Secondary Examination, 2022

Mathematics

Time: $3\frac{1}{4}$ hours **M.M-70**

Instructions:

i) There are seven questions in all in this question paper.
ii) All questions are compulsory.
iii) In the beginning of each question, it has been clearly mentioned that how many parts of it are to be attempted.
iv) Marks allotted to each question are mentioned against it.
v) Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
vi) Write the solution on the pages of both sides of answer-book. Write the steps of solutions of all questions except Question No. 1.
vii) If you need place for rough work, do it on left page of your answer book and cross (x) the page. Do not write the solution on that page.
viii) Do not rub off the lines constructed in a question of construction. Do write the steps of construction in brief, if asked.
ix) Draw neat and correct figure in solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

1. Do all the parts:
Four alternatives of the answer of each part are given, out of which only one is correct. Pick out the correct alternative and write it in your answer-book:
a) For any positive integer p, every positive odd integer will be of the form
i) p
ii) $p + 1$
iii) $2p$
iv) $2p + 1$

Solution:

Every positive odd integer can be expressed in the form 2p+1, where p is a positive integer. This ensures the number remains odd, as it is always one more than an even number (2p).

b) The points $A(-4,0)$, $B(4,0)$ and $C(0,3)$ are the vertices of a triangle. That triangle will be
i) Right angled triangle
ii) Isosceles triangle
iii) Equilateral triangle

iv) Scalene triangle

Solution:

To determine the type of triangle formed by the points $A(-4,0), B(4,0)$, and $C(0,3)$, we calculate the distances between the vertices:

AB = Distance between $(-4,0)$ and $(4,0)$

$AB=\sqrt{(4+4)^2+(0-0)^2}=\sqrt{8^2}=8$

BC = Distance between $(4,0)$ and $(0,3)$

$BC=\sqrt{(0-4)^2+(3-0)^2}=\sqrt{16+9}=\sqrt{25}=5$

CA = Distance between $(0,3)$ and $(-4,0)$

$CA=\sqrt{(0+4)^2+(3-0)^2}=\sqrt{16+9}=\sqrt{25}=5$

Triangle Classification:

Since $BC=CA=5$, the triangle is isosceles.

Check for right-angle:

$BC^2+CA^2=5^2+5^2=25+25=50\neq AB^2(64)$

Since the Pythagorean theorem does not hold, it is not a right-angled triangle.

It is not equilateral as all sides are not equal.

It is not scalene as two sides are equal.

c) The areas of two similar triangles are 20 cm^2 and 45 cm^2 respectively. The ratio of their heights will be

i) 20: 45

ii) 9: 4

iii) 2: 3

iv) 4: 9

Solution:

For similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding heights.

Let the ratio of their heights be $x: y$.

$$\left(\frac{x}{y}\right)^2=\frac{\text{Area }_1}{\text{Area }_2}=\frac{20}{45}$$

$$\left(\frac{x}{y}\right)^2=\frac{4}{9}$$

Taking the square root on both sides:

$$\frac{x}{y}=\frac{2}{3}$$

Thus, the correct answer is 2:3.

d) If $\tan\ \theta=\frac{2ab}{a^2-b^2}$, the value of $\cos\ \theta$ will be

i) 1
ii) $\frac{a^2-b^2}{a^2+b^2}$
iii) $\frac{a^2+b^2}{a^2-b^2}$
iv) $\frac{2ab}{a^2+b^2}$

Solution:

Given: $\tan\ \theta = \frac{2ab}{a^2-b^2}$
Express $\sin\ \theta$ and $\cos\ \theta$ in terms of $\tan\ \theta$:
We use the identity: $\sin^2\ \theta + \cos^2\ \theta = 1$
Let, $\sin\ \theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
From $\tan\ \theta = \frac{\sin\ \theta}{\cos\ \theta}$, assume a right-angled triangle where:
Opposite side $= 2ab$
Adjacent side $= a^2 - b^2$
Hypotenuse $= a^2 + b^2$ (by Pythagoras' theorem)
Now, find $\cos\ \theta$:

$$\cos\ \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a^2 - b^2}{a^2 + b^2}$$

e) The length of minute hand of a clock is 14 cm. The area swept by minute hand in one minute will be
i) 10.26 cm^2
ii) 10.50 cm^2
iii) 10.75 cm^2
iv) 11.0 cm^2

