UP Secondary Examination, 2019

Mathematics

Time: $3\frac{1}{4}$ Hours M.M- 70

GENERAL INSTRUCTIONS TO THE EXAMINEES:

1. There are in all seven questions in this question paper.
2. All the questions are compulsory.
3. In the beginning of each, the number of parts to be attempted has been clearly mentioned.
4. Marks allotted to the questions are indicated against them.
5. Start with the first question and proceed to the last one.
6. Do not waste your time over a question you cannot solve

Q1. a) Find the distance between the points are $(5,0)$ and $(-12,0)$.

Solution:

We know that,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-12 - 5)^2 + (0 - 0)^2}$$
$$= \sqrt{289} = 17$$

Hence, the value is 17 units.

b) The common difference of AP is -4 and its 10^{th} terms is -8. Determine the first terms of AP.

Solution:

We know that,

$Tn = a + (n - 1)d$

$T_{10} = \text{a} + (10 - 1) \times (-4)$

$-8 = a - 36$

$-8 + 36 = a$

$a = 28$

Hence, the value of $a = 28$.

c) Which of the following numbers cannot be the probability of an event?

(i) $\frac{1}{3}$

(ii) $\frac{2}{3}$

(iii) $\frac{3}{2}$

(iv) 15%

Solution:

The probability of an event must lie between 0 and 1.

i. 1/3 is valid (between 0 and 1).

ii. 2/3 is valid (between 0 and 1).

iii. 3/2 is not valid (greater than 1).

iv. 15% is valid (equals 0.15, between 0 and 1).

Therefore, the correct answer is $\mathbf{\frac{3}{2}}$.

d) Evaluate $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$.

(i) cos60°

(ii) tan60°

(iii) sin30°

(iv) sin60°

Solution:

Simplify the expression:

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

$$\frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2\times 3}{\sqrt{3}\times 2}$$

$$= \sqrt{3}$$

$$= tan60^o$$

Hence, the correct option is tan60°.

e) In $\triangle ABC$, if $\mathrm{AB} = 6\sqrt{3}$ cm, $\mathrm{AC} = 12$ cm and $\mathrm{BC} = 6$ cm, find the value of $\angle B$.

(i) 45^o

(ii) 90^o

(iii)120^o

(iv)135^o

Solution:

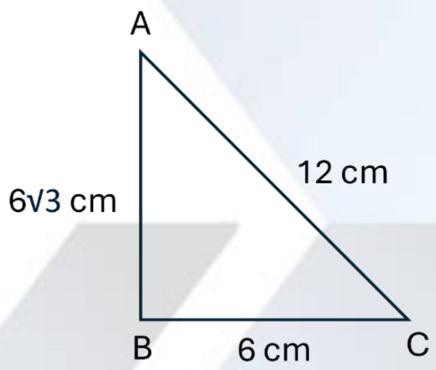
$AC^2 = 12^2 = 144$

$AB^2 + BC^2 = (6\sqrt{3})^2 + 6^2$

$= 108 + 36$

$= 144$

Therefore, $AC^2 = AB^2 + BC^2$



So, from the Pythagoras theorem, $\angle \boldsymbol{B} = \boldsymbol{90^o}$.

f) If the roots of the quadratic equation $3x^2 - 6x + k = 0$ are equal, then the value of k is:

(i)3

(ii)6

(iii)9

(iv)12

Solution:

We know that, in case equal roots:

$D = 0$

$b^2 - 4ac = 0$

$(-6)^2 - 4 \times 3 \times k = 0$

$36 - 12k = 0$

$$k = \frac{36}{12} = 3$$

Hence, the value of k is 3.

Q2. a) Determine the H.C.F of 130 and 280 using Euclid's division algorithm.

Solution:

$a = bq + r$ where $< r < b$

$280 = 130 \times 2 + 20$

$130 = 20 \times 6 + 10$

$20 = 10 \times 2 + 0$

$H.C.F = 10$

Hence, H.C.F = 10.

b) Find the mode of the given distribution frequency.

