Secondary Examination, 2023
Mathematics

Time: $3\frac{1}{4}$ hours **M.M-70**

Instructions:

i) All questions are compulsory.
ii) This question paper has two sections 'A' and 'B'.
iii) Section 'A' contains 20 multiple choice type questions of 1 mark each that has to be answered on OMR Answer Sheet by darkening completely the correct circle with blue or black ballpoint pen.
iv) After giving answer on OMR Answer Sheet do not cut or use eraser, whitener etc.
v) Section 'B' contains descriptive type questions of 50 marks.
vi) Total 5 questions are there in this section.
vii) In the beginning of each question it has been mentioned how many parts of it are to be attempted.
viii) Marks allotted to each question are mentioned against it.
ix) Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
x) If you need place for rough work, do it on left page of your answer book and cross ($\times$) the page. Do not write the solution on that page.
xi) Do not rub off the lines constructed in a question of construction. Do write the steps of construction in brief, if asked.
xii) Draw neat and correct figure in solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A
(Multiple Choice Type Questions)

1. Which one is a pair of co-prime numbers?
(A) $(18,25)$
(B) $(5,15)$
(C) $(7,21)$
(D) $(31,93)$
Solution:
Two numbers are co-prime if they have no common factors other than 1, meaning their Greatest Common Divisor (GCD) is 1.
Let's check each option:
(A) $\text{GCD}(18,25) = 1$ (Co-prime)
(B) $\text{GCD}\ (5,15) = 5$ (Not co-prime)

(C) GCD $(7,21) = 7$ (Not co-prime)
(D) GCD $(31,93) = 31$ (Not co-prime)
Therefore, the correct option is: $(18,25)$

2. The sum of the powers of prime factors in prime factorization of 144 is
(A) 5
(B) 6
(C) 7
(D) 8
Solution:
Find the Prime Factorization of 144:
$144 = 2^4 \times 3^2$
Here, the prime factors are **2** and **3**, with exponents **4** and **2**, respectively.
So, sum of the powers $= 4 + 2 = 6$
Thus, the sum of the powers of prime factors in the prime factorization of 144 is 6.

3. For what value of 'm', pair of equations $x - 2y = 3$ and $3x + \mathrm{m}y = 1$ will have unique solution?
(A) $\mathrm{m} = -6$
(B) $\mathrm{m} = 0$ only
(C) $\mathrm{m} \neq -6$
(D) $\mathrm{m} \neq 0$
Solution:
For unique solution:
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$
$$\Rightarrow \frac{1}{3} \neq \frac{-2}{\mathrm{m}}$$
$$\Rightarrow \mathrm{m} \neq -6$$

4. The solution of $2x + 3y = 18; x - 2y = 2$ will be
(A) $x = 6, y = 2$
(B) $x = 3, y = 4$
(C) $x = 3, y = 8$
(D) $x = 0, y = 6$
Solution:
Given equations:
$2x + 3y = 18$ ...(i)
$x - 2y = 2$ ...(ii)
From equation (ii):

$x = 2 + 2y$ ...(iii)
Substitute the above equation in equation (i):
$2(2 + 2y) + 3y = 18$
$4 + 4y + 3y = 18$
$7y = 14$
$y = 2$
From equation (iii),
$x = 2 + 2(2) = 6$
Therefore, the correct answer is: $x = 6, y = 2$

5. For which value of k, there will be an infinite number of solutions for the pair of linear equations $x + ky = 1$ and $kx + y = k^2$?
(A) +1
(B) ± 1
(C) -1
(D) 5
Solution:
For the system of equations to have an infinite number of solutions, the two equations must be dependent, meaning their coefficients must be proportional:
$$\frac{1}{k} = \frac{k}{1} = \frac{1}{k^2}$$
Solving the above euqation, we get:
$$\frac{1}{k} = \frac{k}{1}$$
$\Rightarrow 1 \times 1 = k \times k$
$\Rightarrow k^2 = 1$
$\Rightarrow k = \pm 1$
Thus, the correct option is ± 1.

