Grade 10 Kerala Mathematics 2023

INSTRUCTIONS:

Read the following instructions carefully and follow them:

1. Read each question carefully before answering.
2. Give explanations wherever necessary:
3. First 15 minutes is cool-off time. You may use this time to read the questions and plan your answers.
4. No need to simplify irrationals like $\sqrt{2}$, $\sqrt{3}$, π etc, using approximations unless you are asked to do so.

Answer any 3 questions from 1 to 4 . Each question carries 2 scores.

Q1. 7,13,19, ... is an arithmetic sequence.

(a) What is its common difference?

(b) Find its 11^{th} term.

Solution:

(a) $d = a_2 - a_1$

$= 13 - 7$

$d = 6$

(b) $a_1 = 7, d = 6,$

$n = 11$

$a_n = a + (n-1)d$

$a_{11} = 7 + (11-1)6$

$a_{11} = 7 + 10 \times 6$

$a_{11} = 67$

Q2. Weights of 11 players of a football team are given in kilograms:

55, 65, 56, 70, 62, 54, 64, 58, 68, 65, 60

Find the median of the weights of players.

Solution:

n is odd

54, 55, 56, 58, 60, 62, 64, 65, 65, 68, 70

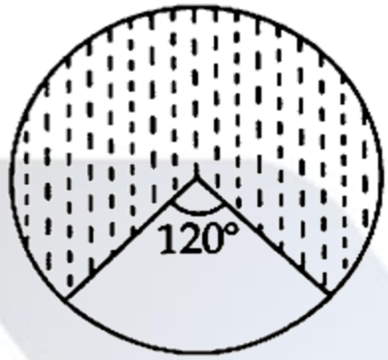
Median $= \left(\frac{n+1}{2}\right)$th term

$= \frac{11+1}{2} = \frac{12}{2} = 6$ th term

6th term is 62

Median $= 62$.

Q3. A dot is put inside the circle without looking it.



(a). What is the probability that the dot to be within the unshaded part?

(b). What is the probability that the dot to be within the shaded part?

Solution:

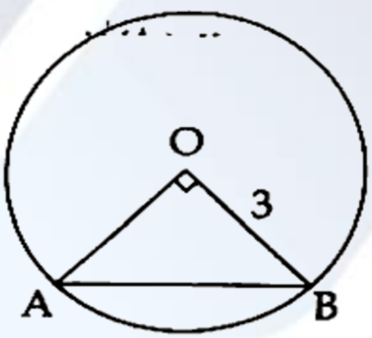
(a) The probability that the dot lies within the unshaded part

$= \frac{\text{Area of sector with } 120^\circ \text{ sectorial angle}}{\text{Area of circle}}$

$= \frac{\pi r^2 \times \frac{120^\circ}{360^\circ}}{\pi r^2} = \frac{1}{3}$

(b) The probability of the dot to be within shaded Part $= 1 - \frac{1}{2} = \frac{2}{3}$

Q4. AB is a chord of circle of radius 3 cm . Chord AB makes a right angle at the centre What is the length of AB ?



Solution:

Since the chord makes 90° at the centre,

Therefore, $\mathrm{AB}^2 = \mathrm{OA}^2 + \mathrm{OB}^2$

$\mathrm{AB}^2 = 3^2 + 3^2$

$\mathrm{AB}^2 = 18 \mathrm{~cm}^2$

$\mathrm{AB} = \sqrt{18} \mathrm{~cm}$

$\mathrm{AB} = 3\sqrt{2} \mathrm{~cm}$

Answer any 4 questions from 5 to 10 . Each question carries 3 scores.

Q5. A(3,9), C(8,12) are the coordinates of two opposite vertices of a rectangle whose sides are parallel to the coordinate axes.

(a) Find the coordinates of other two vertices of the rectangle.

(b) Find the lengths of the sides of the rectangle.

Solution:

(a) Let the coordinates of B be (x_1, y_1) and the coordinates of D be (x_2, y_2)

Now, since sides are parallel to the coordinate axes

AD is parallel to y axis and AB is parallel to x axis

Thus, Coordinates of B are (8,9)

Coordinates of D are: (3,12)

(b) The lengths of sides of rectangle

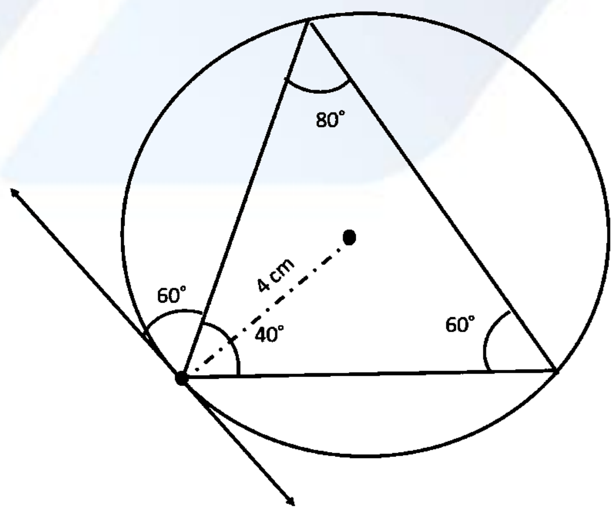
$\mathrm{AB} = \mathrm{CD} = \sqrt{(8-3)^2 + (9-9)^2} = 5$ units

$\mathrm{AD} = \mathrm{BC} = \sqrt{(8-8)^2 + (12-9)^2} = 3$ units

Q6. Draw a circle of radius 4 centimetres.

