Grade 10 Kerala Mathematics 2024

TIME : $2\frac{1}{2}$Hours **Total Score : 80**

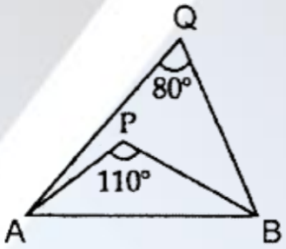
INSTRUCTIONS:

Read the following instructions carefully and follow them:

1. Read each question carefully before answering.
2. Give explanations wherever necessary.
3. First 15 minutes is cool-off time. You may use this time to read the questions and plan your answers.
4. No need to simplify irrationals like $\sqrt{2}$, $\sqrt{3}$, π etc, using approximations unless you are asked to do so.

Answer any 3 questions from 1 to 4 . Each question carries 2 scores.

Q1. See the figure.
If a circle is drawn with AB as diameter, what are the positions of the points P and Q with respect to this circle.



Solution:

Point P is inside the circle (since ∠APB > 90°).
Point Q is outside the circle (since ∠AQB < 90°).

Q2. The hemoglobin levels in grams per decilitres of seven students are given below:
12.9, 12.0, 12.6, 12.5, 14.1, 13.7, 13.4
Find the median hemoglobin level.

Solution:

Arrange the given data in increasing order:
12.0, 12.5, 12.6, 12.9, 13.4, 13.7, 14.1
Here there are 7 observations. So, the median is the 4th observation.
Median = 12.9

Q3. The sequence of perimeters of squares of sides 1 centimetre, 2 centimetres, 3 centimetres and so on the form an arithmetic sequence.

(a) Write the sequence.

(b) What is the common difference?

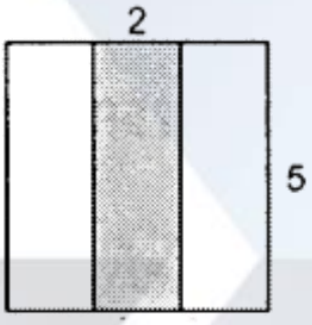
Solution:

Perimeters of square = $4 \times Length\ of\ the\ side$

(a) Sequence of perimeters of squares of sides 1 cm, 2 cm, 3 cm, etc. = 4, 8, 12,...

(b) Common difference = 4 $(8 - 4 = 4)$

Q4. A rectangular portion is shaded in a square of side 5 centimeters as shown in the figure. A dot is put inside the square without looking. Find the probability of the dot to be in the shaded region.



Solution:

Total area of the square = 5^2 = 25 cm^2

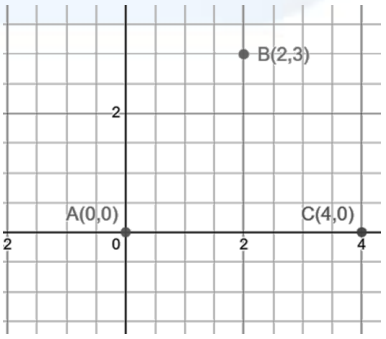
Area of the shaded rectangular region = 2 × 5 = 10 cm^2

∴ Probability of dot to be in the shaded region = $\frac{10}{25} = \frac{2}{5}$

Answer any 4 questions from 5 to 10 . Each question carries 3 scores.

Q5. Draw the coordinate axes and mark the points A(0,0), B(2,3) and C(4,0). What is the perpendicular distance from B to AC ?

Solution:



The perpendicular distance from B to AC = 3 units

Q6. Ajay is 10 years older than Renuka. The product of their ages is 144.
(a) Taking the age of Renuka as x, what is the age of Ajay in terms of x ?
(b) Find their ages.

Solution:

(a) Ajay's age = $x + 10$

(b) Product of their ages = 144

$x(x + 10) = 144$

$x^2 + 10x = 144$

$x^2 + 10x + 5^2 = 144 + 5^2$

$(x + 5)^2 = 13^2$

$x = 8$

So, Renuka's age is 8 and Ajay's age is 18.

Q7. Draw a rectangle of sides 4 centimetres and 3 centimetres. Draw a square of the same area.

Solution:

Draw a rectangle $PQRS$ in the given measurement.