Class interval	Frequency
$10 - 20$	14
$20 - 30$	13
$30 - 40$	12
$40 - 50$	20
$50 - 60$	11
$60 - 70$	15

Solution:

We know that:

$$Mode = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 40 + \frac{20 - 12}{2 \times 20 - 12 - 11} \times 10$$

$$= 40 + \frac{8}{17} \times 10$$

$= 40 + 4.70$

$= 44.70$

Hence, the mode is 44.70.

c) If $8\tan A = 15$, then find the value of sinA.

Solution:

$8\tan A = 15$

$\tan A = \frac{15}{8}$

$AC^2 = AB^2 + BC^2$

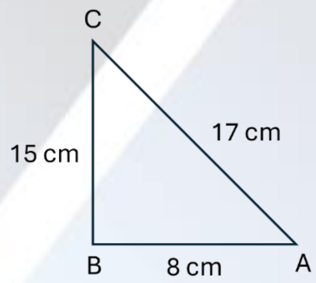
$= 8^2 + 15^2$

$= 64 + 225$

$= 289$

$= 17$

Therefore,



Then, $\sin A = \frac{15}{17}$

Hence, the value of sinA is $\frac{15}{17}$.

d) The sides of two similar triangles are in the ratio of 4:9. Find the ratio of their areas.

Solution:

We know that:

$$\frac{Area\ of\ triangle\ 1}{Area\ of\ triangle\ 2} = \left(\frac{Side\ of\ triangle\ 1}{Side\ of\ triangle\ 2}\right)^2$$

$$\frac{Area\ of\ triangle\ 1}{Area\ of\ triangle\ 2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Q3. a) Show that $5\sqrt{2}$ is an irrational number.

Solution:

Let the $5\sqrt{2}$ is a rational number.

If $5\sqrt{2}$ is rational, then it can be expressed as $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

$$5\sqrt{2} = \frac{p}{q}$$

$$\sqrt{2} = \frac{p}{5\text{q}}$$

$\frac{p}{q}$ a rational number.

$\frac{p}{5q}$ a rational number.

$\sqrt{2}$ a rational number.

However, it is well known that $\sqrt{2}$ is an irrational number

The assumption that $5\sqrt{2}$ is rational leads to the conclusion that $\sqrt{2}$ is rational, which contradicts the fact that $\sqrt{2}$ is irrational.

Since the assumption that $5\sqrt{2}$ is rational leads to a contradiction, $5\sqrt{2}$ must be irrational.

b) If the points $(1,2), (4, y), (x, 6),$ and $(3,5)$ are taken in order as the vertices of a parallelogram, find the values of x and y.

Solution:

Let $A(1,2), \text{B}(4, y), \text{C}(x, 6),$ and $\text{D}(3,5)$

Mid-point of BD $= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$$= \left[\frac{3+4}{2}, \frac{5+y}{2}\right]$$

$$= \left[\frac{7}{2}, \frac{5+y}{2}\right]$$

Mid-point of AC$= \left[\frac{1+x}{2}, \frac{2+6}{2}\right] = \left[\frac{1+x}{2}, 4\right]$

For the points (1,2), (4,y), (x,6), and (3,5) to form a parallelogram, the diagonals must bisect each other. Hence, the midpoints of the diagonals must be equal.

$$\frac{7}{2} = \frac{1+x}{2}$$
$$x = 6$$
$$\frac{5+y}{2} = 4$$
$$5 + y = 8$$
$$y = 3$$

Therefore, $x = 6$ and $y = 3$.

c) A box contains 90 discs, each marked with a number from 1 to 90. If one disc is randomly drawn from the box, the probability of the number on the disc being:

(i)A two-digit number

(ii)A number divisible by 5

Solution:

(i) **Probability of a two-digit number:**

Two-digit numbers are from 10 to 90, so there are 81 two-digit numbers (90 - 10 + 1).

Total numbers = 90

Probability$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{81}{90} = \frac{9}{10}$

(ii) **Probability of a number divisible by 5:**

Numbers divisible by 5 between 1 and 90 are 5, 10, 15, ..., 90. The count of such numbers is $\frac{90}{5} = 18$.

Total numbers = 90

Probability$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{18}{90} = \frac{1}{5}$

d) In △ABC, prove that sec $\left(\frac{B+C}{2}\right) = \text{cosec}\,\frac{A}{2}$.

