Secondary Examination, 2016

Mathematics

Time: $3\frac{1}{4}$ hours **M.M: 70**

GENERAL INSTRUCTIONS TO THE EXAMINEES:

1. There are in all seven questions in this question paper.
2. All the questions are compulsory.
3. In the beginning of each, the number of parts to be attempted has been clearly mentioned.
4. Marks allotted to the questions are indicated against them.
5. Start with the first question and proceed to the last one.
6. Do not waste your time over a question you cannot solve.

Q1. a) If $x = 2 - \sqrt{3}$, find the value of $x^2 + \frac{1}{x^2}$.

Solution:

Simplify the expression,

$$x = 2 - \sqrt{3}$$

$$\frac{1}{x} = 2 + \sqrt{3}$$

Therefore,

$$x + \frac{1}{x} = 2 - \sqrt{3} + 2 + \sqrt{3}$$

$$x + \frac{1}{x} = 4$$

Squaring both side, we get:

$$\left(x + \frac{1}{x}\right)^2 = 4^2$$

$$x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = 16$$

$$x^2 + \frac{1}{x^2} + 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14$$

Hence, the required value is 14.

b) If $\sec\ \theta = 2$, find the value of θ.

Solution:

Simplify the expression,

We know that,

$\sec\ \theta = 2$

$\sec\ \theta = 60^\circ$

Therefore,

$\theta = 60^\circ$

$$\theta = \frac{60^\circ}{180^0} \cdot \pi$$

$$\theta = \frac{\pi}{3}$$

Hence, the value of θ is $\frac{\pi}{3}$.

c) If $a + b + c = 0$, find the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$.

Solution:

Simplify the expression,

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$$

We know that:

$a^3 + b^3 + c^3 - 3abc = 0$

$a^3 + b^3 + c^3 = 3abc$

So,

$$\frac{3abc}{abc} = 3$$

Hence, the required value is 3.

d) The arithmetic mean of natural numbers from 1 to 9 will be:

Solution:

1,2,3,4,5,6,7,8,9 are natural numbers,

$$A.M = \frac{1+2+3+4+5+6+7+8+9}{9}$$

$$= \frac{45}{9}$$

$$= 5$$

Hence, the value of 5.

e) What is the language of withdrawing bank fund from the account?

(i) Debit (ii) Credit (iii) Saving (iv) None of these.

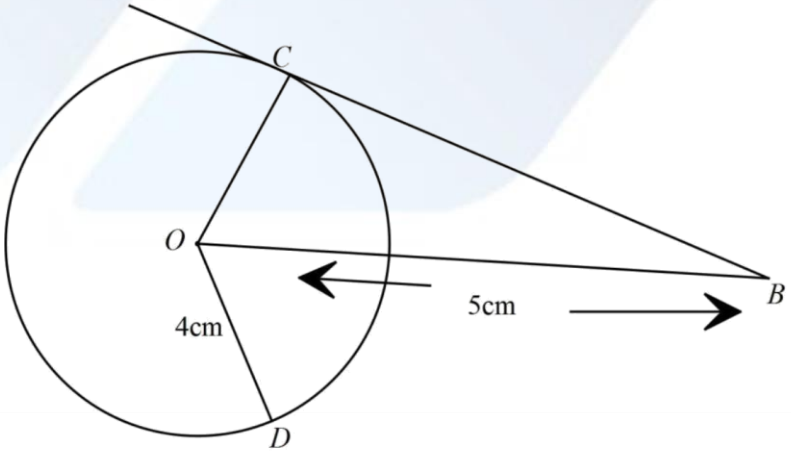
Solution:

Debit refers to the process of withdrawing funds from a bank account.

When money is withdrawn, the account is "debited," which reduces the account balance.

e) In the figure, O is the centre of the circle with radius OD = 4 cm. If OB = 5 cm, then the length of the tangent BC will be?

(i) 3 cm (ii) 4 cm (iii) 2 cm (iv) 3.5



Solution:

Simplify the expression,

$OD = OC = 4$ cm

Therefore,

$OB^2 = OC^2 + BC^2$

$5^2 = 4^2 + BC^2$

$BC^2 = 5^2 - 4^2$

$BC^2 = 25 - 16$

$BC^2 = 9$

$BC = 3$

Hence, the value of BC is 3 cm.

Q2. a) If $A = x + \frac{1}{x}$, find the value of $\frac{1}{A}$.