6. If $\frac{1}{x^2-2} = \frac{1}{7}$, then the value of x will be
(A) ± 2
(B) ± 1
(C) ± 3
(D) ± 5
Solution:
We have the equation:
$$\frac{1}{x^2 - 2} = \frac{1}{7}$$
After doing the cross multiplication, we get:
$x^2 - 2 = 7$

$x^2 = 9$
$x = \pm 3$

7. The centre $(2, -3)$ of a circle has diameter as AB. The coordinate of B is $(1,4)$. The coordinate of A will be:
(A) $(3,10)$
(B) $(10,3)$
(C) $(-10,3)$
(D) $(3,-10)$
Solution:
The centre C of the circle is the midpoint of diameter AB.
Given:
Centre $C(2,-3)$,
Point $B(1,4)$.
Using the midpoint formula:
$$C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
Substituting values:
$$(2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$
Solving for x and y:
$$2 = \frac{x+1}{2}$$
$\Rightarrow x + 1 = 4$
$\Rightarrow x = 3$
Also, $-3 = \frac{y+4}{2}$
$\Rightarrow y + 4 = -6$
$\Rightarrow y = -10$
Thus, $A(3,-10)$

8. If roots of the equation $3x^2 - 12x + k = 0$ are equal, then value of k will be
(A) 12
(B) 4
(C) 7
(D) 9
Solution:
For the quadratic equation $ax^2 + bx + c = 0$, the roots are equal if the discriminant is zero.
Given equation: $3x^2 - 12x + k = 0$
Step 1: Find the discriminant:

$\Delta = b^2 - 4ac$
$= (-12)^2 - 4(3)(k)$
$= 144 - 12k$
Step 2: Set discriminant to zero:
$144 - 12k = 0$
$12k = 144$
$k = 12$

9. Two figures whose shapes are same but the dimensions are not essentially same are called
(A) equal figures
(B) similar figures
(C) symmetrical figures
(D) congruent figures
Solution:
Two figures are **similar** if they have the same shape but not necessarily the same size. This means their corresponding angles are equal, and their corresponding sides are in proportion.
So, the correct answer is: similar figures

10. If the corresponding sides of two similar triangles are in the ratio $3:5$, the ratio of their areas will be
(A) $9:25$
(B) $6:10$
(C) $3:5$
(D) $25:9$
Solution:
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.
Given that the ratio of corresponding sides is $3:5$, the ratio of their areas will be:
$$\left(\frac{3}{5}\right)^2 = \frac{9}{25}$$
Thus, the correct answer is 9:25.

11. If the sides of a triangle are 3 cm, 4 cm and 5 cm. then the triangle will be
(A) right angle triangle
(B) acute angle triangle
(C) obtusc angle triangle
(D) triangle is not possible
Solution:

To determine the type of triangle with sides 3 cm, 4 cm, and 5 cm, we use the Pythagorean theorem: $a^2 + b^2 = c^2$
where c is the longest side (hypotenuse).
Step 1: Check the Pythagorean theorem
$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$
Since the equation holds true, the given triangle satisfies the Pythagorean theorem, meaning it is a **right-angled** triangle.

12. The perimeter of two similar triangles arc 10 cm and 15 cm respectively, find ratio of their areas.
(A) $3:4$
(B) $4:9$
(C) $3:2$
(D) $2:1$
Solution:
For two similar triangles, the ratio of their perimeters is equal to the ratio of their corresponding sides. Let the ratio of their corresponding sides be:
$$\frac{\text{Perimeter of first triangle}}{\text{Perimeter of second triangle}} = \frac{10}{15} = \frac{2}{3}$$
We know that, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides:
$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$
Thus, the ratio of their areas is $4:9$.