Solution:

We know that:

$A + B + C = 180$

$B + C = 180 - A$

$$\frac{B+C}{2} = \left(90 - \frac{A}{2}\right)$$

Taking sec both sides, we get:

$$sec\left(\frac{B+C}{2}\right) = \sec\left(90 - \frac{A}{2}\right)$$

$$sec\left(\frac{B+C}{2}\right) = \text{cosec}\,\frac{A}{2}$$

Hence proved.

Q4. a) Without performing the long division process, state which of the following rational numbers will have terminating decimal expansions and which will have non-terminating recurring decimal expansions?

(i) $\frac{13}{3125}$

(ii) $\frac{15}{7}$

(iii) $\frac{29}{2^3 \times 5^2}$

(iv) $\frac{77}{210}$

Solution:

To determine whether a rational number has a terminating decimal expansion or a non-terminating recurring decimal expansion, we check the prime factorization of the denominator:

$(i)\ \frac{13}{3125}$:

The denominator is 3125=5^5, which is a power of 5. So, this has a **terminating decimal expansion**.

$(ii)\ \frac{15}{7}$:

The denominator is 7, which is a prime number other than 2 or 5. So, this has a **non-terminating recurring decimal expansion**.

(iii) $\frac{29}{2^3 \times 5^2}$

The denominator is $2^3 \times 5^2$, which consists only of the prime factors 2 and 5. So, this has a **terminating decimal expansion**.

(iv) $\frac{77}{210}$:

The denominator is 210=2×3×5×7, which includes a prime factor other than 2 or 5. So, this has a **non-terminating recurring decimal expansion**.

(b). Find the value of k, if the points $(8,1), (k, -4)$, and $(2, -5)$ are collinear.

Solution:

For collinear:

$$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$$

$$8(-4 + 5) + k(-5 - 1) + 2(1 + 4) = 0$$

$$8 \times 1 + k \times (-6) + 2 \times 5 = 0$$

$$8 - 6k + 10 = 0$$

$$-6k = 1 - 8$$

$$k = \frac{18}{6} = 3$$

$$k = 3$$

Hence, the value of k is 3.

(c) Find out whether the system of linear equations $2x - 3y = 8$ and $4x - 6y = 9$ is consistent or inconsistent.

Solution:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{c_1}{c_2} = \frac{8}{9}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{1}{2} = \frac{1}{2} \neq \frac{8}{9}$$

We observe that the second equation is exactly twice the first equation, except for the constant term.

Thus, the system represents two parallel lines that never intersect, meaning the **system is inconsistent**.

(d) Find the median of the following distribution:

Class Interval	Frequency

$0 - 10$	5
$10 - 20$	8
$20 - 30$	20
$30 - 40$	15
$40 - 50$	7
$50 - 60$	5

Solution:

Class Interval	Frequency	Cumulative frequency
$0 - 10$	5	5
$10 - 20$	8	13
$20 - 30$	20	33
$30 - 40$	15	48
$40 - 50$	7	55
$50 - 60$	5	60

The total frequency (N) is the sum of all the frequencies:

N = 5 + 8 + 20 + 15 + 7 + 5 = 60

The median class is the class where the cumulative frequency is greater than or equal to $\frac{N}{2}$.

Since N = 60, we calculate:

$$\frac{N}{2} = \frac{60}{2} = 30$$

Looking at the cumulative frequency, we see that the cumulative frequency first exceeds 30 in the class interval **20-30**. Therefore, the **median class** is 20-30.

The formula to find the median is:

$$M_e = l + \left\{h \times \frac{\left(\frac{N}{2} - c\right)}{f}\right\}$$

$$= 20 + \left\{10 \times \frac{(30-13)}{20}\right\}$$

$$= 20 + \left\{10 \times \frac{17}{20}\right\}$$

$$= 20 + 8.5$$

$$= 28.5$$

Hence, median is 28.5.

Q5. a) Solve the following pair of linear equations.

$$\frac{5}{x+y} + \frac{1}{x-y} = 2$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

Solution:

$$\frac{5}{x+y} + \frac{1}{x-y} = 2 \ldots \text{(i)}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \ldots \text{(ii)}$$

Let, $\frac{1}{x+y} = u, \frac{1}{x-y} = \text{v}$

$5u + v = 2 \ldots\ldots (iii)$

$15u - 5v = -2 \ldots\ldots (iv)$

Equation (iii) multiply by 3.