Draw a triangle whose vertices are on this circle and two of the angles 40° and 60°.

Solution:



Steps of construction:

Here are the steps to construct a triangle whose vertices lie on a circle of radius 4 cm and two of the angles are $40^{\circ}$and 60°.

Step 1: Draw a circle of radius 4 cm using a compass. Place the point of the compass at a fixed point and draw the circle by rotating the compass around the fixed point.

Step 2: Choose a point A on the circle to be the first vertex of the triangle. Mark it with a dot.

Step 3: Draw a line tangent to the circle at the point A . To do this, draw an angle of 90°angle on the point A and the radius.

Step 4: Draw an angle of 60° angle on the point A and the tangent.

Step 5: Now, extend the line from A to B.

Step 6: On B draw an angle of 80°.

Step 7: Extend B till the point C

Step 8: Join C to A.

Q7. Find the length of the sides of the rectangle whose perimeter is 80 centimeters and area 351 square centimeters.

Solution:

Perimeter of rectangle $= 2(l + b) = 80$ cm

Area of Rectangle $= l \times b = 351 \text{ cm}^2$

So, $l = \frac{351}{b}$

Let's keep the value in perimeter formula

$2\left(\frac{351}{b} + b\right) = 80$

$\frac{351}{b} + b = 40$

$351 + b^2 = 40b$

$b^2 - 40b + 351 = 0$

$b^2 - 27b - 13b + 351 = 0$

$b(b - 27) - 13(b - 27) = 0$

$$(b - 27)(b - 13) = 0$$

So, $b = 27,13$

If we take $b = 27$

Than, rectangle $= 2(l + 27) = 80$ cm

$2l + 54 = 80$

$2l = 80 - 54$

$l = 13$

So, If breadth $= 27$ cm than length $= 13$ cm

If breadth $= 13$ cm than length $= 27$ cm

Q8. (4,5) and (8,11) are coordinates of two points on a line.

(a) Find the slope of the line.

(b) Find the equation of the line.

Solution:

(a) Slope of the line with A(4,5) B(8,11) as coordinates of two prints.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{11-5}{8-4}=\frac{6}{4}=\frac{3}{2}$$

(b) Equation of line: $y-y_1=m(x-x_1)$

$$y-5=\frac{3}{2}(x-4)$$

$2y-10=3x-12$ or, $3x-2y-2=0$

Q9. 6^{th} term of an arithmetic sequence is 46. Its common difference is 8.

(a) What is its 16^{th} term?

(b) Find its 21^{st} term.

Solution:

According to Question

$a+5d=46$

Given $d=8$

$a+5\times 8=46$

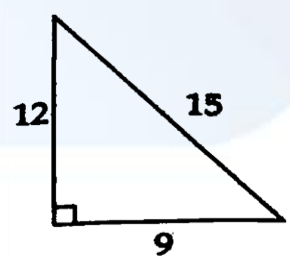
$a=46-40=6$

So,

(a) 16^{th} term $=a+15d=6+15\times 8=126$

(b) 21^{st} term $=a+20d=6+20\times 8=166$

Q10. The sides of a right triangle are 9 centimeters, 12 centimeters and 15 centimeters.



(a) Find the area of the triangle.

(b) Calculate the inradius of the triangle.

Solution:

(a) Sides of a triangle are 9 cm, 12 cm, 15 cm

Here, a $= 9$ cm, b $= 12$ cm, c $= 15$ cm

$\therefore$ Semi-perimeter, $s = \frac{a+b+c}{2} = \frac{9+12+15}{2} = \frac{36}{2} = 18$

$\therefore$ Using Heron's Formula, Area of the given triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-9)(18-12)(18-15)}$

$= \sqrt{18 \times 9 \times 6 \times 3}$

$= \sqrt{9 \times 2 \times 9 \times 2 \times 3 \times 3}$

$= 9 \times 3 \times 2 = 54\ \text{cm}^2$

(b) In radius of triangle $= \frac{\text{Area}}{\text{s}}$

Area $= 54\ \text{cm}^2$

Semi perimeter (s) $= 8$ cm

In radius of triangle $= \frac{54}{8} = 3$ cm

Answer any 8 questions from 11 to 21 . Each question carries 4 scores.

Q11. $P(x) = x^2 - 4x + 4$

(a) What is $P(1)$?

(b) Write a first degree factor of $P(x) - P(1)$.

(c) Write the polynomial $P(x) - P(1)$ as the product of two first degree polynomials.

Solution:

(a) Given, $P(x) = x^2 - 4x + 4$

Put value of $x = 1$ in $P(x)$, we get

$P(1) = 1^2 - 4(1) + 4 = 1 - 4 + 4$

$P(1) = 1$

(b) $P(x) - P(1) = (x^2 - 4x + 4) - 1 = x^2 - 4x + 3$

$P(x) - P(1) = x^2 - x - 3x + 3$

$P(x) - P(1) = x(x-1) - 3(x-1) = (x-1)(x-3)$

A First degree factor of $P(x) - P(1)$ is $x - 1$.

(c) Polynomial $P(x) - P(1)$ as the product of two first degree polynomials is

$P(x) - P(1) = (x-1)(x-3)$

Q12. A cone is made by rolling up a semicircle of radius 20 cm

(a) What is the slant height of a cone?

(b) Find the radius of the cone?

(c) Calculate the curved surface area of the cone?

Solution:

(a) When a semi-circle is used to make a cone then the radius of the semi-circle becomes slant height for the cone

So, slant height of a cone = 20 cm

(b) When a semi-circle is used to make a cone then the circumference of the semi-circle becomes the circumference for the base of the cone.