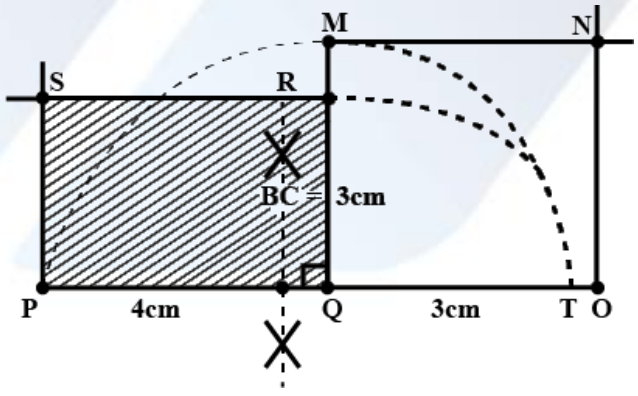
PQ produced up to T as $QT = 3$ cm.

Draw a perpendicular bisector of the lime PT, meet Q on PT.

Produce QR meet the semicircle at M.

Draw the square $QMNO$ as $QM = MN = ON = QO$.

Hence $QMNO$ be the required square.



Q8. Prove that the points (3,5), (6,7) and (9,9) are on the same line.

Solution:

Slope of the line with A(3,5) B(6,7) as coordinates of two points.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-5}{6-3} = \frac{2}{3}$

Equation of line: $y - y_1 = m(x - x_1)$

$y - 5 = \frac{2}{3}(x - 3)$

$3y - 15 = 2x - 6$ or $2x - 3y + 9 = 0$...(1)

Now, Verify the point C(9,9) lies on (1) or not,

$2(9) - 3(9) + 9 = 18 - 27 + 9 = 0$

Hence, A, B, and C lie on the same line.

Q9. The n^{th} term of an arithmetic sequence is $4n + 1$.

(a) Write the common difference of the sequence.

(b) Write the first term of the sequence.

(c) What is the remainder obtained when the terms of this sequence is divided by 4?

Solution:

(a) nth term = 4n + 1

1st term = 4 × 1 + 1 = 5

2nd term = 4 × 2 + 1 = 9

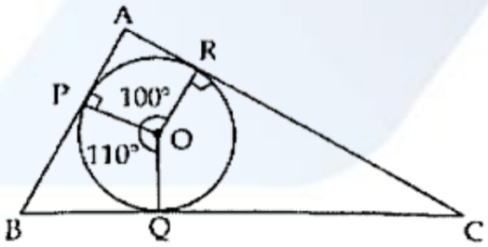
Common difference = 9 – 5 = 4.

(b) First term = 5

(c) Terms of the sequence are 5, 9, 13,.....

When each term is divided by 4, we get the remainder 1.

Q10. AB, BC and CA are tangents to the circle centred at O, touching the circle at P, Q and R respectively, as shown in the figure.



(a) Find $\angle QOR$.

(b) Find the angles of triangle ABC.

Solution:

(a)∠QOR = 360° – (110° + 100°) [Complete angle]

= 360° – 210°

= 150°

Property: Sum of opposite angles in the quadrilateral formed by two tangents = 180°

(b) $\angle POQ + \angle B = 180°$

$\Rightarrow 110° + \angle B = 180°$

$\Rightarrow \angle B = 180° - 110° = 70°$

$\angle POR + \angle A = 180°$

$\Rightarrow 100° + \angle A = 180°$

$\Rightarrow \angle A = 180° - 100° = 80°$

$\angle ROQ + \angle C = 180°$

$\Rightarrow 150° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 150° = 30°$

Answer any 8 questions from 11 to 21 . Each question carries 4 scores.

Q11. Numbers from 1 to 50 are written on slips of paper and put in a box. Without looking, a slip is to be drawn from it.

(a) What is the probability that it is a multiple of 4 ?

(b) What is the probability that it is a multiple of 6 ?

(c) What is the probability that it is a multiple of 4 and 6 ?

