Secondary Examination, 2017

Mathematics

Time: $3\frac{1}{4}$ hours **M.M-70**

GENERAL INSTRUCTIONS TO THE EXAMINEES:

1. There are in all seven questions in this question paper.
2. All the questions are compulsory.
3. In the beginning of each, the number of parts to be attempted has been clearly mentioned.
4. Marks allotted to the questions are indicated against them.
5. Start with the first question and proceed to the last one.
6. Do not waste your time over a question you cannot solve.

Q1. (i) Evaluate adding of inverse $\frac{x^2+2}{x-1}$.

Solution:

Simplify the expression,

$$\frac{x^2+2}{x-1}$$

$$=-\left(\frac{x^2+2}{x-1}\right)$$

$$=\frac{x^2+2}{-x+1}$$

$$=\frac{x^2+2}{1-x}$$

Hence, the required value is $\frac{x^2+2}{1-x}$.

(ii) Find the median of 2,12,0,9,5,15,7,4.

Solution:

Total terms = 8

$n=8$, is the even number.

We know that:

$$\text{median} = \frac{1}{2}\left[\left(\frac{n}{2}\right)^{\text{th}}\text{ term} + \left(\frac{n+2}{2}\right)^{\text{th}}\text{ term}\right]$$

$$= \frac{1}{2}\left[\left(\frac{8}{2}\right)^{\text{th}}\text{ term} + \left(\frac{8+2}{2}\right)^{\text{th}}\text{ term}\right]$$

$$= \frac{1}{2}\left[4^{\text{th}}\text{ term} + 5^{\text{th}}\text{ term}\right]$$

$$= \frac{1}{2}\left[\frac{5+7}{2}\right] = \frac{12}{2} = 6$$

Hence, the median is 6.

(iii) Evaluate $\cos 15^{\circ}$.

(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(b) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$

(d) $\frac{\sqrt{3}\pm 1}{2\sqrt{2}}$

Solution:

Simplify the expression,

$$\cos 15^{\circ}$$
$$= \cos(45^{\circ} - 30^{\circ})$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
$$\cos(A-B) = \cos 45^{\circ}\cos 30^{\circ} + \sin 45^{\circ}\sin 30^{\circ}$$
$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Hence, the value of $\cos 15^{\circ}$ is $\frac{\sqrt{3}+1}{2\sqrt{2}}$.

(iv) Find the gradient of the line perpendicular to $x - y + 3 = 0$.

(a) -1

(b) $\frac{-1}{2}$

(c) $\frac{1}{2}$

(d) 1

Solution:

Simplify the expression, $x - y + 3 = 0$.

$y = x + 3$

Comparing the above equation with $y = mx + c$.

$m_1 = 1$

Let slope is m_2.

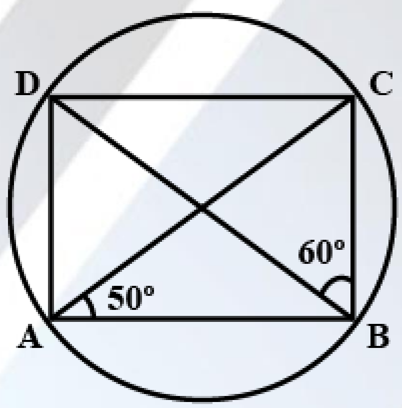
$m_1 \times m_2 = -1$

$1 \times m_2 = -1$

$m_2 = -1$

Hence, the gradient of the line perpendicular to $x - y + 3 = 0$ is -1.

(v) ABCD is a cyclic quadrilateral. If ∠BAC = 50° and ∠DBC = 60° then find ∠BCD.



Solution:

Here ∠BDC = ∠BAC since angles in same segment are equal.