Formula used:

Circumference of the base of the cone = $2\pi R$

Circumference of the semi-circle = Circumference of the base of the cone

$\Rightarrow \pi r = 2\pi R$

$\Rightarrow \frac{22}{7} \times 20 = 2 \times \frac{22}{7} \times \mathrm{R}$

$\Rightarrow \mathrm{R} = 10 \mathrm{~cm}$

∴ The radius of the cone is 10 cm

(c) Curved surface area of cone = $\pi R l$

$= \frac{22}{7} \times 10 \times 20$

$= 628.57$ sq. cm.

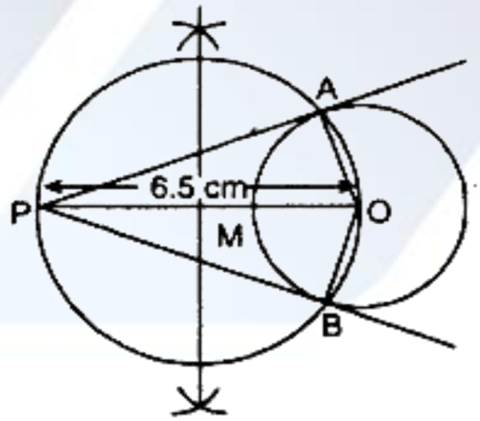
Q13. Draw a circle of radius 2.5 centimeters. Mark a point 6.5 centimeters away from the center.

Draw the tangents to the circle from this point

Measure and write the lengths of the tangents.

Solution:

Steps of construction:



Step 1 First of all, we draw a circle with center O and radius 2.5 cm

Step 2 Take a point P a part 6.5 cm from the center of circle and join OP.

Step 3 Draw a perpendicular bisector of PO and mark the point M .

Step 4 Taking M as center and radius and draw a circle which intersects their

circle center O at A and B .
Step 5 Join PA and PB. Hence PA and PB are the required tangents. On measuring we get PA = PB = 6.0 cm.
Calculation to find the length of tangents:
In △ POA, ∠PAO = 90°
So △ POA is a right angled triangle,
So by the Pythagoras Theorem, $PO^2 = AO^2 + PA^2$
$\Rightarrow PA^2 = PO^2 - AO^2$
$\Rightarrow PA^2 = (6.5) - (2.5)^2 = 42.25 - 6.25$
$\Rightarrow PA^2 = 36$
$\therefore PA = \sqrt{36} = 6$ cm
Hence, the length of each tangent is 6 cm.

Q14. Sum of the first 7 terms of an arithmetic sequence is 140.
Sum of the first 11 terms of the same arithmetic sequence is 440.
(a) What is the 4^{th} term of this arithmetic sequence?
(b) Find its 6^{th} term.
(c) What is the common difference?
(d) Find the first term of this sequence.

Solution:

Let the first term of the arithmetic sequence be 'a' and common difference be 'd '.
Then the seventh term of the sequence will be $a + 6d$, and the eleventh term will be $a + 10d$.
We are given that the sum of the first 7 terms is 140 , so:
$\left(\frac{7}{2}\right)(2a + 6d) = 140$
$2a + 6d = 40$
We are also given that the sum of the first 11 terms is 440 , so:
$\left(\frac{11}{2}\right)(2a + 10d) = 440$
$2a + 10d = 80$
We can solve these two equations to get $a = -10$ and $d = 10$
(a) Therefore, the fourth term of the sequence is:
$a + 3d = -10 + 3(10) = 20$
So the fourth term is 20 .
(b) The Sixth term of the sequence is:
$a + 5d = -10 + 5(10) = 40$
(c) Common difference = 10
(d) The first term of this sequence , $a = -10$

Q15. A box contains 4 slips numbered 1, 2, 3, 4 and another contains 5 slips numbered 1, 2, 3, 4, 5. One slip is taken from each box without looking it.

(a) In how many different ways we can choose the slips?

(b) What is the probability of both numbers being odd?

(c) What is the probability of both numbers being the same?

Solution:

(a) To find the number of different ways we can choose one slip from each box, we multiply the number of choices in each box. Thus, there are 4 choices in the first box and 5 choices in the second box, giving us a total of $4 \times 5 = 20$ different ways we can choose the slips.

(b) There are two odd numbers in the first box (1 and 3) and three odd numbers in the second box (1, 3, and 5). The probability of choosing an odd number from the first box is $\frac{2}{4} = \frac{1}{2}$, and the probability of choosing an odd number from the second box is $\frac{3}{5}$. To find the probability of both events occurring together (i.e, both numbers being odd), we multiply the probabilities:

P(both numbers are odd) = P(odd from box 1) × P (odd from box 2)

$$= \frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$$

Therefore, the probability of both numbers being odd is $\frac{3}{10}$.

(c) There are two ways in which both numbers can be both slips are numbered 1,2,3 , or 4 .

The probability of both slips being 1, 2, 3, or 4 is

$$\frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{1}{5} = \frac{1}{5}$$

Thus, the probability of both numbers being the same is $\frac{1}{5}$.

Q16. In a right triangle, one of the perpendicular sides is 2 centimeters more than that of the other.

Area of the triangle is 24 square centimeters.

Find the lengths of the perpendicular sides of the right triangle.

Solution:

Let one perpendicular side be x cm.