Solution:

To find the area swept by the minute hand in one minute, we use the formula for the area of a sector:

$$\text{Area of sector } = \pi r^2 \times \frac{\theta}{360^\circ}$$

Given:
Length of the minute hand $r = 14$ cm
In one minute, the minute hand moves by $\theta = \frac{360^\circ}{60^\circ} = 6^\circ$
Now, substituting the values:

$$\text{Area} = \pi \times (14)^2 \times \frac{6^\circ}{360^\circ}$$

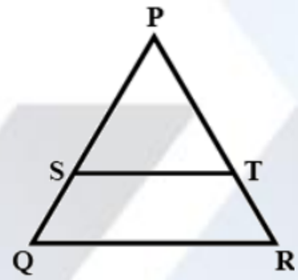
$$= 3.1416 \times 196 \times \frac{1}{60}$$

$$= 3.1416 \times 3.2667$$

$\approx 10.26\ \text{cm}^2$
Thus, the correct answer is $10.26\ \text{cm}^2$.

f) For a frequency distribution, the relation between mean, median and mode is
i) Mode = 3 Mean - 2 Median
ii) Mode = 2 Median - 3 Mean
iii) Mode = 3 Median - 2 Mean
iv) Mode = 3 Median + 2 Mean
Solution:
The correct relation between the mean, median, and mode for a frequency distribution is:
Mode = 3 Median - 2 Mean

2. Do all the parts:
a) In the figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that $\triangle PQR$ is an isosceles triangle.



Solution:
We have, $\frac{\mathrm{PS}}{\mathrm{SQ}} = \frac{\mathrm{PT}}{\mathrm{TR}}$
$\Rightarrow$ ST $\parallel$ QR [By using the converse of Basic Proportionality Theorem]
$\Rightarrow \angle\mathrm{PST} = \angle\mathrm{PQR}$ [Corresponding angles]
$\Rightarrow \angle\mathrm{PRQ} = \angle\mathrm{PQR}$ [$\because \angle\mathrm{PST} = \angle\mathrm{PRQ}$(Given)]
$\Rightarrow$ PQ = PR [$\because$ Sides opposite to equal angles are equal]
$\Rightarrow \triangle$ PQR is isosceles.

b) Find the value of $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$.
Solution:
$$\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$
$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$
$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}}$$
$$= \frac{15 + 64 - 12}{12}$$
$$= \frac{67}{12}$$

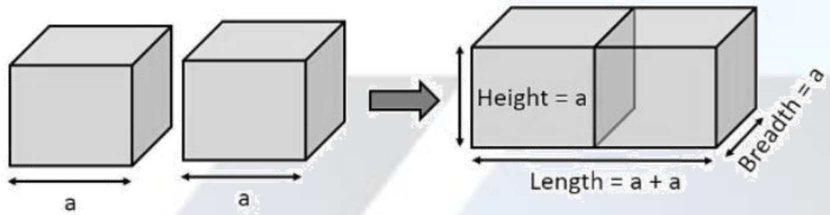
c) Two cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution:

The volume of cube 64 cm^3.

Side of cube = $\sqrt[3]{64} = 4$ cm

Length of resulting cuboid $4 + 4 = 8$ cm



Surface area $= 2(lb + bh + hl)$

$= 2[8(4) + 4(4) + 4(8)]$

$= 2[32 + 16 + 32]$

$= 2[80]$

$= 160\ cm^2$

d) The daily incomes of 50 workers of a factory are given in the following table:

Daily income(in Rs.)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

Find the lower limit and upper limit of the modal class.

Solution:

To find the modal class, we first identify the class interval with the highest frequency from the given data.

Given Data:

Daily Income (Rs.)	100-120	120-140	140-160	160-180	180-200

Number of Workers	12	14	8	6	10

Step 1: Identify the Modal Class
The modal class is the class interval with the highest frequency.
The highest frequency is 14, which corresponds to the class $120 - 140$.
Step 2: Find the Lower and Upper Limits of the Modal Class
Lower limit $= 120$
Upper limit $= 140$
Thus, the modal class is $120 - 140$, with a lower limit of 120 and an upper limit of 140.