13. If $\tan\,\theta = \frac{8}{15}$, then the value of cosecθ will be
(A) $\frac{17}{8}$
(B) $\frac{8}{17}$
(C) $\frac{4}{3}$
(D) $\frac{15}{17}$
Solution:
Given: $\tan\theta = \frac{8}{15} = \frac{\text{Perpendicular}}{\text{Base}}$
Using Pythagoras theorem, Hypotenuse = $\sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17$
So, $\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{17}{8}$

14. If $2\cos\ 3\theta = 1$, then value of θ will be:
(A) 10°
(B) 15°
(C) 20°
(D) 25°
Solution:
Given: $2\cos\ 3\theta = 1$
$\cos\ 3\theta = \frac{1}{2}$
$\cos\ 3\theta = \cos 60^{\circ}$
Comparing the equation both side, we get:
$3\theta = 60^{\circ}$
$\theta = 20^{\circ}$

15. The circumferences of two circles are in the ratio $3:2$, then the ratio of their areas will be:
(A) $7:9$
(B) $4:9$
(C) $2:3$
(D) $9:4$
Solution:
We know that the circumference of a circle is given by: $C = 2\pi r$
Let the radii of the two circles be r_1 and r_2. Given that their circumferences are in the ratio $3:2$, we write:
$\frac{2\pi r_1}{2\pi r_2} = \frac{3}{2}$
$\frac{r_1}{r_2} = \frac{3}{2}$
The area of a circle is given by: $A = \pi r^2$
Thus, the ratio of their areas is: $\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$
So, the correct answer is 9:4.

16. If two hemispheres of equal radius 'r' are joined by their bases, then the curved surface area of this new solid will be
(A) $4\pi r^2$
(B) $8\pi r^2$
(C) $6\pi r^2$
(D) $2\pi r^2$
Solution:
The curved surface area (CSA) of a hemisphere is given by:

CSA of one hemisphere $= 2\pi r^2$

Since two hemispheres are joined at their bases, the total curved surface area of the new solid is:

$2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Thus, the correct answer is $4\pi r^2$.

17. Which one of the following is not a measure of central tendency?

(A) Mean

(B) Median

(C) Mode

(D) Standard deviation

Solution:

Standard deviation is not a measure of central tendency; it is a measure of dispersion.

18. The arithmetic mean of natural numbers 1 to 9 will be-

(A) 5

(B) 4

(C) 3

(D) 6

Solution:

The arithmetic mean (average) of natural numbers from 1 to 9 is calculated as:

$$\text{Mean } = \frac{\text{Sum of terms}}{\text{Number of terms}}$$

Sum of numbers from 1 to 9:

$1+2+3+4+5+6+7+8+9=45$

Number of terms $= 9$

$\text{Mean} = \frac{45}{9} = 5$

19. In a given frequency distribution. if the mean is 15 and median is 16, then its mode will be

(A) 16

(B) 18

(C) 15

(D) 17

Solution:

We use the empirical relation between mean, median, and mode:

$\text{Mode} = 3 \times \text{ Median} - 2 \times \text{Mean}$

Substituting the given values:

$\text{Mode} = 3(16) - 2(15) = 48 - 30 = 18$

Thus, the correct answer is 18.

20. Median of the first 10 natural numbers will be
(A) 5
(B) 5.2
(C) 5.4
(D) 5.5
Solution:
The first 10 natural numbers are: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$
Since the total count is even (10), the median is the average of the 5th and 6th terms:
Median $=\frac{5+6}{2}=\frac{11}{2}=5.5$

Section - B
(Descriptive questions)

1. Do all the parts:
(a) Prove that $5+\sqrt{3}$ is an irrational number.
Solution:
To prove that $5+\sqrt{3}$ is irrational, assume it is rational.
Proof by Contradiction:
Suppose $5+\sqrt{3}$ is rational.
Let $5+\sqrt{3}=\frac{p}{q}$, where p, q are integers and $q \neq 0$.
Rearranging the equation:
$$\sqrt{3}=\frac{p}{q}-5$$
$$\sqrt{3}=\frac{p-5q}{q}$$
Since p and q are integers, $\frac{p-5q}{q}$ is rational.
But $\sqrt{3}$ is irrational, which contradicts the assumption.
Conclusion:
Since our assumption leads to a contradiction, $5+\sqrt{3}$ must be irrational.