According to the question, Other side of perpendicular side $= x + 2$ cm

We know that,

Area of right triangle $= \frac{1}{2} \times$ product of perpendicular sides

24 sq. cm $= \frac{1}{2} \times x(x + 2)$

$$\frac{x^2 + 2x}{2} = 24$$

$x^2 + 2x = 2 \times 24$

$x^2 + 2x = 48$

$x^2 + 2x - 48 = 0$

By middle term splitting method,

$x^2 + 8x - 6x - 48 = 0$

$x(x + 8) - 6(x + 8) = 0$

$(x - 6)(x + 8) = 0$

$x = 6$ or -8

Rejecting negative value of x,

$x = 6$cm

$x + 2 = 8$ cm

Hence, lengths of the perpendicular sides of the right triangle are 6 cm and 8 cm respectively.

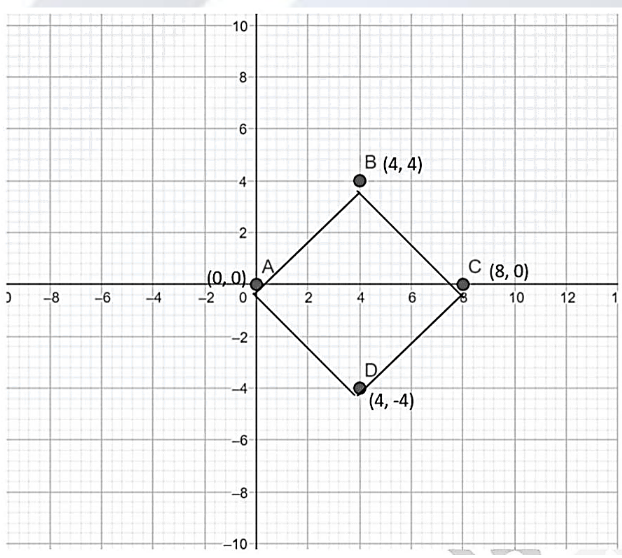
Q17. Draw the co-ordinate axes and mark the points A(0,0), B(4,4), C(8,0) and D(4, -4).

(a) Write the suitable name of the quadrilateral ABCD.

(b) Find the length of the diagonal BD.

Solution:

(a) Plotting the points A, B, C and D on the cartesian plane.



The quadrilateral ABCD thus formed is a square.

(b) The length of the diagonal BD:

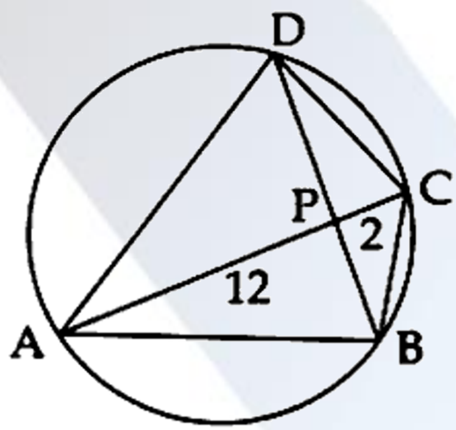
Distance of BD $= \sqrt{(4-4)^2 + (4+4)^2}$

$= \sqrt{64}$

$= 8$ units

Therefore, the length of the diagonal BD is 8 units.

Q18. Diagonals AC and MD of the cyclic quadrilateral ABCD cuts at P.
PA = 12 cm, PC = 2 cm, BD = 11 cm :



(a) If PB $= x$, then write PD in terms of x.

(b) Find the length of PB and PD.

Solution:

(a) If PB $= x$cm, then PD $= 11 - x$cm

(b) Triangles PAB and PDC are similar,

$$\frac{PA}{PD} = \frac{PB}{PC} = \frac{AB}{DC}$$

$$\frac{12}{11 - x} = \frac{x}{2}$$

$$24 = 11x - x^2$$

$$x^2 - 11x + 24 = 0$$

$$x^2 - 3x - 8x + 24 = 0$$

$$x(x - 3) - 8(x - 3) = 0$$

$$(x - 3)(x - 8) = 0$$

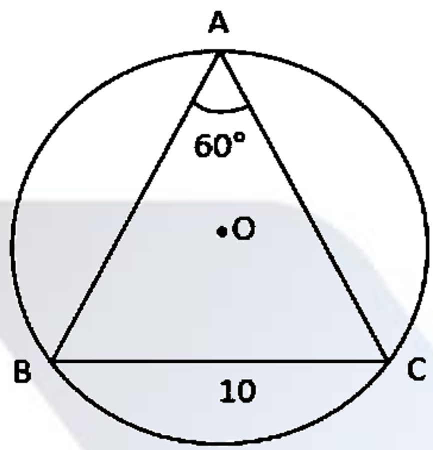
$$x = 3,8$$

If $x = 3$, then PD $= 8$ cm

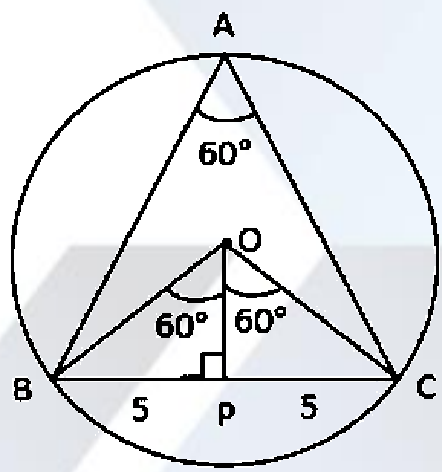
If $x = 8$, then PD $= 3$ cm

Therefore, the length of PB and PD are 8 cm and 3 cm.

