Secondary Examination, 2015

Mathematics

Time: $3\frac{1}{4}$ hours **M.M: 70**

GENERAL INSTRUCTIONS TO THE EXAMINEES:

1. There are in all seven questions in this question paper.
2. All the questions are compulsory.
3. In the beginning of each, the number of parts to be attempted has been clearly mentioned.
4. Marks allotted to the questions are indicated against them.
5. Start with the first question and proceed to the last one.
6. Do not waste your time over a question you cannot solve.

Q1. (a) The simplest form of $\frac{50x^2-98y^2}{10x-14y}$ will be:

(i) $5x-7y$

(ii) $5x+7y$

(iii) $2(5x+7y)$

(iv) $5(x-y)$

Solution:

We have the expression $\frac{50x^2-98y^2}{10x-14y}$.

Taking 2 as common from numerator and denominator, we get:

$$\frac{2(25x^2-49y^2)}{2(5x-7y)}$$

$$=\frac{(25x^2-49y^2)}{(5x-7y)}$$

We know that: $(a)^2-(b)^2=(a+b)(a-b)$

$$\frac{(5x+7y)(5x-7y)}{(5x-7y)}=(5x+7y)$$

Hence, the required solution is $(5x+7y)$.

(b) The value of $\frac{\sin 20^\circ}{\cos 70^\circ}$ will be:

Solution:

Given: $\frac{\sin 20^\circ}{\cos 70^\circ}$

We know that: $\sin(90-\theta)=\cos\theta$

Therefore,

$$\frac{\sin 20^\circ}{\cos 70^\circ} = \frac{\sin(90^\circ - 70^\circ)}{\cos 70^\circ}$$

$$= \frac{\cos 70^\circ}{\cos 70^\circ} = 1$$

Hence, the solution is 1.

(c) The arithmetic mean of positive odd numbers from 1 to 10 will be:

(i)2

(ii)3

(iii)4

(iv)5

Solution:

We know that 1 to 10 positive odd number is 1,3,5,7.

$$\text{Mean} = \frac{\text{Total number of sum}}{\text{Number of term}}$$

$$= \frac{1+3+5+7}{4}$$

$$= \frac{16}{4} = 4$$

Hence, the required value is 4.

(d) Under Section 80G, exemption is allowed for:

(i) National Saving Certificates

(ii) National Security Fund

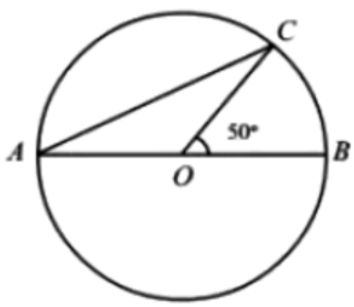
(iii) General Provident Fund

(iv) Life Insurance Premium

Solution:

Under Section 80G of the Income Tax Act, exemption is allowed for donations made to specified funds and charitable organizations. Among the given options, the **National Security Fund** qualifies for exemption under Section 80G.

(e) In the figure, AB is the diameter of the circle and O is the centre of the circle. If ∠COB = 50°, then what is the value of ∠CAB?



(i) 20°

(ii) 25^0

(iii) 40^0

(iv) 60°

Solution:

Given: $\angle COB = 50^\circ$

Then $\angle AOC = 180^\circ - 50^\circ = 130^\circ$

In ΔAOC,

$\angle AOC + \angle CAO + \angle ACO = 180^\circ$

$130^\circ + \angle CAO + \angle CAO = 180^\circ$ [Since OA = OC]

$2\angle CAO = 50^\circ$

$\angle CAO = 25^\circ$

Therefore, $\angle CAB = 25^\circ$

(f) If $x + \frac{1}{x} = 7$, then find the value of $x^2 + \frac{1}{x^2}$.

(i) 47

(ii) 49

(iii) 51

(iv) 50

Solution:

We have: $x + \frac{1}{x} = 7$

We know that: $(a+b)^2 = a^2 + b^2 + 2ab$

Then,

$$x + \frac{1}{x} = 7$$

$$\left(x + \frac{1}{x}\right)^2 = 7^2$$

$$x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = 49$$

$$x^2 + \frac{1}{x^2} + 2 = 49$$

$$x^2 + \frac{1}{x^2} = 49 - 2$$

$$x^2 + \frac{1}{x^2} = 47$$

Hence, the required value of $x^2 + \frac{1}{x^2}$ is 47.