Solution:

Simplify the expression,

$$A = x + \frac{1}{x}$$

$$\text{A} = \frac{x^2 + 1}{x}$$

Then,

$$\frac{1}{A} = \frac{x}{x^2 + 1}$$

Hence, the value of $\frac{1}{A} = \frac{x}{x^2+1}$.

b) If $A + B = 45°$, So find the value of $\sin A \cos B + \cos A \sin B$.

Solution:

We know that:

$\sin A \cos B + \cos A \sin B$ this is the formula of $\sin(A + B)$.

Then, $\sin 45^0 = \frac{1}{\sqrt{2}}$

Hence, the required value is $\frac{1}{\sqrt{2}}$.

c) Find the mode of the given numbers 6,3,2,6,5,6 and 8.

Solution:

We know that:

Mode is the value or observation that occurs most frequently in a data set.

The number given has the highest frequency of 6.

Hence, the mode of the given number is 6.

d) Udit bought a printer for ₹3,500 and sold it at a profit of 12%. Find the selling price of the printer.

Solution:

We are given the cost price of printer is 35,00.

$$12\% = \frac{12}{100}$$

$$3500 \times \frac{12}{100} = 35 \times 12 = 420$$

When, $3500 + 420 = \text{Rs. } 3920$

We get the selling price of printer is Rs. 3920.

Hence, the selling price of the printer is Rs. 3920.

Q3. a) Prove that the line joining the centre of a circle to the midpoint of a chord is perpendicular to the chord.

Solution:

We know that AB is a chord of the circle and C is the midpoint of it.

So, in △ACO and △BCO,

AC = BC (as C is mid-point of AB)

OC = OC (common side)

OA = OB (radius of the circle)

∴ By SSS congruency condition △ACO≅△BCO

Now, as we know that Corresponding Parts of Congruent triangles are equal

∴ ∠OCA = ∠OCB

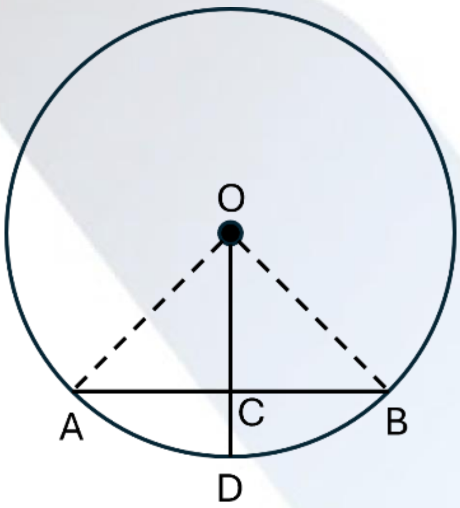
But ∠OCA + ∠OCB = 180° (∵ they make a linear pair)

∴2×∠OCA = 180°

⇒ ∠OCA = ∠OCB = 90°

⇒ OC ⊥ AB.

Hence proved.



b) Prove that: $\frac{\sin 36^0}{\cos 54^0}+\frac{\cos 36^0}{\sin 54^0}=2$

Solution:

Taking L.H.S

$$\frac{\sin 36^\circ}{\cos 54^0}+\frac{\cos 36^\circ}{\sin 54^0}$$

$$=\frac{\sin (90^\circ-54^\circ)}{\cos 54^0}+\frac{\cos (90^\circ-54^0)}{\sin 54^0}$$

We know that,

$\sin (90^\circ-\theta)=\cos \theta$

$\cos (90^\circ-\theta)=\sin \theta$

Then,

$$=\frac{\sin (90^\circ-54^\circ)}{\cos 54^\circ}+\frac{\cos (90^\circ-54^\circ)}{\sin 54^0}$$

$$=\frac{\cos 54^\circ}{\cos 54^\circ}+\frac{\sin 54^0}{\sin 54^0}$$

$= 1 + 1$

$= 2$

Hence, the proved it.

c) The radius and height of the base of a cylinder are 4 cm and 14 cm, respectively. Find the curved surface area of the cylinder.

Solution:

We know that the cylinder of curved surface area is $2\pi rh$.

$r = 4$ cm

$h = 14$ cm

$= 2\pi rh$

$= 2\pi \times 4 \times 14$

$= 112\pi$

Hence, the curved suface area of cylinder is 112π.

d) Find the equation of the line whose y-intercept is -4 and which makes an angle of $60°$ with the positive x-axis.

Solution:

y-axis on $= -4$

$\theta = 60°$

We know that,

$(y - y_1) = m(x - x_1)$

$y - axis = -4$

$x - axis = 0$

$(x_1, y_1) = (0, -4)$

$m = \tan\ 60^0$

$m = \sqrt{3}$

$y - (-4) = \sqrt{3}(x - 0)$

$y + 4 = \sqrt{3}x$

$\sqrt{3}x - y = 4$

Q4. a) If the height of a right circular cone is 3.0 cm and the radius of its base is 4.0 cm, find its volume.