3. Do all the parts:
a) Prove that $\sqrt{2}$ is an irrational number.
Solution:
To prove that $\sqrt{2}$ is an irrational number, we will use the contradiction method.
Let us assume that $\sqrt{2}$ is a rational number with p and q as co-prime integers and $q \neq 0$
$\Rightarrow \sqrt{2} = \frac{p}{q}$
On squaring both sides we get, $\Rightarrow 2q^2 = p^2$
Here, $2q^2$ is a multiple of 2 and hence it is even. Thus, p^2 is an even number.
Therefore, p is also even.
So we can assume that $p = 2x$ where x is an integer.
By substituting this value of p in $2q^2 = p^2$, we get:
$\Rightarrow 2q^2 = (2x)^2$
$\Rightarrow 2q^2 = 4x^2$
$\Rightarrow q^2 = 2x^2$
$\Rightarrow q^2$ is an even number. Therefore, q is also even.
Since p and q both are even numbers, they have 2 as a common multiple which means that p and q are not co-prime numbers as their HCF is 2.
This leads to the contradiction that root 2 is a rational number in the form of $\frac{p}{q}$ with "p and q both co-prime numbers" and $q \neq 0$.

b) Solve the following system of equations by elimination method:
$\frac{x}{2} + \frac{2y}{3} + 1 = 0, x - \frac{y}{3} = 3$
Solution:
Given system of equations are:

$\frac{x}{2} + \frac{2y}{3} = -1$ ...(1)
$x - \frac{y}{3} = 3$ ...(2)
From eq 1 will have:
$\Rightarrow 3x + 4y = -6$ ...(3)
From eq 2 will have:
$\Rightarrow 3x - y = 9$ ...(4)
Now using elimination method first we will eliminate y:
Now multiplying eq 4 from 4 will have:
$\Rightarrow 12x - 4y = 36$
Now from eq 3 and 4 will get x:
$3x + 4y = -6$
$12x - 4y = 36$
$\Rightarrow 15x = 30$
$\Rightarrow x = 2$
Now substituting $x = 2$ in eq 2 will get y:
$\Rightarrow (2)3 - y = 9$
$\Rightarrow 6 - y = 9$
$\Rightarrow -y = 9 - 6$
$\Rightarrow y = -3$
$\Rightarrow x = 2, y = -3$

c) Find the ratio in which x-axis divides the line segment joining the points $A(-4,-3)$ and $B(5,2)$.

Solution:

Let the ratio in which the x-axis divides the line segment joining points $A(-4,-3)$ and $B(5,2)$ be $k:1$.
The section formula states that the coordinates of a point dividing a line segment joining two points (x_1, y_1) and (x_2, y_2) in the ratio $k:1$ are:
$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$
Since the x-axis divides the line segment, the y-coordinate of the dividing point must be zero.
Substituting $y_1 = -3$ and $y_2 = 2$:
$$\frac{k(2) + (-3)}{k+1} = 0$$
$2k - 3 = 0$
$2k = 3$
$$k = \frac{3}{2}$$
Therefore, the x-axis divides the line segment in the ratio 3:2.

d) Two solid right circular cones have the same height. The radii of their bases are r_1 and r_2. They are melted and re-cast into a cylinder of same height. Show that the radius of the base of the cylinder is $\sqrt{\frac{r_1^2+r_2^2}{3}}$.

Solution:

Let h be the height of two given cones of base radii r_1 and r_2 respectively. Further, let R be the radius of the cylinder.

It is given that the cylinder is also of height h.

It is also given that the cylinder is casted out of both cones.

Therefore, volume of the cylinder = sum of the volumes of two cones

We know that, volume of a cylinder $= \pi r^2 h$ and volume of a cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \pi \mathrm{R}^2 \mathrm{~h} = \frac{1}{3}\pi \mathrm{r}_1^2 \mathrm{~h} + \frac{1}{3}\pi \mathrm{r}_2^2 \mathrm{~h}$$

$$\Rightarrow \pi \mathrm{R}^2 \mathrm{~h} = \frac{1}{3}\pi \mathrm{~h}(\mathrm{r}_1^2 + \mathrm{r}_2^2)$$

$$\Rightarrow \mathrm{R}^2 = \frac{1}{3}(\mathrm{r}_1^2 + \mathrm{r}_2^2)$$

$$\Rightarrow \mathrm{R} = \sqrt{\frac{(\mathrm{r}_1^2 + \mathrm{r}_2^2)}{3}}$$

Hence proved.