Q2. All questions are compulsory.

(a) If one root of the quadratic equation $x^2 + kx + 3 = 0$ is 1, find the value of k.

Solution:

We have quadratic equation $x^2 + kx + 3 = 0$.

Putting $x = 1$ in given equation, we obtain:

$(1)^2 + k(1) + 3 = 0$

$k + 4 = 0$

$k = -4$

Hence, the value of k is -4.

(b) If the $\sin\ \theta = \frac{3}{5}$, find the value of $\tan\ \theta$.

Solution:

Given: $\sin\theta = \frac{3}{5} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$

By Pythagoras theorem,

$\text{Base}^2 + \text{Perpendicular}^2 = \text{Hypotenuse}^2$

$\text{Hypotenuse}^2 - \text{Perpendicular}^2 = \text{Base}^2$

$5^2 - 3^2 = x^2$
$25 - 9 = x^2$
$x^2 = 16$
$x = 4$

Therefore, $\tan\ \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

Hence, the value of $\tan\ \theta = \frac{3}{4}$.

(c) Devesh bought a digital camera for ₹6,000 and sold it at a 7% profit. Find the selling price of the digital camera.

Solution:

The cost of camera is 6,000.

We know that $x\% = \frac{x}{100}$ then, similarly $7\% = \frac{7}{100}$.

Therefore, profit $= 6{,}000 \times \frac{7}{100} = \text{Rs.}\ 420$

So, selling price $= 6{,}000 + 420 = 6420$

Hence, the selling prince of digital camera is Rs. 6420.

(d) If the arithmetic mean of the numbers $27, 23, x - 4, x + 4, 15, 3$ and 7 is 15, find the value of x.

Solution:

We know that:

$$\text{Mean} = \frac{\text{Total number of sum}}{\text{Number of term}}$$

$$15 = \frac{27 + 23 + x - 4 + x + 4 + 15 + 3 + 7}{7}$$

$105 = 75 + 2x$

$2x = 105 - 75$

$2x = 30$

$x = 15$

Hence, the value of mean is 15.

Q3. All questions are compulsory.

(a) Find the lengths of the intercepts on the axes made by the line $3x + 4y = 12$.

Solution:

Given: $3x + 4y = 12$

Putting $x = 0$, to solve the x-intercept.

$3(0) + 4y = 12$

$4y = 12$

$y = 3$

Putting $y = 0$, to solve y -intercept.

$3x + 4y = 12.$

$3x + 4(0) = 12$

$3x = 12$

$x = 4$

Hence, the intercept points are $(0, 3)$ and $(4, 0)$.

(b) Prove that the line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Solution:

Let the chord AB of the circle have a midpoint M.

Draw the radius OA, OB, and OM, where O is the centre of the circle.

In triangles △OMA and △OMB:

OA = OB (radii of the circle).

AM = MB (since M is the midpoint).

OM = OM (common side).

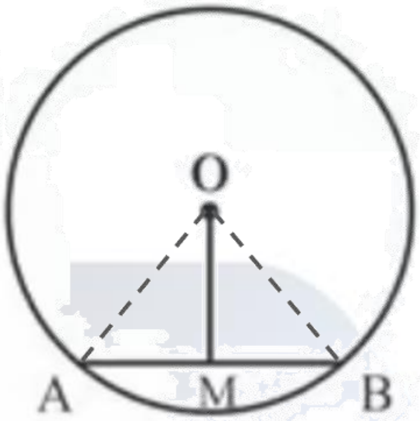
By the **SSS congruence criterion**, △OMA≅△OMB.

This implies ∠OMA = ∠OMB.

Since these two angles form a linear pair, ∠OMA + ∠OMB = 180°.

Thus, ∠OMA = ∠OMB = 90°.

Hence, OM is perpendicular to the chord AB.



(c) Find the value of $\tan\left(\frac{13\pi}{3}\right)$.