Solution:

We know that the volume of cone is $v = \frac{1}{3}\pi r^2 h$.

$h = 3.0$ cm

$r = 4.0$ cm

$$= \frac{1}{3}\pi \times 4^2 \times 3$$

$= 64\pi$

Hence, the volume of cone is 64π cm^3.

b) If $\tan\ A = \frac{1}{\sqrt{3}}$, $\tan\ B = \sqrt{3}$. Then find the value of $A + B$.

Solution:

$$\tan\ A = \frac{1}{\sqrt{3}} = \tan\ 30^\circ$$

Therefore, $A = 30^o$

$\tan\ B = \sqrt{3} = \tan\ 60^\circ$

Therefore, $B = 60^o$

Hence, $A + B = 30^o + 60^o = 90^o$

c) If $x = \frac{a}{a+b}$ and $y = \frac{b}{a-b}$, then prove that $\frac{1}{x} + \frac{1}{y} = \frac{a^2+b^2}{ab}$.

Solution:

$$x = \frac{a}{a+b}$$

$$\frac{1}{x} = \frac{a+b}{a}$$

$$y = \frac{b}{a-b}$$

$$\frac{1}{y} = \frac{a-b}{b}$$

Simplify further,

$$\frac{1}{x}+\frac{1}{y}=\frac{a+b}{a}+\frac{a-b}{b}$$

$$\frac{1}{x}+\frac{1}{y}=\frac{b(a+b)+a(a-b)}{ab}$$

$$\frac{1}{x}+\frac{1}{y}=\frac{ab+b^2+a^2-ab}{ab}$$

$$\frac{1}{x}+\frac{1}{y}=\frac{a^2+b^2}{ab}$$

Hence, proved it.

d) Find the slope of the line passing through the points (-2, 5) and (6, 4).

Solution:

We know that:

$$M=\frac{y_2-y_1}{x_2-x_1}$$

The given points are $(-2,5),(6,4)$.

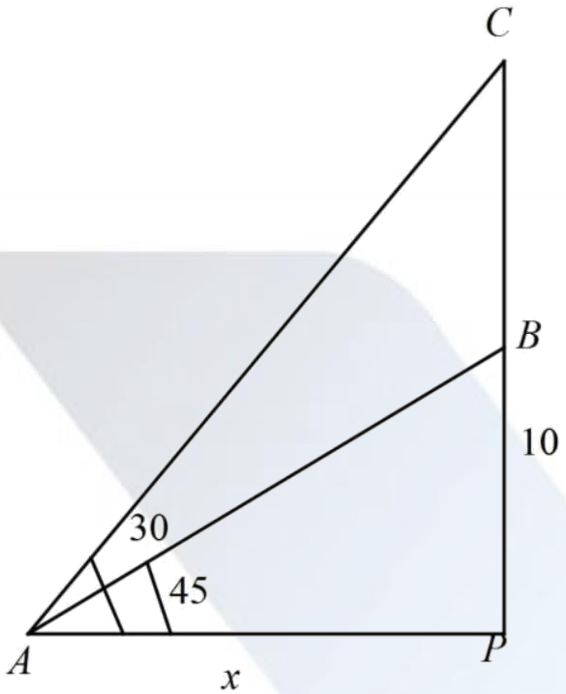
So, $\mathrm{M}=\frac{4-5}{6-(-2)}$

$$\mathrm{M}=\frac{-1}{8}$$

Hence, the value of gradient is $\frac{-1}{8}$.

Q5. a) An observer standing 10m away from a building notice that the angle of elevation of the top and bottom of a flagstaff on the building on the building are 60° and 45° respectively Then the height of flagstaff is.

Solution:



In $\triangle APB$,

$$\tan\ 30^\circ = \frac{10}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{10}{x}$$

$$x = 10\sqrt{3}$$

In $\triangle APC$,

$$\tan\ 45^\circ = \frac{10 + h}{10\sqrt{3}}$$

$$1 = \frac{10 + h}{10\sqrt{3}}$$

$$10\sqrt{3} = 10 + h$$

$$h = 10(\sqrt{3} - 1)$$

$$h = 10(1.732 - 1)$$

$$h = 10(.732)$$

$$h = 7.32 \text{ m}$$

Hence, the flag height is 7.32 cm.

b) Find the mean age from the following frequency distribution.