In △BCD, we have

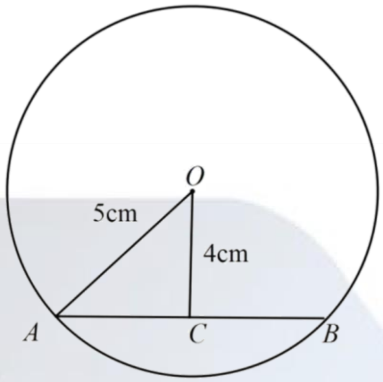
∠BCD = 180° – (∠BDC+∠DBC)

= 180° – (50° + 60°)

= 180° – 110° = 70°

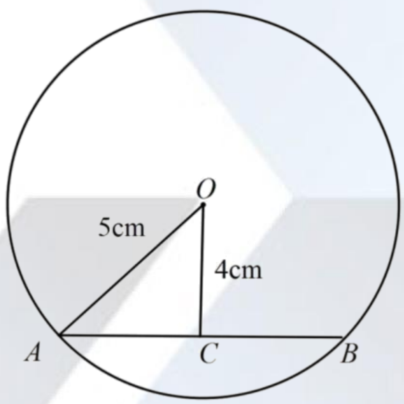
∴ ∠BCD = 70°

(vi) In the given figure, $OA = 5$ cm, $OC \perp AB$, find AB.



Solution:

Simplify the expression,



We know that:

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Therefore, AC = CB

In triangle ACO,

$(AO)^2 = (OC)^2 + (AC)^2$

$(5)^2 = (4)^2 + (AC)^2$

$25 = 16 + AC^2$

$AC^2 = 9$

$AC = 3$ cm

Therefore, $AB = AC + BC = 3 + 3 = 6$ cm

Hence, the value of $AB = 6$ cm

Q2. (i) If the arithmetic mean of 4, 5, x, and 9 is 7, then find the value of x.

Solution:

mean $= \frac{x_1+x_2+x_3+\cdots\ldots\ldots x_n}{n}$

$7 = \frac{4+5+x+9}{4}$

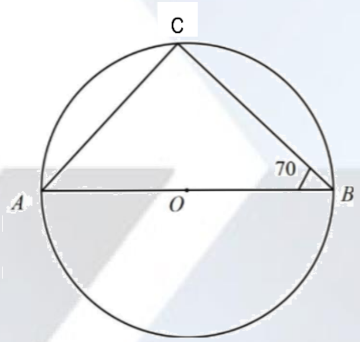
$28 = 18 + x$

$x = 28 - 18 = 10$

$x = 10$

Hence, the value of $x = 10$.

(ii) Given the angle $\angle ABC = 70°$, then find $\angle BAC$.



Solution:

Simplify the expression,

In $\triangle ABC$

$\angle A + \angle B + \angle C = 180°$

$\angle A + 70° + 90° = 180°$

$\angle A + 160° = 180°$

$\angle A = 20°$

Hence, the value of $\angle A = 20°$.

(iii) If $\cos 30° = \frac{\sqrt{3}}{2}$, then find the value of $\sin 15°$.

Solution:

We know that:

$\cos\ 2\theta = 1 - 2\sin^2\ \theta$
$2\sin^2\ \theta = 1 - \cos\ 2\theta$
$\sin\ \theta = \sqrt{\frac{1-\cos\ 2\theta}{2}}$

Putting the value of $\theta = 15^0$, we get:

$\sin\ 15° = \sqrt{\frac{1-\cos\ 2\theta}{2}}$

$= \sqrt{\frac{1-\cos\ 30^0}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

$= \frac{1}{2}\sqrt{2-\sqrt{3}}$

Hence, the value of sin15° is $\frac{1}{2}\sqrt{2-\sqrt{3}}$.

Q3. (i) $x^2 + p^2x + q = 0$ of root α, β, then find the value of $\alpha^2\beta + \alpha\beta^2$.