Solution:

Given: $\tan\left(\frac{13\pi}{3}\right)$

Since 2π is one full revolution, subtract 2π from $\frac{13\pi}{3}$, and simplify further.

$$\frac{13\pi}{3} - 2\pi = \frac{7\pi}{3}$$

Again,

$$\frac{7\pi}{3} - 2\pi = \frac{\pi}{3}$$

Therefore, $\tan\frac{\pi}{3} = \tan 60^\circ = \sqrt{3}$

Hence, the value of $\tan\left(\frac{13\pi}{3}\right)$ will be $\sqrt{3}$.

(d) The ratio of the heights of two cylinders with the same base radius is 3:2. Find the ratio of their curved surface areas.

Solution:

We know that the curved surface area (SA) of cylinder is $2\pi rh$.

$$\frac{SA_1}{SA_2} = \frac{2\pi r h_1}{2\pi r h_2} = \frac{2\pi r \times 3}{2\pi r \times 2} = \frac{3}{2}$$

Hence, the ratio of their curved surface areas is 3:2.

Q4. All questions are compulsory.

(a) Find the slope of a line perpendicular to the line $3x - 4y + 8 = 0$.

Solution:

$3x - 4y + 8 = 0$

$4y = 3x + 8$

$$y = \frac{3x+8}{4}$$

$$y = \frac{3}{4}x + 2$$

We know that when two lines are perpendicular, the product of their slopes is -1.

Therefore,

$$\frac{3}{4} \times m = -1$$

$$m = \frac{-4}{3}$$

Hence, the required slop will be $\frac{-4}{3}$.

(b) Evaluate the value of $\frac{2\tan\ 15^\circ}{1+\tan^2\ 15^\circ}$.

Solution

Given: $\frac{2\tan\ 15^\circ}{1+\tan^2\ 15^\circ}$

We know that $\tan 2A = \frac{2\tan A}{1+\tan^2 A}$

Therefore, $\frac{2\tan\ 15^\circ}{1+\tan^2\ 15^\circ} = tan2 \times 15^o = tan30^o = \frac{1}{\sqrt{3}}$

(c) In the figure, O is the centre of circle and TPQ is the tangent. If $\angle RPQ = 50^\circ$, find the measure of $\angle PSR$.

Solution:

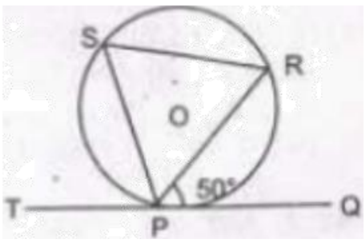
Given: ∠RPQ = 50°

We know that:

The angle between a tangent and the radius at the point of contact is 90∘90^\circ90∘.

So, ∠OPT = 90°.

∠OPR = 90° - 50° = 40°



Therefore,

$$\angle OPR + \angle POR + \angle ORP = 180^\circ$$

$40^\circ + 40^\circ + \angle POR = 180^\circ$

$\angle POR = 100^\circ$

$\angle PSR = \frac{1}{2}\angle POR$

$\angle PSR = 50^\circ$

(d) The height and diameter of the base of a cone are 12 cm and 18 cm, respectively. Find its slant height.

Solution:

Given: Height (h) = 12 cm

Diameter (d) = 18 cm; Radius (r) = 18/2 = 9 cm

Using the formula of slant height:

$L = \sqrt{r^2 + h^2}$

$$L = \sqrt{9^2 + 12^2}$$
$$L = \sqrt{81 + 144}$$
$$L = \sqrt{225}$$
$$L = 15$$

Hence, the slant height of the cone is 15 cm.

Q5. All questions are compulsory.

(a) Prove that: $\cos 4A = 1 - 8\sin^2 A + 8\sin^4 A$

Solution:

Given: $\cos 4A = 1 - 8\sin^2 A + 8\sin^4 A$

L.H.S:

$$\cos 4A = \cos 2(2A)$$
$$= \cos^2 2A - \sin^2 2A$$
$$= 1 - \sin^2 2A - \sin^2 2A$$
$$= 1 - 2\sin^2 2A$$
$$= 1 - 2(2\sin A\cos A)^2$$
$$= 1 - 2(4\sin^2 A\cos^2 A)$$
$$= 1 - 8\sin^2 A\cos^2 A$$
$$= 1 - 8\sin^2 A(1 - \sin^2 A)$$
$$= 1 - 8\sin^2 A + 8\sin^4 A = \text{R.H.S}$$

Hence proved.