Class-interval	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Frequency	7	14	28	26	16	9

Solution:

Simplify the value,

Interval	Frequency (f_i)	Midpoint (x_i)	$f_i x_i$
$0-10$	7	5	35
$10-20$	14	15	210
$20-30$	28	25	700
$30-40$	26	35	910
$40-50$	16	45	720
$50-60$	9	55	495

$\sum f_i x_i = 3070$

$\sum f_i = 100$

$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{3070}{100} = 30.70$

Hence, the value of mean is 30.70.

c) If the perpendicular drawn from the origin to the line $ax + by + a + b = 0$ has a length p, prove that $p^2 = 1 + \frac{2ab}{a^2+b^2}$.

Solution:

$ax + by + a + b = 0$

Perpendicular, $(p) = \frac{c}{\sqrt{a^2+b^2}}$

$$p = \frac{a+b}{\sqrt{a^2+b^2}}$$

Squaring on both sides, we get:

$$p^2 = \left(\frac{a+b}{\sqrt{a^2+b^2}}\right)^2$$

$$p^2 = \frac{a^2+b^2+2ab}{a^2+b^2}$$

$$p^2 = \frac{a^2+b^2}{a^2+b^2} + \frac{2ab}{a^2+b^2}$$

$$p^2 = 1 + \frac{2ab}{a^2+b^2}$$

Hence proved.

d) In the financial year 2014-15, Kuldeep's annual income (excluding house rent allowance) is ₹5,80,000. He deposits ₹9,000 per month into his General Provident Fund (GPF) and ₹60,000 into his Public Provident Fund (PPF) account. Calculate the income tax paid by Kuldeep in the last month of the financial year. The maximum exemption limit on total savings is ₹1,50,000. The income tax rates are as follows:

Apart from this 3% education levy is levied on the income tax payable.

	Taxable Income	Income Tax
(i)	UP to Rs 2,00,000	0%
(ii)	Rs 2,00,001 to Rs 5,00,000	10% of the income exceeding Rs 2,00,000

Solution:

Yearly income $=$ Rs $5{,}580{,}000$.

Future saving income $=$ Per monthly Rs $9{,}000 \times 12$ = Rs 108,000

PPF saving account income $=$ Rs $60{,}000 + 108{,}000 + 1{,}50{,}000 =$ Rs $2{,}28{,}000$.

Total saving income $=$ Rs $5{,}80{,}000 - 2{,}28000 =$ Rs $3{,}52{,}000$

Calculate the income tax

$3{,}52{,}000 \times 10\%$

$= 3{,}52{,}000 \times \frac{10}{100} = 52{,}000$

Total income tax $= 52{,}000$

$= 52{,}000 \times 3\%$

$= 52{,}000 \times \frac{3}{100}$

$= 1560$

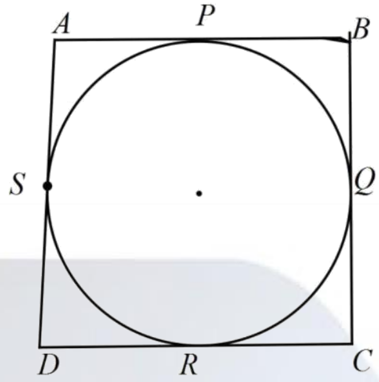
Total pay income tax $= 52{,}000 + 1560 = 53{,}560$

Hence, the value of total pay income tax is Rs 53,560.

Q6. a) A circle touches all four sides of a quadrilateral ABCD. Prove that $AB + CD = BC + DA$.

Solution:

A circle touches all sides of quadrilateral ABCD at P,Q,R,S, respectively.



Using the property of tangents from an external point:

AP=AS, BP = BQ, CR = CQ, DR = DS.

Lengths of sides:

AB = AP+BP = AS+BQ,

BC = BQ + CQ,

CD = CR + DR = CQ+DS,

DA = DS + AS.

Adding opposite sides:

AB + CD = (AS+BQ) + (CQ+DS),

AB + CD = AS + BQ + CQ + DS,

BC + DA = (BQ + CQ) + (DS + AS),

BC + DA = BQ + CQ + DS + AS.

Hence, AB + CD = BC + DA.

Hence proved.

b) Find the height of the cone, whose curved surface area is $188\frac{4}{7}m^2$ and base of diameter 12 m.