Q19. BC is a chord of the circle centred at O.
BC = 10 centimetres, $\angle A = 60^\circ$. Find the radius of the circle.



Solution:



In the given figure,

Perpendicular from centre O will bisect chord BC at point P.

Also, $\angle BOP = \angle COP = \angle BAC = 60^\circ$

Now In $\triangle$ BOP,

$$\frac{BP}{OB} = \sin 60^\circ$$

$$\frac{5}{OB} = \frac{\sqrt{3}}{2}$$

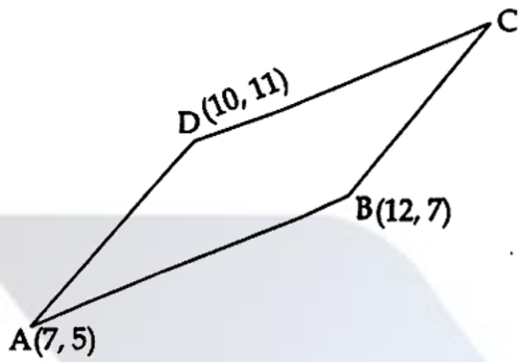
$$OB = \frac{10}{\sqrt{3}} \text{ cm}$$

Q20. In the figure, co-ordinates of 3 vertices of the parallelogram ABCD are given

(a) Write the co-ordinates of C

(b) Calculate the length of the diagonal AC.

(c) Find the co-ordinates of the point of intersection of the diagonals.



Solution:

(a) Three vertices of a parallelogram taken in order are A(7,5), B(12,7) and D(10,11)

We need to find the coordinates of C.

We know that the diagonals of a parallelogram bisect each other.

Let (x, y) be the coordinates of C.

$\therefore$ Mid-point of diagonal AC $= \left(\frac{7+x}{2}, \frac{5+y}{2}\right)$

And midpoint of diagonal BD $= \left(\frac{12+10}{2}, \frac{7+11}{2}\right) = (11,9)$

Thus we have

$\frac{7+x}{2} = 11$ and $\frac{5+y}{2} = 9$

$\Rightarrow 7 + x = 22$ and $5 + y = 18$

$\Rightarrow x = 15$ and $y = 13$

$\therefore \mathrm{C} = (15,13)$

(b) Length of diagonal AC $= \sqrt{(15-7)^2 + (13-5)}$

$= \sqrt{(8)^2 + (8)^2}$

$= \sqrt{64 + 64}$

$= \sqrt{128}$

Length of diagonal AC $= 8\sqrt{2}$

(c) In a parallelogram the diagonals bisect each other. That is the point of intersection of the diagonals is the midpoint of either of the diagonals. Here, the vertices of a parallelogram are A(7,5), B(12,7),C(15,13) and D(10,11).

We see that ' AC ' and ' BD ' are the diagonals of the parallelogram.

The midpoint of either one of these diagonals will give us the point of intersection of the diagonals.

So, midpoint of diagonal BD $= \left(\frac{12+10}{2}, \frac{7+11}{2}\right) = (11,9)$

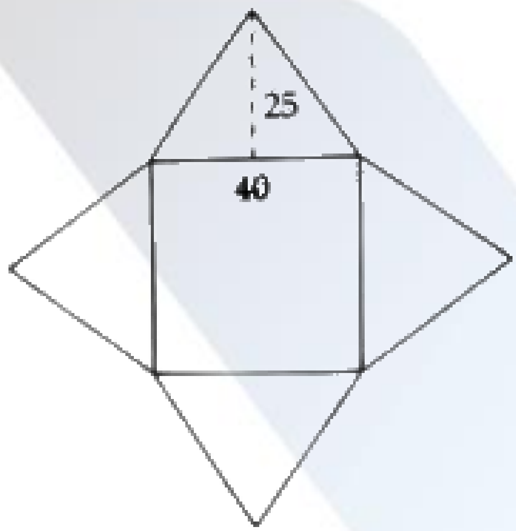
Hence the co-ordinates of the point of intersection of the diagonals of the given parallelogram are (11, 9).

Q21. A square pyramid is made by cutting out a paper as in the figure. Side of the square is 40 centimeters. Height of the triangle is 25 centimeters.

(a) What is the slant height of the square pyramid?

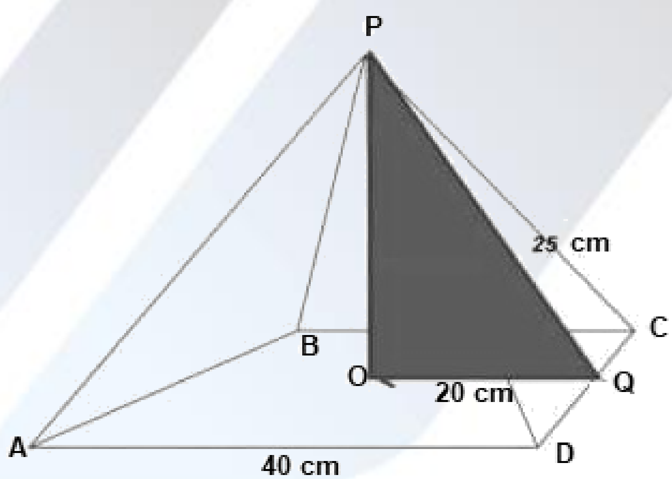
(b) Find the height of the pyramid.

(c) Calculate the volume of the pyramid.



Solution:

On folding the cuttings of paper we will get the square pyramid as following



(a) Slant height of the square pyramid will be height PQ of the triangle as shown in the above figure.