(b) If the length of the perpendicular drawn from the origin to the line that intercepts the axes at a and b is p, prove that:

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

Solution:

Equation of line AB in intercept form, $\frac{x}{a}+\frac{y}{b}=1$.

Perpendicular (⊥) distance form (0,0) is p.

Therefore ⊥ distance = $\left|\frac{Ax_1+By_1+c}{\sqrt{A^2+B^2}}\right|$

$$p=\left|\frac{\frac{1}{a}\times 0+\frac{1}{b}\times 0}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

$$p=\left|\frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}\right|$$

$$\frac{1}{p}=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

Squaring both side, we get:

$$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

Hence proved.

(c) Find the median from the following frequency distribution.

Class interval	$0-12$	$10-20$	$20-30$	$30-40$	$40-50$
Frequency	6	9	12	8	15

Solution:

Class Interval (CI): 0-10, 10-20, 20-30, 30-40, 40-50

Frequency (f): 6, 9, 12, 8, 15

Cumulative Frequency (CF): 6, 15, 27, 35, 50

Total Frequency (N): 50

Median Class: N/2 = 25, so the median class is 20-30.

Formula: Median= $L+\left(\frac{\frac{N}{2}-CF_{previous}}{f_{median\ class}}\right)\times h$

Where:

L = 20 (lower boundary of median class)

N = 50 (total frequency)

$CF_{previous}$ = 15 (cumulative frequency before median class)

$f_{median\ class} = 12$

h = 10 (class width)

Substitute values:

$\text{Median} = 20 + \left(\frac{25-15}{12}\right) \times 10$

$\text{Median} = 20 + \left(\frac{10}{12}\right) \times 10$

$\text{Median} = 20 + 8.33$

$\text{Median} \approx 28.33$

(d) In the financial year 2013-14, X has an annual income of ₹6,20,000 (excluding house rent allowance). He deposits ₹8,000 per month into his General Provident Fund (GPF) account and ₹80,000 into his Public Provident Fund (PPF) account. Calculate the income tax payable by X, given that the maximum deduction limit for savings is ₹1,00,000. The income tax rates are as follows:

Apart from this 3% education levy is levied on the income tax payable.

	Taxable Income	Income Tax
(i)	UP to Rs. 2,00,000	0%
(ii)	Rs. 2,00,001 to Rs. 5,00,000	10 % of the income exceeding Rs. 2,00,000.
(iii)	Rs. 5,00,001 to Rs. 10,00,000	Rs. 30,000 + 20% of the income exceeding Rs. 5,00,000

Solution:

Yearly income of X $= Rs, 20{,}000$

Future saving income $=$ per monthly Rs $8{,}000 \times 12 =$ Rs. $96{,}000$

PPF saving account income $=$ Rs $80{,}000 + 96{,}000 + 1{,}00{,}000 =$ Rs $27{,}6000$.

Total saving income $=$ Rs $6{,}20{,}000 - 2{,}76000 =$ Rs $3{,}44000$

Calculate the income tax $= 3{,}44{,}000 \times 10\%$

$$= 3{,}44{,}000 \times \frac{10}{100} = 34{,}000$$

Total income tax $= 34{,}000$

$= 34,000 \times 3\%$

$= 34,000 \times \dfrac{3}{100}$

$= 1020$

Total pay income tax $= 34,000 + 1020 = 35,020$

Hence, the value of total pay income tax 35,020.

Q6. All questions are compulsory.

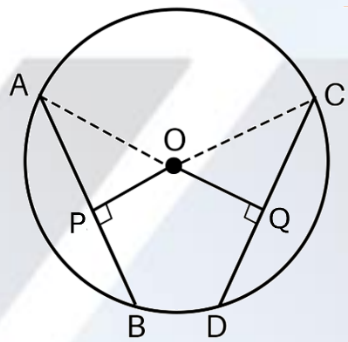
(a) Prove that the chords in a circle which are equidistant from the centre are equal in length.

Solution:

Let the centre of the circle be O and let two chords AB and CD be equidistant from the centre.

Let the perpendicular distances from O to the chords AB and CD be equal.

Let the perpendiculars from O to AB and CD meet the chords at points P and Q, respectively.