(b) If $\cos A=\frac{\sqrt{3}}{2}$, then find value of $\sin 2A$.
Solution:
Given: $\cos A=\frac{\sqrt{3}}{2}$
Step 1: Find $\sin A$
Since $\sin^2 A+\cos^2 A=1$,

$\sin^2 A = 1 - \left(\frac{\sqrt{3}}{2}\right)^2$

$\sin^2 A = 1 - \frac{3}{4} = \frac{1}{4}$

$\sin A = \frac{1}{2}$ (assuming A is in first or second quadrant)

Step 2: Use the identity for $\sin 2A$

$\sin 2A = 2\sin A\cos A$

$\sin 2A = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$\sin 2A = \frac{\sqrt{3}}{2}$

(c) Prove that the twice of the volume of a cylinder is equal to the product of its radius of base and curved surface.

Solution:

We need to prove that: 2 × Volume of Cylinder = Radius × Curved Surface Area

Step 1: Formula for Volume of Cylinder

The volume of a cylinder is given by: $V = \pi r^2 h$

So, twice the volume: $2V = 2\pi r^2 h$

Step 2: Formula for Curved Surface Area (CSA)

The curved surface area of a cylinder is: $\text{CSA} = 2\pi rh$

Step 3: Multiply Radius and CSA

$r \times \text{CSA} = r \times 2\pi rh = 2\pi r^2 h$

Conclusion: $2V = r \times \text{CSA}$

Thus, the twice of the volume of the cylinder is equal to the product of the radius and the curved surface area.

(d) Find the median of following data:

Class Interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	2	8	30	15	5

Solution:

To find the median of the given data, follow these steps:

Given Data:

Class Interval	Frequency (f)	Cumulative Frequency (CF)
$0 - 10$	2	2
$10 - 20$	8	10

$20-30$	30	40
$30-40$	15	55
$40-50$	5	60

Steps:
Find Total Frequency (N):
$N=2+8+30+15+5=60$
Find Median Class:
$\frac{N}{2}=\frac{60}{2}=30$
The cumulative frequency just greater than 30 is 40 , corresponding to the class $20-30$.
So, median class $=20-30$.

Use the Median Formula: Median $=L+\left(\frac{\frac{N}{2}-CF}{f}\right)\times h$

Where:

- $L=20$ (lower boundary of median class)
- $N=60$ (total frequency)
- $CF=10$ (cumulative frequency before median class)
- $f=30$ (frequency of median class)
- $h=10$ (class width)

Substituting Values:
Median $=20+\left(\frac{30-10}{30}\times 10\right)$
$=20+\left(\frac{20}{30}\times 10\right)$
$=20+\left(\frac{200}{30}\right)$
$=20+6.67$
$=26.67$

(e) Find the coordinates of the point which divides the line segment formed by joining the points $(-1,7)$ and $(4,-3)$ in the ratio $2:3$.
Solution:
The section formula states that the coordinates of a point dividing the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio $m:n$ are:
$\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$
Given points: $A(-1,7),B(4,-3)$
Ratio: $2:3$ (i.e., $m=2,n=3$)

Applying the formula:

$x = \frac{(2 \times 4) + (3 \times -1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$

$y = \frac{(2 \times -3) + (3 \times 7)}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$

Thus, the required point is $(1,3)$.

(f) For what value of K, points $(K, -1), (2,1)$ and $(4,5)$ will be on same line?

Solution:

The given points $(K, -1), (2,1)$, and $(4,5)$ will be collinear if the area of the triangle formed by them is zero.

The area of a triangle formed by three points $(x_1, y_1), (x_2, y_2)$, and (x_3, y_3) is given by:

$\text{Area} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Substituting the given points:

$\frac{1}{2}|K(1 - 5) + 2(5 + 1) + 4(-1 - 1)| = 0$

$\frac{1}{2}|K(-4) + 2(6) + 4(-2)| = 0$

$\frac{1}{2}| - 4K + 12 - 8| = 0$

$\frac{1}{2}| - 4K + 4| = 0$

$-4K + 4 = 0$

$K = 4$

Thus, the required value of K is **1**.

2. Answer any five of the following parts:

(a) Show through graphical method that linear equation system $3x - y = 2$ and $9x - 3y = 6$ have infinite number of solutions.