Solution:

(a) Multiple of 4 from 1 to 50 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

$\therefore$ Probability that it is a multiple of 4 = $\frac{12}{50} = \frac{6}{25}$

(b) Multiple of 6 from 1 to 50 are 6, 12, 18, 24, 30, 36, 42, 48

$\therefore$ Probability that it is a multiple of 6 = $\frac{8}{50} = \frac{4}{25}$

(c) Multiple of 4 and 6 from 1 to 50 are 12, 24, 36, 48

$\therefore$ Probability that it is a multiple of 4 and 6 = $\frac{4}{50} = \frac{2}{25}$

Q12. Draw a circle of radius 2.5 centimetres and mark a point 6 centimetres away from the centre of the circle.

(a) How many tangents can be drawn to the circle from this point?

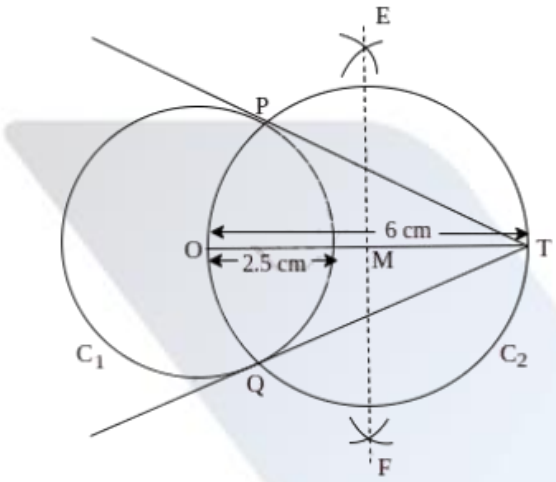
(b) Draw the tangents to the circle from this point.

Solution:

Step of Construction:

1. Draw a line segment OT = 6 cm.
2. Draw a circle of radius 2.5 cm taking O as centre.
3. Draw a perpendicular bisector EF of OT which meets OT at M.
4. Taking MT as radius and M as centre, draw a circle C_2 which intersects C_1 at Pand Q .
5. Join TP and TQ.

6. Hence, TP and TQ are the required tangents.



(a)Two(2) Tangents can be drawn to the circle from the point T(OT=6cm)
(b)TP and TQ are the two tangents from point T

Q13. Consider the arithmetic sequence 8,14,20, …
(a) Is 25 a term of this sequence?
(b) Check if 144 is a term of this sequence ?
(c) Prove that there are no perfect squares in this sequence.

Solution:

The common difference (d) is $14 - 8 = 6$.
The nth term (a_n) is given by $a_n = a_1 + (n-1)d$, where $a_1 = 8$ and $d = 6$.
Therefore, $a_n = 8 + (n-1)6 = 6n + 2$.
(a)Check if 25 is a term in the sequence
Set $6n + 2 = 25$.
Solving for n gives $n = \frac{23}{6}$, which is not an integer.
Therefore, 25 is not a term.
(b)Check if 144 is a term in the sequence
Set $6n + 2 = 144$. Solving for n gives $n = \frac{142}{6}$, which is not an integer.
Therefore, 144 is not a term.
(c)If a term is a perfect square, then $6n + 2 = k^2$, where k is an integer.
This simplifies to $6n = k^2 - 2$.
For n to be an integer, $k^2 - 2$ must be divisible by 6 .
However, k^2 can only leave remainders of 0,1,3, or 4 when divided by 6 .
Therefore, $k^2 - 2$ can only leave remainders of $-2, -1$, or 2 when divided by 6 , none of which are 0 .
Thus, no terms are perfect squares.

Q14. $A(2,3)$, $B(8,5)$ and $C(4,7)$ are the coordinates of the vertices of triangle ABC. P is the midpoint of AB and Q is the midpoint of BC.

(a) Find the coordinates of P and Q .

(b) Find the distance between P and Q .

Solution:

(a) P is the midpoint of AB. So,

$\mathrm{P} = \left(\frac{2+8}{2}, \frac{3+5}{2}\right) = (5,4)$

Q is the midpoint of BC . So,

$P = \left(\frac{8+4}{2}, \frac{5+7}{2}\right) = (6,6)$

(b) Distance between P and Q $= \sqrt{(6-5)^2 + (6-4)^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$ units.

Q15. From a circle of radius 15 centimetres, a sector of central angle 120° is cut out and rolled up to make a cone.

(a)What is the slant height of the cone?

(b)What is the base radius of the cone ?

(c)Calculate the curved surface area of the cone.