Since OP = OQ and both perpendiculars are radii of the circle, the triangles OPA and OQC are congruent by the Hypotenuse-Leg (HL) Theorem (both share the radius OA = OC, and both have equal perpendiculars OP = OQ).

Thus, by CPCT (Corresponding Parts of Congruent Triangles), we have AB = CD.

Hence, the chords AB and CD are equal.

(b) If $Ax + By = C$ and $x\cos\alpha + y\sin\beta = p$ represent the same line, find the value of p.

Solution:

We know that: $y = mx + c$

$Ax + By = C$

$y = \frac{-A}{B}x + \frac{C}{B}$ ....(1)

Also,

$x\cos\alpha + y\sin\beta = p$

$y=\frac{-\cos\alpha}{\sin\beta}x+\frac{P}{\sin\beta}$ ....(2)

From (1) and (2), comparing the constant term, we get:

$$\frac{C}{B}=\frac{\mathrm{p}}{\sin\beta}$$

Hence, the value is $p=\frac{C\sin\beta}{B}$.

(c) The slant height of a conical frustum is 13 cm, and the total surface area is 90π sq. cm. Find the radius of its base.

Solution:

The formula for the total surface area of a cone is: $A=\pi r(l+r)$

$90\pi=\pi r(13+r)$

$90=r(13+r)$

$90=13r+r^2$

$r^2+13-90=0$

Using the formula of quadratic equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r=\frac{-13\pm\sqrt{13^2-4\times1\times(-90)}}{2\times1}$$

$$r=\frac{-13\pm\sqrt{169+360}}{2}$$

$$r=\frac{-13\pm\sqrt{529}}{2}$$

$$r=\frac{-13\pm23}{2}$$

$r=\frac{-13+23}{2}$ or $r=\frac{-13-23}{2}$

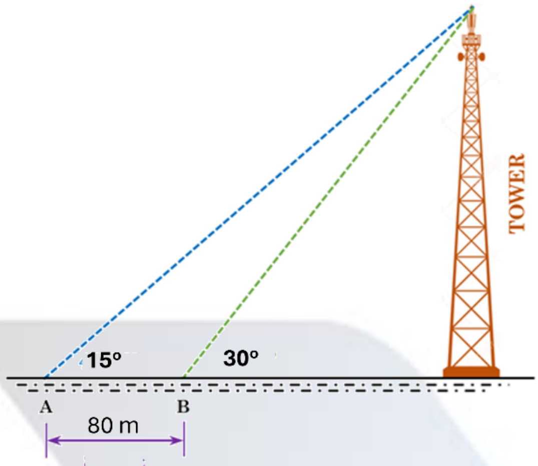
$r=5$ or $r=-18$ (radius cannot be negative)

Thus, the radius of the base is **5 cm**.

(d) From a point O on the horizontal plane, the angle of elevation to the top of a vertical tower is 15°. After moving 80 m towards the tower, the angle of elevation becomes 30°. Find the height of the tower.

Solution:

Let the height of the tower is ***h***.



$\tan\ 30^{\circ} = \frac{h}{x}$

$\tan\ 15^{\circ} = \frac{h}{80 + x}$

We know that: $\tan 15^{0} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = (2 - \sqrt{3})$

$(2 - \sqrt{3}) = \frac{h}{80 + x}$

$(2 - \sqrt{3})(80 + x) = h$

$160 + 2x - 80\sqrt{3} - \sqrt{3}x = h$

Substituting the value of h.

$160 + 2x - 80\sqrt{3} - \sqrt{3}x = \frac{x}{\sqrt{3}}$

$160\sqrt{3} + 2\sqrt{3}x - 240 - 30x = x$

$160\sqrt{3} + 2\sqrt{3}x - 240 = 4x$

$4x - 2\sqrt{3}x = 160\sqrt{3} - 240$

$x = \frac{160\sqrt{3} - 240}{4x - 2\sqrt{3}x} = \frac{80\sqrt{3} - 120}{2 - \sqrt{3}}$

Hence, the height of tower is $\frac{80\sqrt{3}-120}{2-\sqrt{3}}$ m.

Q7. All questions are compulsory.