$15u + 3v = 6$

$15u - 5v = -2$

$\Rightarrow 8v = 8$

$\Rightarrow v = 1$

From equation (iii,

$5u + 1 = 2$

$5u = 1$

$u=\frac{1}{5}$

$\frac{1}{x-y}=1$

$x-y=1.\ldots$ (A)

$\frac{1}{x+y}=\frac{1}{5}$

$x+y=5$........(B)

Adding equation (A) and equation (B), we get:

$x-y=1$

$x+y=5$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

Substitute the value of x in equation (A).

$3-y=1$

$3-1=y$

$y=2$

Hence, the value of x and y are $x=3, y=2$.

b) Find the sum of the odd numbers between 0 and 100.

Solution:

We know that:

0 to 100 between odd numbers are 1,3,5,7,9 ...……99

$a=1$, d $=3-1=2$, l= 99

$$l=a+(n-1)d$$
$$99=1+(n-1)2$$
$$\Rightarrow 99-1=2n-2$$
$$\Rightarrow 98+2=2n$$
$$\Rightarrow 2n=100$$
$$\Rightarrow n=50$$

Now,

$S_n=\frac{n}{2}[2a+(n-1)\text{d}]$

$S_{50}=\frac{50}{2}[2\times 1+(50-1)2]$

$= 25[2 + 98] = 25 \times 100$

$\Rightarrow S_{50} = 2500$

Hence, the value of $S_{50} = 2500$.

c) A solid toy is in the shape of a hemisphere with a conical shape mounted on it. The height of the cone is 2 cm, and the diameter of its base is 4 cm. Find the volume of this toy.

Solution:

Given Dimensions:

Radius of the cone and hemisphere, $r = \frac{4}{2} = 2$ cm.

Height of the cone, h = 2 cm.

Volume of the cone:

$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2)^2(2) = \frac{8\pi}{3}$ cm^3

Volume of the hemisphere:

$V_{hemisphere} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(2)^3 = \frac{16\pi}{3}$ cm^3

Total Volume of the Toy:

$V_{toy} = V_{cone} + V_{hemisphere} = \frac{8\pi}{3} + \frac{16\pi}{3} = \frac{24\pi}{3} = 8\pi$ cm^3

Final Answer:

8π cm^3 or approximately 25.13 cm^3 (taking $\pi \approx 3.14$).

d) The angle of depression of two ships observed from the top of a lighthouse, which is 75 m above sea level, are 30^o and 45^o. If the ships are on the same side of the lighthouse and one ship is directly behind the other, calculate the distance between the two ships.

Solution:

Let the lighthouse height be AB = 75 m. The ships are at C and D on the same side of the lighthouse.

From the right-angled triangle △ABC:

$$tan45^0 = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{75}{BC}$$

$\Rightarrow BC = 75$ cm

From the right-angled triangle △ABD:

$$tan30^0 = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$\Rightarrow BD = 75\sqrt{3}$ cm

Distance between the ships:

CD = BD – BC = $75\sqrt{3} - 75 = 75(\sqrt{3} - 1)$ m.

Thus, the distance between the two ships is $75(\sqrt{3} - 1)$ m.

Q6. a) The following distribution represents the daily pocket money of children in a neighbourhood. The mean pocket money is ₹18. Find the value of the frequency f:

Daily pocket money	Number of students
$11 - 13$	7
$13 - 15$	6
$15 - 17$	9
$17 - 19$	13
$19 - 21$	f
$21 - 23$	5
$23 - 25$	4

Solution:

Daily pocket money	Median value	Frequency	f_x
$11 - 13$	12	7	84
$13 - 15$	14	6	84

15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	20 f
21 – 23	22	5	110
23 – 25	24	4	96
		$\sum f_i = 44 + f$	$\sum(f_1 \times x_1) = 752 + 20f$

We know that:

$$\text{Mean} = \sum \frac{(f_i \times x_i)}{f_i}$$

$$18 = \frac{20f + 752}{f + 44}$$

$$18f + 792 = 20f + 752$$

$$792 - 752 = 20f - 18f$$

$$2f = 40$$

$$f = 20$$

Hence, the mean value of $f = 20$.

b) (i) To create a cuboid with dimensions 5.5 cm × 10 cm × 3.5 cm, how many silver coins, each with a diameter of 1.75 cm and a thickness of 2 mm, need to be melted?