Solution:

We know that:

$$\pi rl = 188\frac{4}{7}$$
$$r = 6$$
$$\pi rl = \frac{1320}{7}$$

$$\frac{22}{7} \times 6 \times l = \frac{1320}{7}$$

$l = 10$

$l^2 = r^2 + h^2$

$10^2 = 6^2 + h^2$

$h^2 = 10^2 - 6^2$

$h^2 = 100 - 36$

$h^2 = 64$

$h = 8\text{ m}$

Hence, the height of the cone is 8 m.

c) Find the lengths of the intercepts made by the line $x\cos\alpha + y\sin\alpha = \sin 2\alpha$ on the axes.

Solution:

$x\cos\alpha + y\sin\alpha = \sin 2\alpha$

Divide by $\sin 2\alpha$ on both sides, we get:

$$\frac{x\cos\alpha}{\sin 2\alpha} + \frac{y\sin\alpha}{\sin 2\alpha} = \frac{\sin 2\alpha}{\sin 2\alpha}$$

$$\frac{x\cos\alpha}{2\sin\alpha\cos\alpha} + \frac{y\sin\alpha}{2\sin\alpha\cos\alpha} = 1$$

$$\frac{x}{2\sin\alpha} + \frac{y}{2\cos\alpha} = 1$$

Therefore, x-intercept $= 2\sin\alpha$

y-intercept $= 2\cos\alpha$

d) Prove that:

$$(\cos A + \cos B)^2 + (\sin A - \sin B)^2 = 4\cos^2\frac{A+B}{2}.$$

Solution:

Taking L.H.S.

$(\cos A + \cos B)^2 + (\sin A - \sin B)^2$

$= \cos^2 A + \cos^2 B + 2\cos A\cos B + \sin^2 A + \sin^2 B - 2\sin A\sin B$

$= (\cos^2 A + \sin^2 A) + (\cos^2 A + \sin^2 A) + 2\cos A\cos B - 2\sin A\sin B$

$= 2 + \cos(A + B) + \cos(A - B) - [\cos(A - B) - \cos(A + B)]$

$= 2 + \cos(A + B) + \cos(A - B) - \cos(A - B) + \cos(A + B)$

$= 2 + 2(1 + \cos(A + B))$

$= 2(1 + \cos(A + B))$

$= 2\left[1 + 2\cos^2 \frac{(A + B)}{2} - 1\right]$

$= 4\cos^2 \frac{(A + B)}{2} = \mathrm{R.H.S}$

Hence proved.

a) The sum of the squares of two consecutive positive even numbers is 244. Find the numbers.

Solution:

Let two even number are x, x + 2.

$(x^2) + (x + 2)^2 = 244$

$x^2 + x^2 + 4 + 4x = 240$

$2x^2 + 4x - 240 = 0$

$2(x^2 + 2x - 120) = 0$

$x^2 + 2x - 120 = 0$

$x^2 + 12x - 10x - 120 = 0$

$x(x + 12) - 10(x + 12) = 0$

$(x - 10)(x + 12) = 0$

$x - 10 = 0$

$x = 10$

$x + 12 = 0$

$x = -12$

Putting the value of x, we get:

$x = 10$

$= x + 2$

$= 10 + 2$

$= 12$

Hence, the numbers are 10 and 12.

OR

Evaluate the value of $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0.$

Solution:

$$\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

Let's, add and subtract 2 in the above this equation.

$$\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 + 2 - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$

Let, $\left(x - \frac{1}{x}\right) = y$

$y^2 - 3y = 0$

$y(y - 3) = 0$

So, $y = 0$ or $y = 3$.

Therefore,

$x - \frac{1}{x} = 0$ and $x - \frac{1}{x} = 3$

$x^2 = 1$ and $x^2 - 3x - 1 = 0$

So, $x = \pm 1$ and $x = \frac{3 \pm \sqrt{3}}{2}$

b) Construct a cyclic quadrilateral PQRS where PQ = 5.0 cm, QR = 6.0 cm, PR = 3.5 cm, and RS = 5.0 cm.

Solution:

To construct the cyclic quadrilateral PQRS:

i. **Draw the base**: Draw PQ = 5.0 cm using a scale.

ii. **Draw an arc for QR**: With Q as the centre, draw an arc of 6.0 cm.

iii. **Draw an arc for PR**: With P as the centre, draw an arc of 3.5 cm to intersect the previous arc at R.

iv. **Join PR and QR**: Connect P to R and Q to R.

v. **Draw an arc for RS**: With R as the centre, draw an arc of 5.0 cm.

vi. **Draw an arc for PS**: With P as the centre, draw an arc to complete the quadrilateral by intersecting at S.

vii. **Complete the quadrilateral**: Join RS and PS.

This forms the required cyclic quadrilateral PQRS.