(a) Solve the equation: $\left(\frac{x-1}{x+1}\right)^{4} - 13\left(\frac{x-1}{x+1}\right)^{2} + 36 = 0$

Solution:

$$\left(\frac{x-1}{x+1}\right)^{4} - 13\left(\frac{x-1}{x+1}\right)^{2} + 36 = 0$$

Let, $\left(\frac{x-1}{x+1}\right)^2 = x$

We get this equation $x^2 - 13x + 36 = 0$.

Factorize $x^2 - 13x + 36 = 0$.

$x^2 - 13x + 36 = 0$
$x^2 - 9x - 4x + 36 = 0$
$x(x - 9) - 4(x - 9) = 0$
$(x - 9)(x - 4) = 0$
$x - 9 = 0$
$x = 9$
$x - 4 = 0$
$x = 4$

Hence, the solution is $x = 4$ and $x = 9$.

OR

The sum of the digits of a two-digit number is 12. When the digits are reversed, the new number is 18 more than the original number. Find the number.

Solution

Let the two-digit number be $10x + y$, where x and y are the digits.

$x + y = 12$ .... (1)

$10x + y = 10y + x + 18$

$9x - 9y = 18$

So, $9(x - y) = 2$

$x - y = 2$ ....(2)

Simplify the equation (1) and equation (2). We get:

Adding (1) and (2), we get:

$2x = 14$

$x = 7$

From (2), we get:

$7 - y = 2$

$y = 5$

Hence, the number is 75.

b) Construct a cyclic quadrilateral ABCD where AB = 4.0 cm, BC = 5.0 cm, AC = 6.0 cm, and CD = 4.0 cm.

Solution:

Draw the diagonal AC: Draw a line segment AC = 6.0 cm.

Construct triangle ABC:

With A as the centre, draw an arc of radius 4.0 cm (AB).

With C as the centre, draw an arc of radius 5.0 cm (BC).

Mark the intersection of the arcs as point B.

Join AB and BC.

Construct triangle ACD:

With A as the centre, draw an arc of radius 4.0 cm (AD).

With C as the centre, draw an arc of radius 4.0 cm (CD).

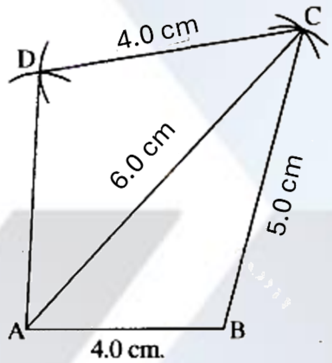
Mark the intersection of the arcs as point D.

Join AD and CD.

Complete the quadrilateral:

Connect points B and D to form the cyclic quadrilateral ABCD.

This completes the construction.

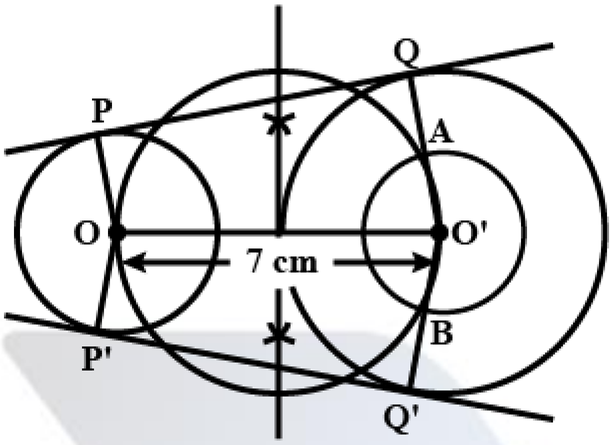


Or

Two circles have radii of 2.0 cm and 3.5 cm, respectively, and the distance between their centres is 7.0 cm. Draw the common tangents to these circles and find their lengths.

Solution:

Let the radii of the two circles be $r_1 = 2.0$ cm and $r_2 = 3.5$ cm, and the distance between their centres be $d = 7.0$ cm.



Use the formula for the length of the direct common tangent:

$L = \sqrt{d^2 - (r_1 - r_2)^2}$

$L = \sqrt{7^2 - (3.5 - 2.0)^2}$

$L = \sqrt{49 - (1.5)^2}$

$L = \sqrt{49 - 2.25}$

$L = \sqrt{46.75} = 6.83 \text{ cm}$

Thus, the length of the direct common tangent is approximately 6.83 cm.