Solution:

Sum of roots, $\alpha + \beta = -\frac{b}{a}$

Product of roots, $\alpha \times \beta = \frac{c}{a}$

$\alpha + \beta = -\frac{p^2}{1}$

$\alpha + \beta = -p^2$

$\alpha \times \beta = \frac{q}{1}$

$\alpha \times \beta = q$

Now,

$\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$
$= \mathrm{q} \times (-p^2)$
$= -p^2q$
$\alpha^2\beta + \alpha\beta^2 = -p^2q$

Hence, the required value is $-p^2q$.

(ii) Mohan purchased a bicycle including sales tax is Rs.1058.40, if the actual price is Rs. 980, then find the rate of sales tax.

Solution:

$M.R.P+$ tax account =C.P

$980+\frac{980\times x}{100}=1058.40$

$\frac{980\times x}{100}=1058.40-980$

$\frac{980\times x}{100}=78.40$

$x=\frac{78.40\times 100}{980}$

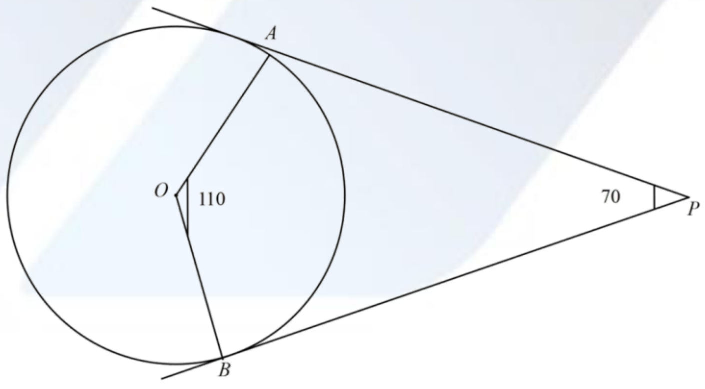
$x=8\%$

Hence, the value of $x=8\%$.

(iii) Draw a circle with a radius of 2.5 cm such that the angle between two tangents to the circle is 70°.

Solution:

1. Draw a circle of radius 2.5 cm and centre O.
2. Take a point A on circle, Join OA.
3. Draw a perpendicular to OA at A.
4. Draw a Radius OB, making on angle 70° with OA.
5. PA and PB are required tangents inclined at the angle of 110°.



(iv). Find the curved surface area of cylinder. If the given is cylinder of height 5 cm and area of base 36π cm^2.

Solution:

Simplify the expression,

$h = 5$ cm

$\pi r^2 = 36\pi$

curved surface area $= 2\pi rh$

$\pi r^2 = 36\pi$

$r = 6$ cm

curved surface area $= 2\pi rh$

$= 2 \times \pi \times 6 \times 5$

$= 60\pi \text{ cm}^2$

Hence, the value of curved surface area is $60\pi \text{ cm}^2$.

Q4. (i) Two right circular cone having same height. If the radius of base of a right circular cone is half of the radius of another cone, then find the ratio of their volume.

Solution:

Simplify the expression,

Radius $= R$

hight $= H$

$v_2 = \frac{1}{3}\pi r^2 h$

$v_2 = \frac{1}{3}\pi r^2 h$

Height $= H$

Radius $= \frac{R}{2}$

$v_1 = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{R}{2}\right)^2 \cdot H$

$= \frac{1}{3}\pi \cdot \frac{R^2}{4} H$

$$\frac{v_1}{v_2}=\frac{\frac{1}{3}\pi\cdot\frac{R^2}{4}H}{\frac{1}{3}\pi R^2\cdot H}=\frac{1}{4}$$

$v_1 : v_2 = 1 : 4$

(ii) Find the line of perpendicular equation $\frac{x}{a}+\frac{y}{b}=1$ such that passing through the root point.

Solution:

Simplify the expression,

$\frac{x}{a}+\frac{y}{b}=1$

$\frac{bx+ay}{ab}=1$

Equation of line perpendicular to given equation.