4. Do all the parts:

a) Without performing the long division procedure show that the rational number $\frac{1351}{1250}$ is terminating decimal.

Solution:

To determine whether the rational number $\frac{1351}{1250}$ is a terminating decimal without performing long division, we follow these steps:

Step 1: Check the Prime Factorization of the Denominator

A rational number $\frac{p}{q}$ has a terminating decimal expansion if and only if the denominator q, in its simplest form, has only the prime factors 2 or 5 or both.

First, factorize 1250:

$1250 = 125 \times 10$

$125 = 5^3,\ 10 = 2 \times 5$

$1250 = 2 \times 5^4$

Since the denominator 1250 consists only of the prime factors 2 and 5 , the fraction $\frac{1351}{1250}$ represents a terminating decimal.

b) Find the roots of quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$ by factorization method.

Solution:

$3x^2 - 2\sqrt{6}x + 2 = 0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2}) = 0$ and $(\sqrt{3}x - \sqrt{2}) = 0$

$\Rightarrow (\sqrt{3}x - \sqrt{2})^2 = 0$

$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$

Therefore, the roots are $\left\{\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right\}$.

c) If the points $(a, 0), (0, b)$ and $(1,1)$ are collinear, prove that $\frac{1}{a} + \frac{1}{b} = 1$.

Solution:

The area of a triangle formed by three points $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) is given by:

Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

If the points are collinear, the area is zero.

Let the points be: $A(a, 0)$; $B(0, b)$; $C(1,1)$

Using the area formula:

$\frac{1}{2}|a(b - 1) + 0(1 - 0) + 1(0 - b)| = 0$

Simplify the expression inside the absolute value:

$\frac{1}{2}|a(b - 1) - b| = 0$

$|a(b - 1) - b| = 0$

$a(b - 1) - b = 0$

$ab - a - b = 0$

$ab = a + b$

$1 = \frac{a}{ab} + \frac{b}{ab}$

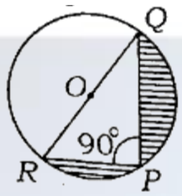
Divide both sides by ab:

$1 = \frac{1}{b} + \frac{1}{a}$

Conclusion: $\frac{1}{a} + \frac{1}{b} = 1$

Thus, we have proved the required result.

d) In the figure, O is the centre of a circle. If $PQ = 24$ cm and $PR = 7$ cm, find the area of shaded portion.



Solution:

We know that, the angle in a semicircle is a right angle.

$\therefore \angle \mathrm{RPQ} = 90^{\circ}$

Thus, $\triangle$ RQP is a right-angled triangle.

Given, $\mathrm{PQ} = 24$ cm, $\mathrm{PR} = 7$ cm

$\therefore$ Using Pythagoras theorem,

$RQ^2 = PR^2 + PQ^2$

$RQ = \sqrt{7^2 + 24^2}$

$\mathrm{RQ} = \sqrt{49 + 576}$

$\mathrm{RQ} = \sqrt{625}$

$\mathrm{RQ} = 25$ cm

Thus, $\mathrm{RQ} = 25$ cm which is the diameter.

Therefore, radius $(r) = \frac{25}{2}$ cm

Area of shaded region = Area of semicircle RPQ - Area of $\triangle RQP$

$= \frac{1}{2} \times \pi r^2 - \frac{1}{2} \times PQ \times RP$

$= \frac{1}{2}[(\frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}) - (24 \times 7)]$

$= \frac{1}{2}\left[\frac{6875}{14} - 168\right]$

$= \frac{1}{2}\left[\frac{(6875 - 2352)}{14}\right]$

$= \frac{1}{2}\left[\frac{(4523)}{14}\right]$

$= \frac{1}{2}\left[\frac{(4523)}{14}\right]$

$= 161.54\ \mathrm{cm}^2$ (approximately)

5. Do all the parts:

a) Show by graphical method that system of linear equations $2x + 4y = 10$ and $3x + 6y = 12$ has no solution.

