x = y$.

$x \times x = 12\ cm^2$

$x^2 = 12\ cm^2$

$x = 2\sqrt{3}$ cm

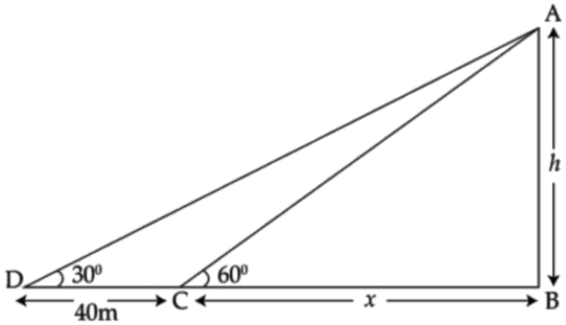


Q24. A boy standing at one bank of a river sees the top of a tree on the other bank directly opposite to the boy at an elevation of 60°. Stepping 40 meters back, he sees the top of the elevation at 30°.

[a] Draw a rough sketch and find the height of the tree.

[b] What is the width of the river?

Solution:



Let AB be h and CB be x .
In $\triangle$ ABC,

$$\tan\ 60^\circ = \left(\frac{AB}{BC}\right)$$

$$\sqrt{3} = \left(\frac{h}{x}\right)$$

$$h = \sqrt{3}x ----(1)$$

In $\triangle$ ABD,

$$\tan\ 30^\circ = \left(\frac{AB}{BD}\right)$$

$$\left(\frac{1}{\sqrt{3}}\right) = \left(\frac{h}{x}\right) + 40$$

$$x + 40 = \sqrt{3}(\sqrt{3}x) --- (2)$$

$$x + 40 = 3x$$

$$40 = 2x$$

$$x = 20$$

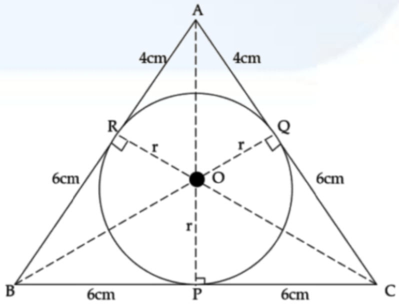
$$h = 20\sqrt{3}\ m$$

Width of the river is 20 m and the height of the tree is $20\sqrt{3}$ m.

Q25. Circle with centre O touches the sides of a triangle at P, Q and R, AB = AC, AQ = 4 cm and CQ = 6 cm.

[a] What is the length of CP?
[b] Find the perimeter and the area of the triangle.
[c] What is the radius of the circle?

Solution:



[a] CP = CQ [Length of external tangents are equal]
CP = 6 cm
[b] Perimeter of triangle = 4 + 6 + 6 + 6 + 4 + 6 = 32 cm
For the area of $\triangle$ ABC,

$$s = \left(\frac{[AB + BC + CA]}{2}\right)$$
$$= \left(\frac{[10 + 12 + 10]}{2}\right)$$
$= 16$ cm

Area of $\triangle$ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(16)(16-10)(16-12)(16-10)}$

$= \sqrt{16 \times 6 \times 4 \times 6}$

$= 48\ \text{cm}^2$

[c] Area of $\triangle$ ABC = area of $\triangle$ AOB + area of $\triangle$ BOC + area of $\triangle$ COA

$$48 = \left(\frac{1}{2}\right) \times 10 \times r + \left(\frac{1}{2}\right) \times 12 \times r + \left(\frac{1}{2}\right) \times 10 \times r$$

$48 \times 2 = r(10 + 12 + 10)$

$48 \times 2 = 32 \times r$

$r = 3$ cm

Q26. Radius of a cylinder is equal to its height. If the radius is taken as ' r ', the volume of the cylinder is $\pi r^2 \times r = \pi r^3$. Like this find the volumes of the solids, with the following measures.

Solids	Measures	Volume
Cone	radius = height = r	
Hemisphere	radius = r	
Sphere	radius = r	

[a] What is the ratio of the volumes of the cone, hemisphere, cylinder and the sphere?

[b] A solid metal sphere of radius 6cm is melted and recast into solid cones of radius 6 cm and height 6 cm . Find the number of cones.

Solution:

[a]

Solids	Measures	Volume
Cone	radius = height = r	$\frac{1}{3}\pi r^2 h \Rightarrow \frac{1}{3}\pi r^2 \times r \Rightarrow \frac{1}{3}\pi r^3$
Hemisphere	radius = r	$\frac{2}{3}\pi r^3$
Sphere	radius = r	$\frac{4}{3}\pi r^3$

[b] $V_c : V_h : V_{cy} : V_s = \left[\left(\frac{\pi r^3}{3}\right)\right] : [\left(\frac{2}{3}\right)]\pi r^3 : \pi r^3 : [\left(\frac{4}{3}\right)]\pi r^3$

$= \left(\frac{1}{3}\right) : \left(\frac{2}{3}\right) : 1 : \left(\frac{4}{3}\right)$