Solution:

Given Data:

Dimensions of the cuboid:

Length (l) = 5.5 cm,

Breadth (b) = 10 cm,

Height (h) = 3.5 cm.

Volume of the cuboid = l×b×h = 5.5×10×3.5 = 192.5 cm^3

Dimensions of each silver coin:

Diameter = 1.75 cm → Radius (r) = 1.75/2 = 0.875 cm

Thickness = 2 mm = 2/10 = 0.2 cm

Volume of a coin = $\pi r^2 h = \pi(0.875)^2(0.2)$ cm^3

Volume of a coin = $\pi(0.875)^2(0.2)$

First, calculate $(0.875)^2$:

$(0.875)^2 = 0.765625$

Now, multiply by 0.2:

$0.765625 \times 0.2 = 0.153125$

Finally, multiply by $\pi \approx 3.1416$:

Volume of a coin = $3.1416 \times 0.153125 \approx 0.4809$ cm^3.

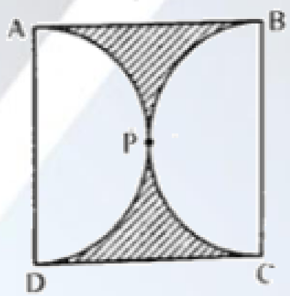
Number of coins= $\frac{\text{Volume of the cuboid}}{\text{Volume of a coin}}$

Substitute the values:

Number of coins= $\frac{192.5}{0.4809} \approx 400.4.$

Approximately **400 coins** are required to create the cuboid.

b) (ii) In the figure, find the area of the shaded region if ABCD is a square with a side length of 14 cm, and APD and BPC are two semicircles.



Solution:

Given:

Side of square ABCD = 14 cm

Radius of each semicircle = 14/2 = 7 cm

Step 1: Calculate the area of the square

Area of square = side^2 = 14^2 = 196 cm^2

Step 2: Calculate the area of the two semicircles

Total area of the semicircles = Area of a full circle

Area of a circle = $\pi r^2 = \pi(7)^2 = 49 \times 3.14 = 153.86$ cm^2

Step 3: Calculate the area of the shaded region

Shaded area = Area of square − Area of semicircles

Shaded area = 196 − 153.86

Shaded area = 42.14 cm^2

Therefore, the area of the shaded region is approximately **42.14** cm^2.

c) Prove that $(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$

Solution:

First, let us consider LHS:

$(\text{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)$

$$= \left(\text{cosec}\theta - \frac{1}{\text{cosec}\theta}\right)\left(\sec\theta - \frac{1}{\sec\theta}\right)$$
$$= \left(\frac{\text{cosec}^2\theta - 1}{\text{cosec}\theta}\right)\left(\frac{\sec^2 - 1}{\sec\theta}\right)$$
$$= \left(\frac{\cot^2\theta}{\text{cosec}\theta}\right)\left(\frac{\tan^2}{\sec\theta}\right)$$
$$= \cot^2\theta \cdot \sin\theta \times \tan^2 \cdot \cos\theta$$
$$= \frac{\cos^2\theta}{\sin^2\theta} \cdot \sin\theta \times \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos\theta$$
$$= \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}$$
$$= \cos\theta \cdot \sin\theta$$

Now, R.H.S:

$$\frac{1}{\tan\theta + \cot\theta}$$
$$= \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$
$$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta}} \quad [\text{Since}, \sin^2\theta + \cos^2\theta = 1]$$
$$= \cos\theta \cdot \sin\theta$$

Therefore, L.H.S. = R.H.S.

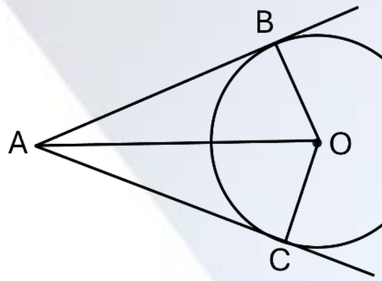
d) Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Let A be a point outside the circle with centre O. Two tangents, AB and AC, are drawn from A to the circle, touching the circle at points B and C, respectively. We need to prove that AB = AC.