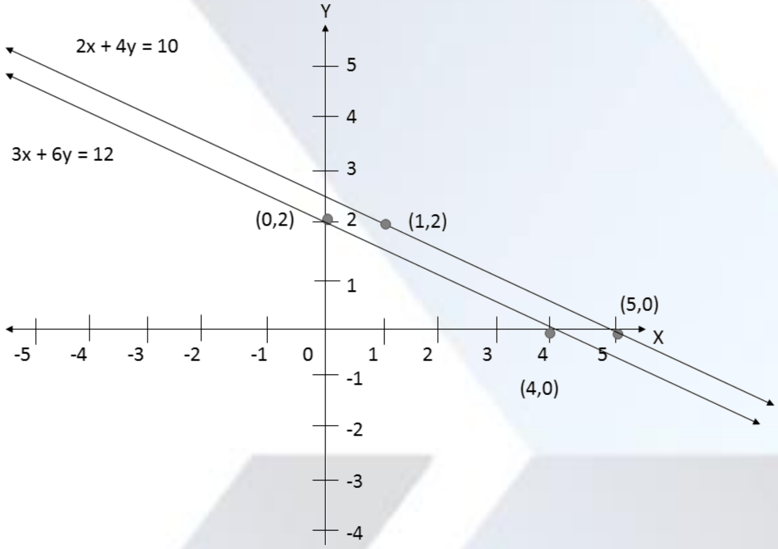
Solution:

Consider, $2x + 4y = 10$

x	5	1
y	0	2

Consider, $3x + 6y = 12$

x	0	4
y	2	0



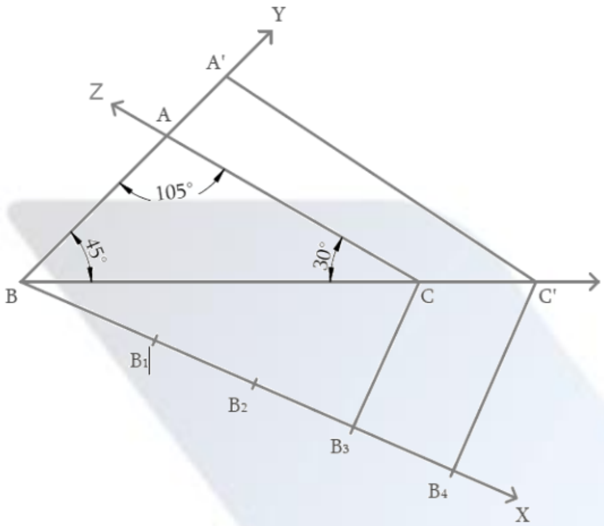
Since the graph consists of a pair of parallel lines, the system of equations has no solution.

Hence proved.

b) Draw a triangle ABC in which $BC = 7$ cm, $\angle B = 45^\circ$ and $\angle A = 105^\circ$. Construct a triangle, similar to triangle ABC, whose sides are $\frac{4}{3}$ times of the corresponding sides of $\triangle ABC$.

Solution:

- Draw the triangle with the given conditions.
- Then draw another line that makes an acute angle with the baseline. Divide the line into m + n parts where m and n are the ratios given.
- Two triangles are said to be similar if their corresponding angles are equal. They are said to satisfy Angle-Angle-Angle (AAA) Axiom.
- The basic proportionality theorem states that "If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally".



Steps of construction:

- Draw BC = 7 cm and at B , make an angle ∠CBY = 45° and at C , make $\angle BCZ = 30°$ [Since, $180° - (45° + 105°)$].
  Both BY and CZ intersect at A and thus $\triangle ABC$ is constructed.
- Draw the ray BX so that $\angle CBX$ is acute.
- Mark 4 (since $4 > 3$ in $\frac{4}{3}$) points B_1, B_2, B_3, B_4 on BX such that $B_1 = B_1B_2 = B_2B_3 = B_3B_4$
- Join B_3 third point on BX, (since $3 < 4$ in $\frac{4}{3}$) to C and draw BC' parallel to BC such that C' lies on the extension of BC.
- Draw C'A' parallel to CA to intersect the extension of BA at A'. Now, triangle $A'BC'$ is the required triangle similar to $\triangle ABC$ where, $\frac{B'A}{BA} = \frac{BC'}{BC} = \frac{C'A'}{CA} = \frac{4}{3}$.

c) i) Prove that $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55° = 1$.