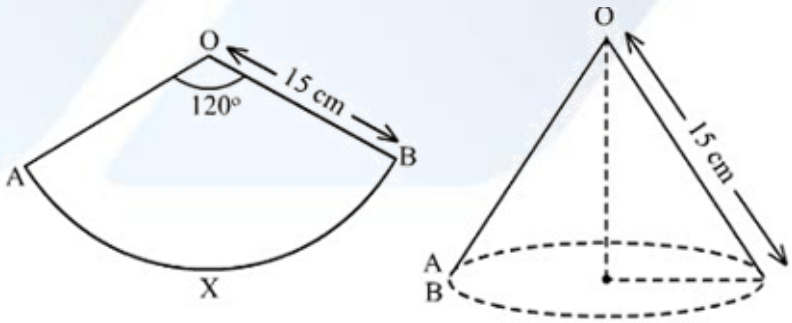
Solution:

Given that,

Radius of circle $R = 15$ cm

The angle of sector $= 120°$

Let slant height and radius of the cone be l, and r respectively.



(a) Slant height (l) = Radius of circle = 15 cm

(b)The curved surface area of the cone = Area of the sector of a circle.

$\pi rl = \pi R^2 \left(\frac{120^\circ}{360^\circ}\right)$

But slant height $l = R$

$\Rightarrow r = \frac{1}{3} \times 15$ cm

$\Rightarrow r = 5$ cm

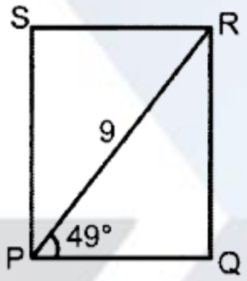
(c) Curved surface area of the cone = πrl

$= \pi \times 5 \times 15$

$= 75\pi$ cm^2

Q16. The diagonal of a rectangle is 9 centimetres and it makes an angle 49° with one side. Find the length of the sides of the rectangle.

$(\sin 49^\circ = 0.75, \cos 49^\circ = 0.66)$



Solution:

The diagonal of the rectangle is 9 centimetres

The angle between the diagonal and one side of the rectangle is 49 degrees

$\sin 49^\circ = 0.75$

$\cos 49^\circ = 0.66$

Let the length of the sides be x and y.

Using the trigonometric identities:

$\sin 49^\circ = \frac{x}{9}$

$\cos 49^\circ = \frac{y}{9}$

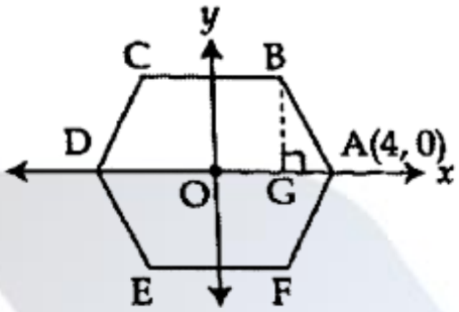
Solving for x and y :

$x = 9 \times \sin 49^\circ = 9 \times 0.75 = 6.75$ cm

$y = 9 \times \cos 49^\circ = 9 \times 0.66 = 5.94$ cm

The length of the sides of the rectangle are approximately 6.75 cm and 5.94 cm .

Q17. ABCDEF is a regular hexagon with origin as centre. The coordinates of the point A is (4,0).



(a) What are the coordinates of the point *D* ?

(b) Find the length of BG.

(c) Write the coordinates of the points B and E.

Solution:

(a) Coordinate of point D = (-4, 0)

(b) Since ABCDEF is a regular hexagon, each angle is 120°.