= 1: 2: 3: 4

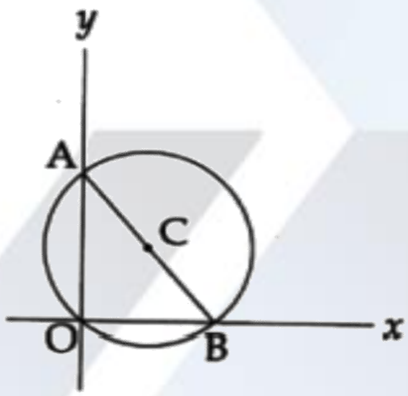
Number of cones = $\frac{\text{Volume of the sphere}}{\text{Volume of the cone}}$

$= \frac{\left(\frac{4}{3}\right)\pi r^3}{\frac{\pi R^2 h}{3}}$

$= \left(\frac{\frac{[4 \times \pi \times 6^3]}{3}}{\frac{[\pi \times 6^2 \times 6]}{3}}\right)$

$= 4$

Q27. C is at the centre of the circle passing through the origin. Circle cuts the y-axis at A(0,4) and the x-axis at B(4,0).



[a] Write the coordinates of C.

[b] Write the equation of the circle.

[c] (0,0) is a point on the circle. There is one more point on the circle with x and y coordinates equal. Which is that?

Solution:

[a] C is the midpoint of AB .

$x = \frac{(4+0)}{2}$

$= \left(\frac{4}{2}\right)$

$x = 2$

$y = \frac{(4+0)}{2}$

$= \left(\frac{4}{2}\right)$

$y = 2$

The coordinates of C are (2,2).

[b] The equation of the circle is given by $(x-a)^2+(y-b)^2=r^2$

$(x-2)^2+(y-2)^2=\left[\sqrt{(4-2)^2+(0-2)^2}\right]^2$

$x^2+4-4x+y^2+4-4y=8$

$x^2+y^2-4x-4y=0$

[c] Let $P(x,x)$ be a point on the circle.

$x^2+x^2-4x-4x=0$

$2x^2-8x=0$

$x=0,4$

The required point is (4,4).

Q28. The table below shows the number of children in a class, sorted according to their heights.

Height (Centimetres)	Number of Children
130 – 140	7
140 – 150	9
150 – 160	10
160 – 170	10
170 – 180	9

If the students are directed to stand in a line according to the order of their heights starting from the smallest, then

[a] The height of the child at what position is taken as the median?

[b] What is the assumed height of the child in the 17th position?

[c] Find the median height.

Solution:

Class interval	frequency	Cumulative frequency
130 – 140	7	7
140 – 150	9	16
150 – 160	10	26
160 – 170	10	36
170 – 180	9	45

[a] $N=45$

Median is taken as $\frac{(N+1)}{2}$

$= \left(\frac{[45+1]}{2}\right)$

$= \left(\frac{46}{2}\right)$

$= 23$

The height of the child at the 23^{rd} position is taken as the median.

[b] Height of the child in the 17^{th} position between 150-160. Assumed height is 152 cm .

[c] Median $= [l_1] + \frac{\left[\left(\frac{N}{2}\right) - C\right]}{cf} \times h$

$= 150 + \left(\frac{[22.5-16]}{10}\right) \times (10)$

$= 150 + 6.5$

$= 156.5$

Q29. Read the following. Understand mathematical concepts in it and answer the questions that follow.

The remainders obtained on dividing the powers of two by 7 have an interesting property. We can understand it from the table given below.

Number	2^1	2^2	2^3	2^4	2^5	2^6	2^7	………
Remainder	2	4	1	2	4	1	2	……..

If the powers are 1,4,7 … … the remainder is 2.

If the powers are 3,6,9 ….. the remainder is 1 .

[a] What is the remainder on dividing 2^8 by 7 ?

[b] Write the sequence of powers of 2 leaving remainder 1 on division by 7.

[c] Check whether 2019 is a term of arithmetic sequence 3, 6,9 … ….

[d] What is the remainder on dividing 2^{2019} by 7 ?

[e] Write the algebraic form of the arithmetic sequence 1,4,7 … …

[f] Write the algebraic form of the sequence $2^1, 2^4, 2^7$ …. [powers of two leaving remainder 2 on division by 7].

Solution:

[a] If 2^8 is divided by 7, then the remainder is 4.

[b] $2^3, 2^6, 2^9$ … … when divided by 7 leaves a remainder 1 .

[c] Yes

$2019 = 3(n - 3)$

$2019 = 3n - 9$

$2019 + 9 = 3n$

$2028 = 3n$

$\left(\frac{2028}{3}\right) = n$

$n = 673$ terms

[d] 1 is the remainder on dividing 2^{2019} by 7 .

[e] $a_n = a + (n - 1)d$
$a_n = 1 + (n - 1)3$
$= 1 + 3n - 3$
$a_n = 3n - 2$
[f] 1,4,7 ..... n^{th} term is $3n - 2$.
So, the algebraic form is 2^{3n-2}.