Solution:

Given: $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$

We know that,

$\tan(90 - \theta) = \cot\theta$ and $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan 35° = \tan(90° - 55°) = \cot 55°$

$\tan 40° = \tan(90° - 50°) = \cot 50°$

$\therefore \tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$

$= \cot 55° \cot 50° \tan 45° \tan 50° \tan 55°$

$= \dfrac{1}{\tan 55°} \dfrac{1}{\tan 50°} \tan 45° \tan 50° \tan 55°$

$= \tan 45°$

$= 1$

Hence proved.

ii) In $\triangle ABC$, prove that $\tan \frac{B+C}{2} = \cot \frac{A}{2}$.

Solution:

In $\triangle ABC$,

$A + B + C = 180°$

$B + C = 180° - A$

$\frac{B+C}{2} = 90° - \frac{A}{2}$

$\tan\left(\frac{B+C}{2}\right) = \tan\left(90° - \frac{A}{2}\right)$

$\tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

Hence proved.

d) If the arithmetic mean of the following given table is 25, find the missing frequency:

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	f	6	10	5

Solution:

To find the missing frequency f, we use the formula for the arithmetic mean:

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

Given that the arithmetic mean is 25:

Find the midpoints (x_i) of each class interval:

$x_i = \frac{\text{lower limit} + \text{upper limit}}{2}$

$x_1 = 5, x_2 = 15, x_3 = 25, x_4 = 35, x_5 = 45$

Form the equation using the mean formula:

$25 = \frac{(6 \times 5) + (f \times 15) + (6 \times 25) + (10 \times 35) + (5 \times 45)}{6 + f + 6 + 10 + 5}$

$25 = \frac{30 + 15f + 150 + 350 + 225}{27 + f}$

$25(27 + f) = 755 + 15f$

$675 + 25f = 755 + 15f$

$10f = 80$

$f = 8$

Therefore, the missing frequency $f = 8$.

6. Do all the parts:

a) Determine the positive value of k for which the equations $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will have both real roots.

Solution:

For a quadratic equation to have real roots, discriminant must be greater than or equal to zero.

For the first equation,

$k^2 - 4(1)(64) \geq 0$ ($\because$ discriminant $= b^2 - 4ac$)

$\Rightarrow k^2 - 256 \geq 0$

$\Rightarrow (k - 16)(k + 16) \geq 0$

$\Rightarrow k \geq 16$ and $k \leq -16$

For the second equation,

$64 - 4k \geq 0$

$\Rightarrow k \leq 16$

Therefore, the value of k that satisfies both the conditions is $k = 16$.

b) A man standing on the deck of a ship, water is 14 metre above water level, observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Find the height of the hill and distance of the hill from the ship. (given $\sqrt{3} = 1.732$)

Solution:

Let:

- h be the height of the hill.
- d be the horizontal distance from the ship to the hill.
- The observer's height above water level is 14 m.

Step 1: Using the angle of depression (30°)

For the base of the hill:

$$\tan 30^\circ = \frac{14}{d}$$

$$\frac{1}{\sqrt{3}} = \frac{14}{d}$$

$d = 14\sqrt{3}$

$d = 14 \times 1.732$

$d = 24.248$

Step 2: Using the angle of elevation (60°)

For the top of the hill:

$$\tan\ 60^\circ = \frac{H}{d}$$

$$\sqrt{3} = \frac{H}{24.25}$$

$H = 24.25 \times 1.732 = 42$ m
Step 3: Total height of the hill
$h = H + 14 = 42 + 14 = 56$ m
Therefore, final Answer is:

- Height of the hill $= 56$ m
- Distance of the hill from the ship $= 24.25$ m

c) A solid toy is in the shape of a hemisphere surmounted by a right circular cone. Height of this cone is 2 cm and the diameter of the base is 4 cm . Find the volume of this toy. If a right circular cylinder circumscribes the toy, find the difference of volumes of the cylinder and toy. (Take $\pi = 3.14$)

Solution:

Given Data:

- Hemisphere: Diameter $= 4$ cm $\rightarrow$ Radius $r = 2$ cm
- Cone: Height $h = 2$ cm, Radius $r = 2$ cm
- Circumscribing Cylinder: Same radius $r = 2$ cm, Height $=$ Total height of toy $=$ Hemisphere's radius + Cone's height $= 2 + 2 = 4$ cm

Step 1: Volume of the Toy
Volume of Hemisphere:
$$V_h = \frac{2}{3}\pi r^3 = \frac{2}{3} \times 3.14 \times 2^3 = \frac{2}{3} \times 3.14 \times 8 = 16.75 \text{ cm}^3$$
Volume of Cone:
$$V_c = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 2^2 \times 2 = \frac{1}{3} \times 3.14 \times 8 = 8.37 \text{ cm}^3$$
Total Volume of Toy:
$$V_t = V_h + V_c = 16.75 + 8.37 = 25.12 \text{ cm}^3$$
Step 2: Volume of the Circumscribing Cylinder
$$V_{cy} = \pi r^2 H = 3.14 \times 2^2 \times 4 = 3.14 \times 4 \times 4 = 50.24 \text{ cm}^3$$
Step 3: Difference in Volume
$$V_{diff} = V_{cy} - V_t = 50.24 - 25.12 = 25.12 \text{ cm}^3$$
Therefore, final Answer is:

- Volume of the toy $= 25.12$ cm^3
- Difference of volumes $= 25.12$ cm^3

d) The marks obtained by the students of a class in Mathematics are as following:

Marks obtained	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of students	2	10	12	15	13	8	4

Find the meadin.

Solution:

To find the median, follow these steps:

Given data:

Marks Range	No. of Students (Frequency)
$0-10$	2
$10-20$	10
$20-30$	12
$30-40$	15
$40-50$	13
$50-60$	8
$60-70$	4

Step 1: Find the cumulative frequency (CF)

Marks Range	Frequency (f)	Cumulative Frequency (CF)
$0-10$	2	2
$10-20$	10	12
$20-30$	12	24
$30-40$	15	39
$40-50$	13	52
$50-60$	8	60
$60-70$	4	64

Step 2: Find the Median Class

Total students $= 64$

Median position $= \frac{64}{2} = 32^{\text{nd}}$ student

The median class is $30-40$, as CF just before it is 24 and next is 39 .

Step 3: Apply Median Formula

$$\text{Median } = L + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$$

Where:

- $L = 30$ (Lower boundary of median class)
- $N = 64$ (Total frequency)
- $CF = 24$ (Cumulative frequency before median class)
- $f = 15$ (Frequency of median class)
- $h = 10$ (Class width)

$$\text{Median} = 30 + \left(\frac{32 - 24}{15} \times 10\right)$$

$$= 30 + \left(\frac{8}{15} \times 10\right)$$

$$= 30 + 5.33$$

$$= 35.33$$

Therefore, the median is 35.33 marks.

7. Do all the parts:

a) A boat, whose speed in still water is 5 km/hr, takes 1 hour more time to go 12 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Let the speed of the stream be x km/hr.

Given data:

- Speed of the boat in still water = 5 km/hr
- Distance = 12 km
- Time taken upstream = Time taken downstream + 1 hour

Formulating equations:

- Speed upstream = $(5 - x)$ km/hr
- Speed downstream = $(5 + x)$ km/hr
- Time = Distance / Speed

So,

$$\frac{12}{5 - x} = \frac{12}{5 + x} + 1$$

$$\frac{12}{5 - x} - \frac{12}{5 + x} = 1$$

$$\frac{12(5 + x) - 12(5 - x)}{(5 - x)(5 + x)} = 1$$

$$\frac{60 + 12x - 60 + 12x}{25 - x^2} = 1$$

$$\frac{24x}{25-x^2}=1$$
$$24x=25-x^2$$
$$x^2+24x-25=0$$
Using the quadratic formula:
$$x=\frac{-24\pm\sqrt{(24)^2-4(1)(-25)}}{2(1)}$$
$$x=\frac{-24\pm\sqrt{576+100}}{2}$$
$$x=\frac{-24\pm\sqrt{676}}{2}$$
$$x=\frac{-24\pm 26}{2}$$
Possible values:
$$x=\frac{-24+26}{2}=\frac{2}{2}=1$$
$x=\frac{-24-26}{2}=\frac{-50}{2}=-25$ (Not possible, as speed connot be negative)
Thus, speed of the stream $=1$ km/hr.