$ax-by=\lambda$ ...(ii)

since , line passes through origin.

$a(0)-b(0)=\lambda$

$\lambda=0$

putting the value of λ in equaton (ii)

$ax-by=0$

$ax=by$

Hence, we get the $ax=by$.

(iii) Prove that $\cos A+\tan A=2\operatorname{cosec}2A$.

Solution:

Simplify the expression,

L.H.S:

$\cot A+\tan A$

$=\frac{\cos A}{\sin A}+\frac{\sin A}{\cos A}$

$=\frac{\cos^2 A+\sin^2 A}{\sin A\times\cos A}$

$=\frac{1}{\sin A\times\cos A}$

Multiply by numerator and denominator in 2.

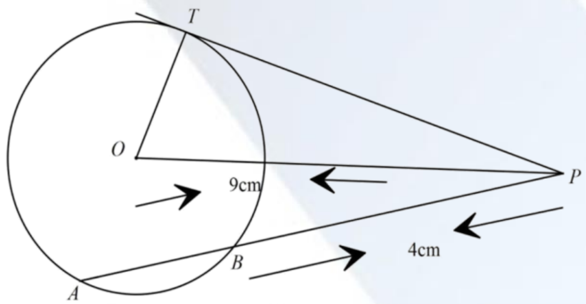
$$= \frac{2 \times 1}{2 \times \sin A \times \cos A}$$

We know that: $\sin 2A = 2\sin A \times \cos A$

$$= \frac{2}{\sin 2A} = 2\operatorname{cosec} 2A$$

Hence, proved.

(iv) PT is a tangent, and PBA is a piercing line. If PB $= 4$ cm, PA $=$ 9 cm, then PT $= ?$



Solution:

Simplify the expression,

$(PT)^2 = PA \times PB$

$(PT)^2 = 9 \times 4$

$PT^2 = 36$

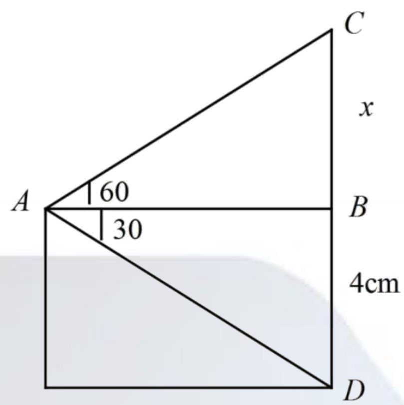
$PT = 6$ cm

Hence, the value PT $= 6$ cm.

Q5. (i) A man on the deck of a ship at a height of 'a' meters above the water level observes that the angle of elevation to the top of a cliff is 60°, and the angle of depression to the base of the cliff is 30°. Calculate the distance of the cliff from the ship and the height of the cliff.

Solution:

Simplify the expression,



In $\triangle ABC$:

$\tan\ 60^\circ = \frac{x}{y}$

$\sqrt{3} = \frac{x}{y}$

$y = \frac{x}{\sqrt{3}}$.

In $\triangle ABD$:

$\tan\ 30^\circ = \frac{a}{y}$

$\frac{1}{\sqrt{3}} = \frac{a}{y}$

Putting the value of y from equation (i)

$\frac{1}{\sqrt{3}} = \frac{a}{x}\sqrt{3}$
$x = a\sqrt{3} \cdot \sqrt{3}$
$x = 3a$
height $= x + a$
$= 3a + a$
$= 4a$
$y = \frac{x}{\sqrt{3}} = \frac{3a}{\sqrt{3}}$
$y = \frac{\sqrt{3}\times\sqrt{3}\times a}{\sqrt{3}}$
$y = \sqrt{3}a$

Hence, the value of $y = \sqrt{3}a$.

(ii) Find the ratio of the height of cone and the hemi-sphere having the same base and having same volume.