Thus, Slant height of the square pyramid = 25 cm

(b) Height of square pyramid = OP

$$OQ = \frac{AD}{2} = \frac{40}{2} = 20 \text{ cm}$$

Now in right triangle POQ ,
Using Pythagoras theorem,
$OP^2 + OQ^2 = PQ^2$
$OP^2 + 20^2 = 25^2$
$OP^2 = 25^2 - 20^2$
$OP^2 = (25 - 20)(25 + 20) = 5 \times 45$
$OP^2 = 225$
$OP = \sqrt{225} = 15$ cm
Height of square pyramid $= 15$ cm
(c) We know that,
Volume of square pyramid $= \frac{1}{3} \times$ Base area $\times$ Height of pyramid
Base area $=$ Area of square ABCD $= 40 \times 40 = 1600\ cm^2$
Height of square pyramid $= 15$ cm
Volume $= \frac{1}{3} \times 1600 \times 15 = 8000\ cm^3$

Answer any 6 questions from 22 to 29 . Each question carries 5 scores.

Q22. The daily wages of 99 workers in a factory is shown in the table.

Daily wages	Number of Workers
500 – 600	8
600 – 700	13
700 – 800	20
800 – 900	25
900 – 1000	19
1000 – 1100	14

(a) If the workers are arranged on the basis of their daily wages, at what position does the median wage fall?
(b) What is the median class?
(c) Find the median of the wages.

Solution:

(a) Median wage falls at 50^{th} position.
(b) Therefore, the median class is the category of workers earning between 800 and 900.

(c) To find Median of the wages

$= \text{value of } \left(\frac{n}{2}\right)^{\text{th}}$ observation

Class Interval	Frequency (f)	cf
$500-600$	8	8
$600-700$	13	21
$700-800$	20	41
$800-900$	25	66
$900-1000$	19	85
$1000-1100$	14	99

$= \text{value of } \left(\frac{99}{2}\right)^{\text{th}}$ observation

$= \text{value of } 49^{\text{th}}$ observation

From the column of cumulative frequency cf, we find that the 49^{th} observation lies in the class $800-900$.

$\therefore$ The median class is $800-900$.

Now,

$\therefore L =$ lower boundary point of median class $= 800$

$\therefore n =$ Total frequency $= 99$

$\therefore cf =$ Cumulative frequency of the class preceding the median class $= 41$

$\therefore f =$ Frequency of the median class $= 25$

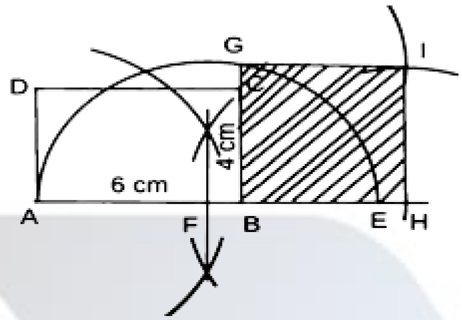
$\therefore c =$ class length of median class $= 100$

Median $M = L + \frac{\frac{n}{2} - cf}{f} \times c$

$= 800 + \frac{49.5-41}{25} \times 100 = 800 + \frac{8.5}{25} \times 100 = 800 + 34 = 834$

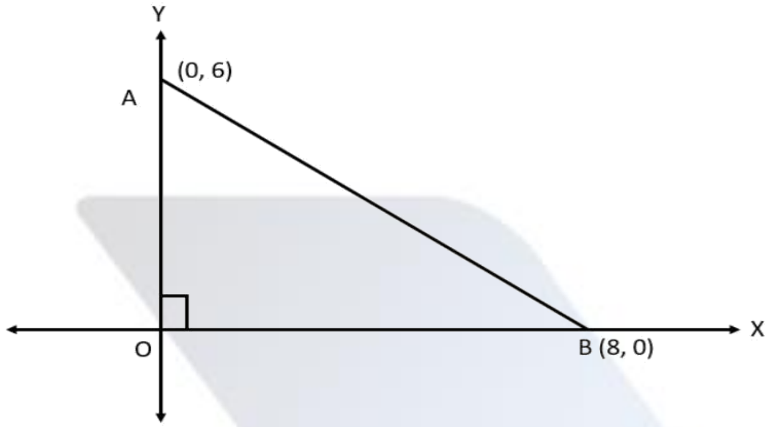
Q23. Draw a rectangle of area 24 square centimeters. Draw a square of area equal to the area of this rectangle.

Solution:



Given : a rectangle of area 24 cm^2
To Find : draw a square of the same area
rectangle area = 24 cm^2
Let say sides are 6 cm & 4 cm
Step 1: Draw line segment AB = 6 cm
Step 2 : Draw a right angle at A & B using setsquare or protractor
Step 3: Take length on both right angle as 4 cm
BC = 4 cm & AD = 4 cm
Step 4 : Join CD
Now to construct Square of same area we need to find side = $\sqrt{(6 \times 4)}$
Step 1: Extend AB as AX
Step 2 : use compass width BC = 4 cm and taking B as center cut AX at E
Step 3: Draw perpendicular bisector of AE with mid point of AE at F
Step 4 : Using compass width = AF or FE , draw a semicircle.
Step 5 : Extend BC to intersect semi-circle at G
BG = $\sqrt{(6 \times 4)}$
Step 6 : Using compass width = BG, taking B as center cut AX at H
Step 7 : Keeping compass width same and taking G & H as center draw arc intersecting at I
Join HI & GI
BHIG is a square of area same as of rectangle

Q24. In the figure, (0,6) and (8,0) are coordinates of the pointe A and B.
A circle at diameter AB is to be drawn.
(a) Find the coordinates of the center of the circle.
(b) Find the radius of the circle.
(c) What is the equation of the circle?