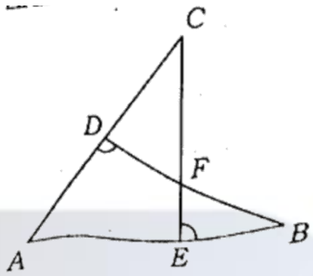
OR

Find the nature of the roots of the quadratic equation $2x^2-6x+3=0$. If the roots are real, find them.

Solution:

To determine the nature of the roots of the quadratic equation $2x^2-6x+3=0$, we use the discriminant formula:
$$D=b^2-4ac$$
For the given equation $2x^2-6x+3=0$:
$$a=2,\ b=-6,\ c=3$$
$$D=(-6)^2-4(2)(3)=36-24=12$$
Since $D>0$, the roots are real and distinct.
The roots are given by:
$$x=\frac{-b\pm\sqrt{D}}{2a}$$
$$x=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm 2\sqrt{3}}{4}=\frac{3\pm\sqrt{3}}{2}$$
Thus, the roots are real and distinct, given by: $x=\frac{3+\sqrt{3}}{2},\ x=\frac{3-\sqrt{3}}{2}$

b) i) In the figure, if $BD\perp AC$ and $CE\perp AB$, prove that $\Delta AEC\backsim\Delta ADB$.



Solution:

In $\triangle AEC$ and $\triangle ADB$, we have:

$\angle AEC = \angle ADB = 90^\circ [\because CE \perp AB$ and $BD \perp AC]$ and,

$\angle EAC = \angle DAB$ [Each equal to $\angle A$]

Therefore, by AA-criterion of similarity, we have,

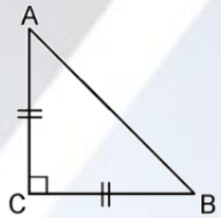
$\triangle AEC \sim \triangle ADB$

Hence proved.

ii) $\triangle ABC$ is an isosceles triangle in which $AC = BC$. If $AB^2 = 2AC^2$, prove that $\triangle ABC$ is a right triangle.

Solution:

We know that, in a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.



In $\triangle ABC$,

It is given that $AC = BC$ and $AB^2 = 2AC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

$\Rightarrow AB^2 = AC^2 + BC^2$[Since $AC = BC$]

As the above equation satisfies Pythagoras theorem, we can say that:

$\Rightarrow \angle ACB = 90^\circ$

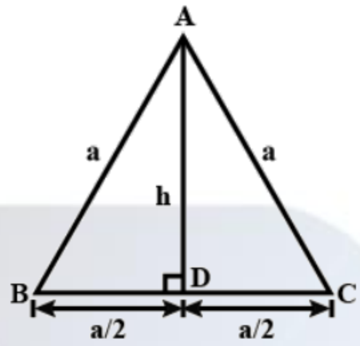
Therefore, ΔABC is a right triangle.

OR

i) In equilateral triangle ABC, whose side is a, prove that its altitude is $\frac{a\sqrt{3}}{2}$.

Solution:

In ΔABD,



$AB^2 = AD^2 + BD^2$

$a^2 = h^2 + \left(\frac{a}{2}\right)^2$

$h^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

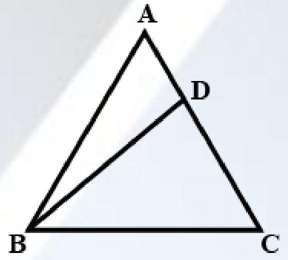
$h = \frac{a\sqrt{3}}{2}$

Hence proved.

ii) The point D is on the side AC of $\triangle ABC$ such that $\angle ACB = \angle ABD$. Prove that $\triangle ABC \sim \triangle ADB$.

Solution:

Given that $\angle ACB = \angle ABD$, we need to prove that $\triangle ABC \sim \triangle ADB$.



Angle Equality:

Given $\angle ACB = \angle ABD$.

$\angle BAC$ is common in both $\triangle ABC$ and $\triangle ADB$.

Angle-Angle (AA) Similarity Criterion:

Since two corresponding angles are equal, $\triangle ABC \sim \triangle ADB$ by the AA similarity criterion.

Hence proved.