∴ ∠OAB = 60°

Also, ∠AOB = 60°

Consider ΔBOG,

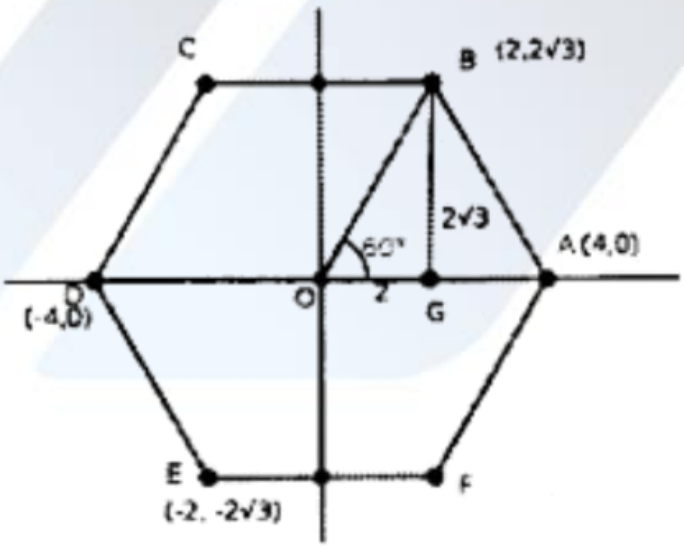
∠GOB = 60°

∠BGO = 90°

∴ ∠GBO = 30°

So, the sides are in the ratio $1 : \sqrt{3} : 2$

∴ BG = $2\sqrt{3}$ units



(c) B(2, $2\sqrt{3}$), E(-2, $-2\sqrt{3}$)

Q18. The square of a number is equal to 12 added to the number. Find the number.

Solution:

Let x be the number.

Given, $x^2 = 12 + x$

$\Rightarrow x^2 - x - 12 = 0$

$\Rightarrow (x - 4)(x + 3) = 0$

$\Rightarrow x = 4, -3$

Q19. Consider the polynomial $p(x) = x^2 - 5x + 6$.

(a) Write $p(x)$ as the product of two first degree polynomials.

(b) Find the solutions of the equation $p(x) = 0$.

Solution:

(a)Let $p(x) = x^2 - 5x + 6 = (x - a)(x - b)$

$x^2 - 5x + 6 = x^2 - (a + b)x + ab$

$\Rightarrow a + b = 5$ and $ab = 6$

$\Rightarrow a = 2, b = 3$

$\therefore x^2 - 5x + 6 = (x - 2)(x - 3)$

(b) $p(x) = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x = 2,3$

Q20. Diameters of two hemispheres are in the ratio 5:3.

(a) Write the ratio of their radii.

(b) Find the ratio of their surface areas.

(c) If the surface area of the first hemisphere is 100 square centimetres, what is the surface area of the other?

Solution:

(a) Ratio of their radii $= 5: 3$

(b) Surface area of hemisphere $= 2\pi r^2$

$2\pi r_1^2 : 2\pi r_2^2$

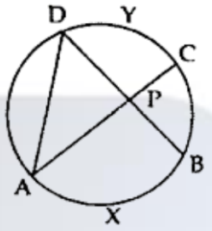
$= 2\pi \times 5^2 : 2\pi \times 3^2$

$= 25: 9$

(c) $25: 9 = 100: x$

$\Rightarrow x = \dfrac{9 \times 100}{25} = 36$sq. cm

Q21. The central angle of the are AXB is 110° and the central angle of the are CYD is 80°. Find the angles of triangle APD.



Solution:

Since the central angle of AXB is 110°, $\angle ADB = 55°$

Since the central angle of CYD is 80°, $\angle DAC = 40°$

$\therefore \angle \text{APD} = 180° - (55° + 40°)$

$= 180° - 95° = 85°$

Answer any 6 questions from 22 to 29 . Each question carries 5 scores.

Q22. Draw a triangle of sides 5 centimetres, 6 centimetres and 7 centimetres. Draw the incircle of the triangle. Measure the radius of the incircle.

Solution:

Steps of Construction:

1.Construct $\triangle$ ABC having sides BC = 7 cm, AC = 5 cm and AB = 6 cm.

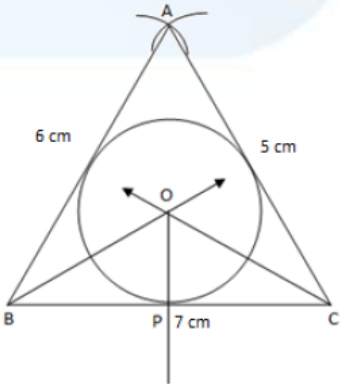
2.Draw angle bisector of any two angles say $\angle B$ and $\angle C$ of $\triangle ABC$ and let these intersect at a point say O.

3.Draw perpendicular from O on any side of triangle, say OP on BC.