Steps:

1. **Join the centre O to the points A, B, and C:**
   - OB and OC are radii of the circle.
   - OA is the distance from the external point A to the centre O.



2. **Properties of tangents:**
   - The tangent to a circle is perpendicular to the radius at the point of contact.
   - Therefore, $OB \perp AB$ and $OC \perp AC$.
3. **Triangles △OAB and △OAC:**
   - In △OAB and △OAC:
     - OB = OC (radii of the circle),
     - OA (common side),
     - $\angle OBA = \angle OCA = 90^\circ$ (tangent is perpendicular to the radius).
   - By the RHS (Right angle-Hypotenuse-Side) congruence rule, △OAB ≅ △OAC.
4. **Conclusion:**
   - Since △OAB ≅ △OAC, their corresponding sides are equal.
   - Therefore, AB=AC.

Thus, the lengths of tangents drawn from an external point to a circle are equal.

Q7. a) Find all the other zeroes of $3x^2 + 6x^3 - 2x^2 - 10x - 5$, given that two of its zeros are $-\sqrt{\frac{5}{3}}$ and $\sqrt{\frac{5}{3}}$.

Solution:

Given that:

$x = -\sqrt{\frac{5}{3}}$ and $x = \sqrt{\frac{5}{3}}$

Therefore,

$$\left(x + \sqrt{\frac{5}{2}}\right)\left(x - \sqrt{\frac{5}{3}}\right) = 0$$

$$x^2 - \frac{5}{2} = 0$$

Now,

$$\frac{3x^2 + 6x^2 - 2x^2 - 10x - 5}{x^2 - \frac{5}{3}} = 3x^2 + 6x + 3$$

Now, $3x^2 + 6x + 3$

$3x^2 + 3x + 3x + 3 = 0$

$3x(x + 1) + 3(x + 1) = 0$

$(3x + 3)(x + 1) = 0$

$\Rightarrow x + 1 = 0$

$x = -1$

$\Rightarrow (3x + 3) = 0$

$\Rightarrow 3x = -3$

$x = -1$

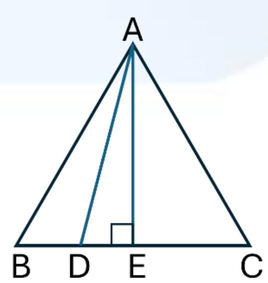
Hence, the other zeros are -1 and -1.

b) In an equilateral triangle ABC, a point D lies on the side BC such that $BD = \frac{1}{3}BC$.

Prove that $9AD^2 = 7AB^2$.

Solution:

We know that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



In ΔABC as shown in the figure above,

AB = BC = CA and BD $= \frac{1}{3}$BC

Draw AE ⊥ BC

We know that in an equilateral triangle, a perpendicular drawn from the vertex to the opposite side bisects the side.

Thus, BE = EC = $\frac{1}{2}$BC

Now, in ΔADE,

$AD^2 = AE^2 + DE^2$ (Pythagoras theorem) …(1)

AE is the height of an equilateral triangle which is equal to side $\frac{\sqrt{3}}{2}$.

Thus, AE = $\frac{\sqrt{3}}{2}$BC

Also, DE = BE - BD [From the diagram]

Substituting these in equation (1) we get,

$$\mathrm{AD}^2 = \left(\frac{\sqrt{3}}{2}\mathrm{BC}\right)^2 + (\mathrm{BE} - \mathrm{BD})^2$$

$$\mathrm{AD}^2 = \frac{3}{4}(\mathrm{BC})^2 + \left(\frac{\mathrm{BC}}{2} - \frac{\mathrm{BC}}{3}\right)^2$$

$$\mathrm{AD}^2 = \frac{3}{4}BC^2 + \left(\frac{\mathrm{BC}}{6}\right)^2$$

$$\mathrm{AD}^2 = \frac{3}{4}BC^2 + \frac{1}{36}BC^2$$

$$AD^2 = \frac{27BC^2 + BC^2}{36}$$

$$36AD^2 = 28BC^2$$

$$9AD^2 = 7BC^2$$

$9AD^2 = 7AB^2$ [Since, AB = BC = CA]

Hence proved.