Solution:

(a) To find the center of the circle, we need to find the midpoint of the line segment AB, which is simply the average of the x −coordinates and the average of the y −coordinates:

Let, $A(x_1, y_1), B(x_2, y_2)$

midpoint $C = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$A(8,0)B(0,6)$

midpoint of $AB = C = \left(\frac{8+0}{2}, \frac{6+0}{2}\right) = \left(\frac{8}{2}, \frac{6}{2}\right) = (4,3)$

∴ Centre of the circle is $(4,3)$

(b) To find the radius of the circle, we need to find the distance between the center of the circle and either of the endpoints of the diameter (A or B). We can use the distance formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let's use point A as the reference point:

$d = \sqrt{(8-0)^2 + (0-6)^2}$

$d = \sqrt{64 + 36}$

$d = \sqrt{100}$

$d = 10$

Therefore, the radius of the circle is $\frac{10}{2} = 5$.

So, the circle has center $(4,3)$ and radius 5 .

(c) General Form Equation of a Circle

$(x-a)^2 + (y-b)^2 = r^2$

Where a and b are the coordinates of the center and r is the radius of the circle.

Here $a = 4, b = 3$ and $r = 5$

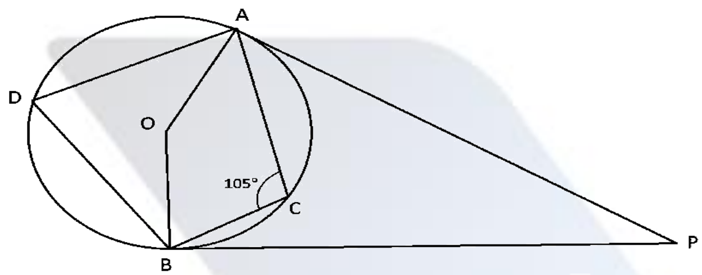
Substituting the given values of radius $r = 5$ and center $(a, b) = (4,3)$, we get:

$(x-4)^2 + (y-3)^2 = 5^2$

Therefore, the equation of the circle is

$(x-4)^2+(y-3)^2=5^2$

Q25. PA and PB are two tangents to the circle centered at O, $\angle ACB=105^{\circ}$. Find the angles given below.



(a) $\angle ADB=$
(b) $\angle AOB=$
(c) $\angle APB=$
(d) $\angle ABP=$
(e) $\angle ABO=$

Solution:

In cyclic quadrilateral ACBD
(a) $\angle ACB=105^{\circ}$
$\angle ADB=180^{\circ}-105^{\circ}$ (opposite angles must be supplementary to one another.)
$\angle ADB=75^{\circ}$
(b) Angle formed by an arc at the centre of a circle is twice the angle formed by the same arc at it's circumference
$\angle ADB=75^{\circ}$
$\therefore \angle AOB=150^{\circ}$
(c) Now, $\angle AOB+\angle APB=180^{\circ}$
$\angle APB=180^{\circ}-\angle AOB$
$\angle APB=180^{\circ}-150^{\circ}=30^{\circ}$
Reflex angle of $\angle AOB=360^{\circ}-150^{\circ}$
Reflex angle of $\angle AOB=210^{\circ}$
Join OB and OA, Now OAB is an isosceles triangle
Then $\angle AOB=150^{\circ}$
$\angle OAB=\angle OBA=x$
Now by angle sum property,
$x+x+150^{\circ}=180^{\circ}$
$x=15^{\circ}$
(d) Since the angle between tangent and radius is 90°
$\therefore \angle ABP=90^{\circ}-15^{\circ}$
$\angle ABP=75^{\circ}$
(e) $\angle APB, \angle ABP, \angle ABO$ are $30^{\circ}, 75^{\circ} \& 15^{\circ}$ respectively.

Q26. There are two cylindrical wooden blocks with diameter 60 centimeters and height 60 centimeters. A largest cone is carved out from one block and a largest sphere from the other

(a) What is the volume of the cylinder?

(b) Find the volume of the cone.

(c) Find the radius of the sphere

(d) Calculate the volume of the sphere

(c) Find the ratio of the volumes of the cone and the sphere

Solution:

(a) The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius and h are the height.

In this case, the diameter of the cylinder is given as 60 cm ,

so the radius is 30 cm . Therefore, the volume of the cylinder is:

$V = \pi(30 \text{ cm})^2(60 \text{ cm}) = 54{,}000\pi\ cm^3$

(b) The largest cone that can be carved out of the first block will have the same height as the cylinder (60 cm), and its base will be a circle with the same diameter (60 cm).

The radius of the base of the cone is therefore $r = 30$ cm, and the volume of the cone is given by the formula $V = \left(\frac{1}{3}\right)\pi r^2 h$, where h is the height.

In this case, $h = 60$ cm, so the volume of the cone is:

$$\text{V} = \frac{1}{3}\pi(30 \text{ cm})^2(60 \text{ cm}) = \frac{54{,}000\pi}{3} = 18{,}000\pi \text{ cm}^3$$

(c) The largest sphere that can be carved out of the second block will have the same diameter as the cylinder (60 cm).

The radius of the sphere is half the diameter, so $r = 30$ cm.

(d) The volume of a sphere is given by the formula $V = \left(\frac{4}{3}\right)\pi r^3$.