Solution:

We know that:

Volume of a cone is $V_c = \frac{1}{3}\pi r^2 h$...(i)

Now, the volume of the hemi-sphere is $V_s = \frac{2}{3}\pi r^3$ ...(ii)

Since, $V_c = V_s$ (Given)

Therefore, from eq(i) and (ii) we get:

$$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$

$$h = 2r$$

So, the ratio of the height of the cone and the hemi-sphere is,

$$\frac{h}{r} = \frac{2}{1}$$

$$h:r = 2:1$$

Hence, the ratio is 2: 1.

(iii) Find the area of the triangle if one vertex (referred to as the pick point) is at (2, 1), and the base of the equilateral triangle lies along the line $x + y = 2$.

Solution:

Draw perpendicular on BC,

Area of the triangle ABC= $\frac{1}{2} \times b \times h = \frac{1}{2} \times BC \times AD$

We know that length of perpendicular drawn at $x + y = 2$ from the point (2,1) is:

$$\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

$$= \frac{2+1-1}{\sqrt{1^2+1^2}}$$

$$= \frac{1}{\sqrt{2}}$$

Now, in triangle ACD,

$$\tan 30^\circ = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}} = \frac{CD}{AD}$$

$$\frac{1}{\sqrt{3}}=\frac{CD}{\frac{1}{\sqrt{2}}}$$

$$CD=\frac{1}{\sqrt{6}}$$

$\because BC=2CD$

$$BC=2\times\frac{1}{\sqrt{6}}$$

$$=\frac{2}{\sqrt{6}}$$

Therefore, the area of triangle,

$=\frac{1}{2}\times BC\times AD$

$=\frac{1}{2}\times\frac{1}{\sqrt{2}}\times\frac{2}{\sqrt{6}}$

$=\frac{1}{2\sqrt{3}}$

Hence, the area of the triangle is $\frac{1}{2\sqrt{3}}$.

(iv) If two circles with centers O and O′ touch each other externally at point P, and a common tangent AB is drawn at P, prove that ∠APB = 90°.

Solution:

Prove the expression,

For circle whose center is O .

CP = CA (The tangent drawn from same point)

$\angle CAP=\angle CPA$

$\angle CPA=\angle CPA=\alpha$

For another circle whose center is CB = CP (Tangent from same point).

$\angle CBP=\angle CPB$ (opposite angle of equal side)

$\angle CBP=\angle CPB=\beta$(let)

In △ APB,

$\angle A+\angle B+\angle C=180^{\circ}$

$\alpha+(\alpha+\beta)+\beta=180^{0}$

$2\alpha+2\beta=180^{0}$

$\alpha + \beta = 90°$

$\angle APB = 90°[\because \alpha + \beta = \angle APB]$

Hence, proved it.

Q6. (a) The monthly income person is Rs.50,000. He deposited Rs. 35,000 prime minister fund and in which 100% discount and he denoted Rs.15,000 in a hospital on which 50% discount, then find his saving for future per month 50,000 saving for future per month 50,000 and he deposited Rs10,000 half-yearly life insurance also he bought national saving draft Rs-25,000. If he paid sell tax a year then the find amount of sell tax paid by him in the last month. (Rs-15,0,000 on total saving discount 100%) and education tax of 3%.

Tax of income	tax
Rs. 2,00,000 to	zero
Rs. 2,00,001 to 5,00,000	2,00,000 to maximum income 10%

Solution:

Simplify the expression,

Income $= 50{,}000 \times 12 = 60{,}000$

Donation in prime minister relief fund $= 35{,}000$

100% rebate at the tax $= 35{,}000$

Donate in hospital $= 15{,}000$

(50% debate of hospital) $= 15{,}000 \times 50\%$

$= 15{,}000 \times \frac{50}{100} = 7{,}500$

Rest amount $= 6{,}00{,}000 - 42{,}500 = 5{,}57{,}500$

L.I.C premium $= 10{,}000 \times 2 = 20{,}000$

National income certificate $= 25{,}000$

Total saving $= 1{,}05{,}000$

Taxable amount $= 5{,}57{,}500 - 1{,}05{,}000 = 4{,}52{,}500$

$(4,52,500 - 2,00,000) \times 10\%$

$= 2,52,000 \times 10\% = 25,250$

Tax $= 25,200$

Charge of education $= 25,200 \times 3\% = 757.50$

Total tax $= 25250 + 757.50 = 26007.50$

paid tax $= 2000 \times 11 = 22,000$

Had to pay tax in 12th in 12th months

$= 26007.50 - 22,000$

$= 4007.50$

Hence, the final result of tax is Rs.4007.50

(ii) Find the median daily wages from the following frequency distribution:

Class-interval	$10-15$	$15-20$	$20-25$	$25-30$	$30-35$	$35-40$
Frequency	5	6	8	12	6	4

Solution:

Interval	Frequency (f_i)	Mid-point (x_i)	$f_i x_i$
$10-15$	5	12.5	62.5
$15-20$	6	17.5	105.0
$20-25$	8	22.5	180.0
$25-30$	12	27.5	330.0

30 – 35	6	32.5	195.0
35 – 40	4	37.5	130.0

$\sum f_i x_i = 1022.5$

$\sum f_i = 41$

Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{1022.5}{41} = 24.9$

Hence, the value of 24.9.

(iii) Evaluate the equation $\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$.

Solution:

Simplify the equation,

$$\frac{x^2 + (x+1)^2}{x(x+1)} = \frac{34}{15}$$

$$\frac{x^2 + x^2 + 1 + 2x}{x^2 + x} = \frac{34}{15}$$

$$\frac{2x^2 + 2x + 1}{x^2 + x} = \frac{34}{15}$$

$30x^2 + 30x + 15 = 34x^2 + 34x$

$4x^2 + 4x - 15 = 0$

$4x^2 + 10x - 6x - 15 = 0$

$2x(2x + 5) - 3(2x + 5)$

$(2x - 3)(2x + 5) = 0$

$2x - 3 = 0$

$2x = 3 \Rightarrow x = \frac{3}{2}$

$2x + 5 = 0$

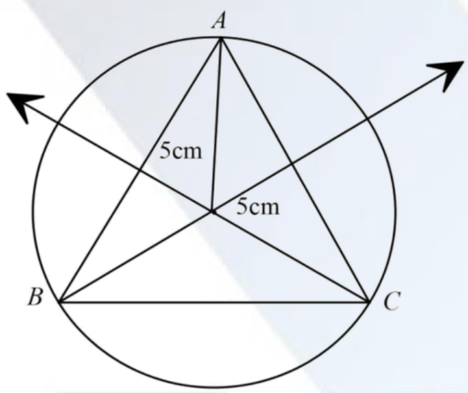
$2x = -5$

$x = \frac{-5}{2}$

Hence, the value of $x = \frac{3}{2}$ and $x = \frac{-5}{2}$.

(iv) Drawn the 5 cm equilateral triangle and construction of outscribed. Give the reason.

Solution:



Step 1: Draw a line segment BC = 5 cm.

Step 2: With B as center and radius 5 cm .

Step 3: with C as center and radius 5 cm draw another are intersecting previous are at point A .

Step 4: Join AB & AC

Step 5: ABC is required equilateral triangle.

Step 6: Draw are perpendicular bisector of the side of triangle.

Step 7: Mark the point of intersection of the circum center of triangle.

Step 8: with O center, OA ,OB , & OC are radius, now draw a circle .

Thus the circum circle of the equilateral triangle is constracted.

Q7. (i) The two term of expansion HCF and LCM $(x-5)$ and $(x-5)(x^2-7x+12)$. If one term of expansion is $(x^2-8x+15)$ then find the second term of expansion.