4.Taking O as centre and OP as radius, draw a circle.

5.The circle touches the other two sides of triangle. This will be the required in circle of the triangle.

Radius, OP of incircle is 1.6 cm.



Q23. The ages of the workers of an organization are arranged as follows :

Age	Number of workers
$20-30$	9
$30-40$	10
$40-50$	8
$50-60$	5
$60-70$	1

(a) If the workers are arranged in order of their wages, the age of which worker is taken as the median age?

(b) Find the median age.

Solution:

Age	Number	Age	Number
$20-30$	9	Below 30	9
$30-40$	10	Below 40	19
$40-50$	8	Below 50	27
$50-60$	5	Below 60	32
$60-70$	1	Below 70	33
Total	33		

(a) $\frac{33+1}{2} = 17$

So, the age of the 17th worker is taken as the median age.

(b) $d = \frac{40-30}{10} = 1$

Age of 10th worker $= \frac{30+31}{2} = 30.5$

Age of 17th worker $= 30.5 + 7 \times 1 = 37.5$

So, the median age $= 37.5$

Q24. From a point on the ground at a distance of 100 metres away from a tower, the top of the tower is seen at an angle of elevation 45°. From the top of the tower, a car is seen on the opposite side of the tower at an angle of depression 25°.

(a) Draw a rough figure showing the details given in the question.

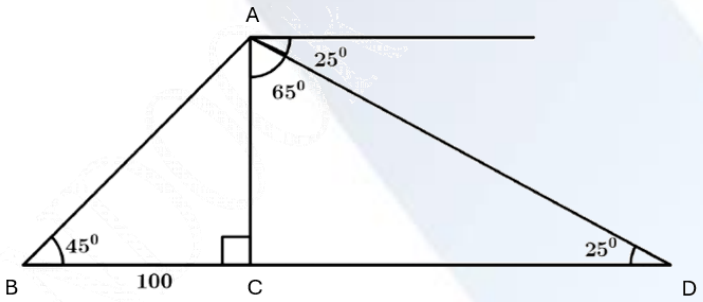
(b) Find the height of the tower.

(c) What is the distance of the car from the tower?

(sin 65° = 0.91, cos 65° = 0.42, tan 65° = 2.14)

Solution:

(a)



(b) In ΔABC, angles are 45°, 45°, 90°

So sides are in the ratio $1 : 1 : \sqrt{2}$

So, the height of the tower AC = 100 m

(c) $\tan 65° = \frac{CD}{100}$

CD = 100 × tan 65°

= 100 × 2.14

= 214 m

Q25. The third term of an arithmetic sequence is 26 and its eighth term is 61.

(a) Find the common difference of the sequence.

(b) What is its first term?

(c) Write the algebraic form of the sequence.

(d) Find the sum of the first 15 terms of the sequence.

Solution:

(a) 3rd term = 26

8th term = 61

$$\text{Common difference (d)} = \frac{\text{Term difference}}{\text{Position difference}}$$

$$= \frac{61 - 26}{8 - 3}$$

$= 7$

(b) First term(a) $=$ 3rd term $-2d$

$= 26 - 2 \times 7$

$= 26 - 14$

$= 12$

(c) Algebraic form $= a + (n - 1)d$

$= 12 + (n - 1)7$

$= 7n + 5$

(d) Sum of first 15 terms $= \frac{n}{2}[2a + (n - 1)d]$

$= \frac{15}{2}[2 \times 12 + (15 - 1)7]$

$= \frac{15}{2}[24 + 98]$

$= 915$

Q26. A vessel (without lid) in the shape of a square pyramid, made from a metallic sheet with a base perimeter of 80 centimeters and a slant height of 26 centimeters.

(a) How many square centimeters of the metallic sheet was needed to make the vessel?

(b) Calculate the height of the vessel.

(c) What is the capacity of the vessel in liters?