In this case, $r = 30$ cm, so the volume of the sphere is:

$$V = \frac{4}{3}\pi(30\ cm)^3 = 36{,}000\pi \text{ cm}^3$$

(e) The ratio of the volumes of the cone and the sphere is:

$$\frac{\text{V(cone)}}{\text{V(sphere)}} = \frac{(18{,}000\pi)}{(36{,}000\pi)} = \frac{1}{2}$$

Therefore, the volume of the cone is half the volume of the sphere.

Q27. (a) Find the sum of the first 20 natural numbers.

(b) Write the algebraic expression of the arithmetic sequence 5,9,13,

(c) Find the sum of first 20 terms of the arithmetic sequence 5,9,13,

Solution:

(a) Sum of first n natural numbers

$\frac{n(n+1)}{2}=\frac{20\times21}{2}=210.$

Here $a=5, d=4$

(b) $a_n=a+(n-1)d$

$a_n=5+(n-1)4$

$a_n=5+4n-4$

$\Rightarrow a_n=1+4n$

(c) $S_{20}=\frac{20}{2}(a+l)$, here $a=5$,

l is the 20^{th} term

$a_{20}=a+19d$

$a_{20}=5+19\times4$

$a_{20}=81$

$S_{20}=\frac{20}{2}(5+81)$

$=10\times86=860$

Q28. A child sees the top of a telephone tower at an elevation of 80°. Stepping 20 meters back, he sees it at an elevation of 40°.

(a) Draw a rough figure

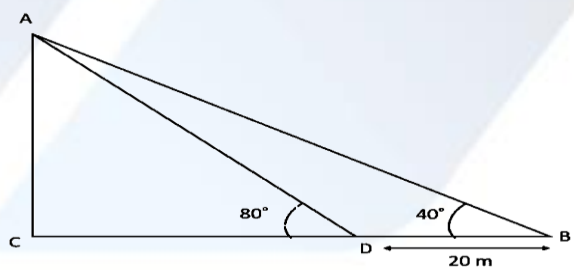
(b) Calculate the height of the tower.

$(\sin 40^\circ=0.64, \cos 40^\circ=0.77, \tan 40^\circ=0.84)$

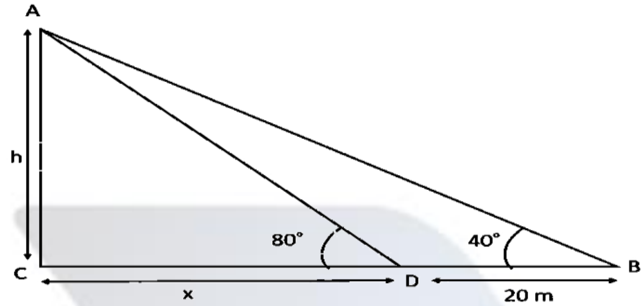
$(\sin 80^\circ=0.98, \cos 80^\circ=0.77, \tan 80^\circ=5.7)$

Solution:

(a)



(b) Let the height of the tower AC $=h$ and CD $=x$



$tan80^{\circ} = \frac{h}{x}$

$x = \frac{h}{tan80^{\circ}}$

Now, $tan40^{\circ} = \frac{h}{20 + x}$

$20 + x = \frac{h}{tan40^{\circ}}$

$20 + \frac{h}{tan\ 80^{\circ}} = \frac{h}{tan\ 40^{\circ}}$

$20^{\circ} = h\left(\frac{1}{tan\ 40^{\circ}} - \frac{1}{tan\ 80^{\circ}}\right)$

$20 = h\left(\frac{1}{tan\ 40^{\circ}} - \frac{1}{tan\ 80^{\circ}}\right)$

$20 = h\left(\frac{1}{0.84} - \frac{1}{5.7}\right)$

$20 = h(1.1904 - 0.1754)$

So, $20 = h \times 1.015$

$h = 19.70\ m$

Therefore, the height of the tower is 19.70 meters

Q29. Diagonals of a quadrilateral are the lines joining its opposite vertices.

What about the diagonals of a polygon? The lines from one vertex to the adjacent two vertices are not diagonals. They are the sides of the polygon. Lines to all other vertices are diagonals. In a quadrilateral, only one diagonal can be drawn from one vertex. If we draw from all 4 vertices, we get 4 diagonals. But 2 among them are the same. In a pentagon, from one vertex, 2 diagonals can be drawn. Therefore, total number of lines is $5 \times 2 = 10$. But 5 among them are the same.

So number of diagonals in a pentagon $= \frac{5 \times 2}{2} = 5$.

Now complete the table given below:

Polygon	Number of sides	Number of diagonals from one vertex	Total number of diagonals
Quadrilateral	4	1	$\frac{4\times 1}{2}=2$
Pentagon	5	2	$\frac{5\times 2}{2}=5$
Hexagon	6	3	$\frac{6\times 3}{2}=9$
Heptagon	7	… … … … …	… … … … …
Decagon	10	… … … …	… … … … …
n sided polygon	n	$n-3$	… … … …

Solution:

Polygon	Number of sides	Number of Diagonals from one vertex	Number of Diagonals
Quadrilateral	4	1	$\frac{4\times 1}{2}=2$
Pentagon	5	2	$\frac{5\times 2}{2}=5$
Hexagon	6	3	$\frac{6\times 3}{2}=9$
Heptagon	7	4	$\frac{7\times 4}{2}=14$
Decagon	10	7	$\frac{10\times 7}{2}=35$
n sided polygon	n	$n-3$	$\frac{n\times(n-3)}{2}$ $=\frac{n^2-3n}{2}$