Solution:

Simplify the equation,

HCF $= x - 5$

LCM $= (x-5)(x^2 - 7x + 12)$

Polynomial-1 $= (x^2 - 8x + 15)$

(ii) Polynomial-2 = ?

LCM × HCF =polynomial-1 ×polynomial-2

$(x-5) \times (x^2 - 7x + 12)(x-5) = (x^2 - 8x + 15) \times \text{polynomial} - 2$

polynomial-2 $= \frac{(x-5)(x^2-7x+12)(x-5)}{(x^2-8x+15)}$

polynomial-2$= \frac{(x-5)^2(x^2-4x-3x+12)}{(x^2-8x+15)}$

polynomial-2 $= \frac{(x-5)^2[x(x-4)-3(x-4)]}{(x^2-5x-3x+15)}$

polynomial-2 $= \frac{(x-5)^2(x-4)(x-3)}{x(x-5)-3(x-5)}$

polynomial-2$= \frac{(x-5)^2(x-4)(x-3)}{(x-5)(x-3)}$

polynomial-2$= (x-5)(x-4) = x^2 - 9x + 20$

polynomial-2$= x^2 - 9x + 20$

Hence, the second polynomial is $x^2 - 9x + 20$.

(ii) Prove that $(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 4\cos^2\left(\frac{A-B}{2}\right)$.

Solution:

Prove the expression,

L.H.S:

$(\cos A + \cos B)^2 + (\sin A + \sin B)^2$

We know that,

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

L.H.S

$\left[\cos\ C+\cos\ D=2\cos\ \left(\frac{C+D}{2}\right)\cdot\cos\ \left(\frac{C-D}{2}\right)\right]^2+\left[\sin\ C+\sin\ D=2\sin\ \left(\frac{C+D}{2}\right)\cdot\cos\ \left(\frac{C-D}{2}\right)\right]^2$

$=4\cos^2\ \left(\frac{A+B}{2}\right)\cos^2\ \left(\frac{A-B}{2}\right)+\sin^2\ \left(\frac{A+B}{2}\right)\cos^2\ \left(\frac{A-B}{2}\right)$

$=4\cos^2\ \left(\frac{A-B}{2}\right)\left[\cos^2\ \left(\frac{A+B}{2}\right)+\sin^2\ \left(\frac{A+B}{2}\right)\right]$

$=4\cos^2\ \left(\frac{A-B}{2}\right)\Rightarrow 1$

$\Rightarrow 4\cos^2\ \left(\frac{A-B}{2}\right)=\text{R.H.S}$

Hence, proved.

(iii) Prove that $\sin\alpha\times\sin\ (60-\alpha)\times\sin\ (60+\alpha)=\frac{1}{4}\sin\ 3\alpha$.

Solution:

Prove the expression,

We, know that: $\sin\ 3\alpha=3\sin\ \alpha-4\sin^3\ \alpha$

L.H.S

$\sin\alpha\times\sin\ (60-\alpha)\times\sin\ (60+\alpha)$

Using the formula:

$[\sin^2\ A-\sin^2\ B=\sin\ (A+B)\cdot\sin\ (A-B)]$

$=\sin\ \alpha\left[\left(\frac{\sqrt{3}}{2}\right)^2-\sin^2\ \alpha\right]$

$=\sin\ \alpha\left[\frac{3}{4}-\sin^2\ \alpha\right]$

$=\sin\ \alpha\left[\frac{3-4\sin^2\ \alpha}{4}\right]$

$=\left[\frac{3\sin\ \alpha-4\sin^3\ \alpha}{4}\right]$

$\Rightarrow\sin\ 3\alpha=3\sin\ \alpha-4\sin^3\ \alpha$

$\Rightarrow\frac{\sin\ 3\alpha}{4}=\frac{1}{4}\sin\ 3\alpha$

$\frac{1}{4}\sin\ 3\alpha=\ \text{R.H.S}$

Hence, proved.