Solution:

(a) Base perimeter $= 80$ cm

$\Rightarrow 4a = 80$

$\Rightarrow a = 20$ cm

Slant height, $l = 26$ cm

$\therefore$ Lateral surface area $= 2al$

$= 2 \times 20 \times 26$

$= 1040\ \text{cm}^2$

(b) Height h $= \sqrt{l^2 - \left(\frac{a}{2}\right)^2}$

$= \sqrt{26^2 - 10^2}$

$= \sqrt{676 - 100}$

$= \sqrt{576}$

$= 24$ cm

(c) Volume of the vessel $= \frac{1}{3}a^2$ h

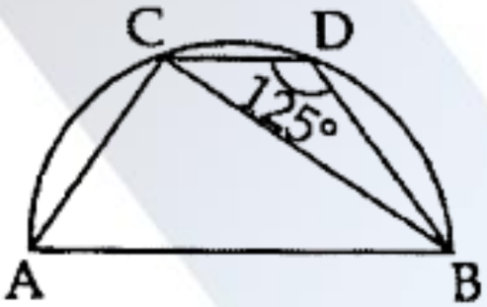
$= \frac{1}{3} \times 20 \times 20 \times 24$

$= 3200\ \text{cm}^3$

$= \frac{3200}{1000}\ Litres$

$= 3.2$ Litres

Q27. C and D are points on a semicircle with AB as diameter. ∠BDC = 125°. The CD is parallel to AB.



Find the measures of:

(a) ∠BAC

(b) ∠ACB

(c) ∠ACD

(d) ∠ABD

Solution:

(a) ∠BAC = 180° – 125° = 55°(Sum of opposite angles in cyclic quadrilateral)

(b) ∠ACB = 90°(Angle in Semi-Circle)

(c) In ΔACB, ∠BAC = 55°

∠ACB = 90°

∴ ∠ABC = 180° – (55° + 90°) = 35°

AB parallel to CD.

∴ ∠BCD = 35°

∴ ∠ACD = ∠ACB + ∠BCD

= 90° + 35°

= 125°

(d) In ΔBCD,

∠BCD = 35°

∠CDB = 125°

∴ ∠CBD = 180° – (35° + 125°) = 20°

∴ ∠ABD = ∠ABC + ∠CBD

= 35° + 20°

= 55°

Q28. The equation of a line is $2x - y - 2 = 0$.

(a) Check whethor the point (3,4) is on this line.

(b) Find the coordinates of the points where this line cuts the x and y axes.

Solution:

(a) Equation of the line is 2x – y – 2 = 0 .....(1)

Consider the point (3, 4).

Substitute x = 3, y = 4 in (1),

2 × 3 – 4 – 2 = 6 – 4 – 2 = 0

So, (3, 4) is a point on the line 2x – y – 2 = 0.

(b) If the line cuts the x-axis its y-coordinate is 0.

2x – y – 2 = 0

⇒ 2x – 0 – 2 = 0

⇒ 2x = 2

⇒ x = 1

So, the coordinates of the point where the line cuts the x-axis is (1, 0).

If the line cuts the y-axis its x-coordinate is 0.

∴ 2x – y – 2 = 0

⇒ 2 × 0 – y – 2 = 0

⇒ -y = 2

⇒ y = -2

So, the coordinates of the point where the line cuts the y-axis is (0, -2).

Q29. Consider the sequence : 2,6,18,54, ...

First term = 2

Second term = $2 \times 3 = 6$

Third term = $6 \times 3 = 18$

Fourth term = $18 \times 3 = 54$ and so on.

Sequences starting with a non-zero number, and each succeeding term got by multiplying the preceding term by a fixed number except zero, are called geometric sequences. The fixed number multiplied is the common ratio of the sequence. Thus, in the geometric sequence 2,6,18,54, ... the first term is 2 and the common ratio is 3 .

(a) The first term of a geometric sequence is 3 and common ratio is 2 . Find its second and third terms.

(b) Which of the following is a geometric sequence ?

(i) 2,4,6,8, ...

(ii) 2,4,8,16, ...

(iii) 1,4,9,16, ...

(c) What is the common ratio of the geometric sequence 5,20,80,320, …

(d) Write the next term of the geometric sequence 3,9,27, …

Solution:

(a) Second term $= 3 \times 2 = 6$

Third term $= 6 \times 2 = 12$

(b) (ii) 2,4,8,16, ….. is a geometric sequence with a common ratio of 2 .

(c) Common ratio $= \frac{20}{5} = 4.$

(d) Next term $= 27 \times 3 = 81.$