Solution:

To show that the system of equations:

$3x - y = 2$ ...(i)

$9x - 3y = 6$ ...(ii)

has infinite solutions using the graphical method, follow these steps:

Step 1: Convert to Slope-Intercept Form (y = mx + c)

Rearrange both equations:

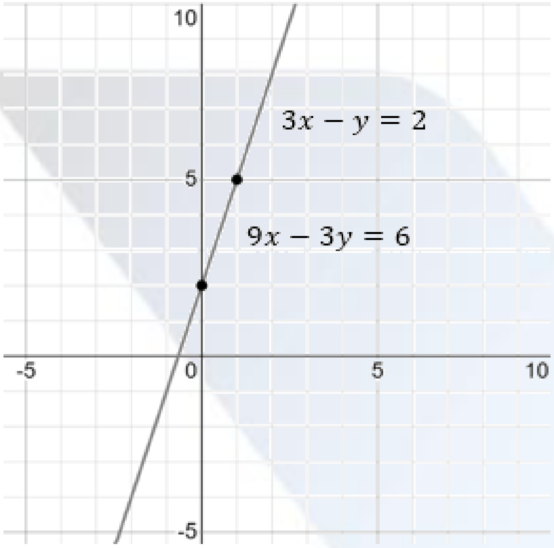
$y = 3x - 2$ ....(iii)

$3y = 9x - 6$

$y = 3x - 2$ ....(iv)

Both equations are identical, meaning they represent the same line.

Step 2: Graphing the Equations
Since both equations represent the same line, every point on this line is a solution. When graphed, they overlap completely, confirming an infinite number of solutions.



Since both equations are the same, the system is dependent and has **infinitely many solutions**.

(b) The sum of the digits of a two digit number is 9. If the digits of the number are interchanged then the new number will exceed the original number by 27. Find the number.

Solution:

Let the two-digit number be $10x + y$, where x is the tens digit and y is the units digit.

Given conditions:

$x + y = 9$ ...(i)

When the digits are interchanged, the new number is $10y + x$.

$10y + x = (10x + y) + 27$

Simplifying:

$10y + x - 10x - y = 27$

$9y - 9x = 27$

$y - x = 3$

$y = x + 3$ ...(ii)

From (i) and (ii):

$x + (x + 3) = 9$

$2x + 3 = 9$

$2x = 6$

$x = 3$

Therefore, $y = \text{x} + 3 = 3 + 3 = 6$

Thus, the number is 36.

(c) Construct $\triangle$ ABC in which BC $= 6$ cm, AB $= 3$ cm and $\angle$ABC $= 45°$. Construct a similar triangle 'ABC' whose corresponding sides are $\frac{3}{4}$ of sides of $\triangle ABC$. Write the steps of construction in brief.

Solution:

Steps to Construct $\triangle ABC$:

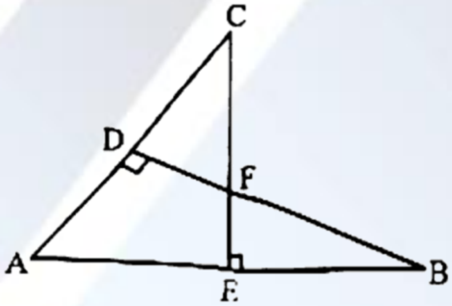
1. Draw $BC = 6$ cm.
2. Construct $\angle$ABC $= 45°$ using a protractor.
3. Mark point A on the ray such that $AB = 3$ cm.
4. Join AC to complete $\triangle ABC$.

Steps to Construct Similar $\Delta A'B'C'$ (Scale Factor $= \frac{3}{4}$):

5. Draw a ray BX making an acute angle with BC.
6. Mark 4 equal points on BX (since 4 is the greater denominator in $3/4$).
7. Join B4C (4th point to C).
8. Draw $B'C'$ parallel to $B4C$ (since 3 is the numerator in $3/4$).
9. From C', draw $A'C'$ parallel to AC.
10. Join $A'B'$ to form the required $\triangle A'B'C'$.

Thus, $\triangle A'B'C'$ is the required similar triangle with a scale factor of $\frac{3}{4}$ of $\triangle ABC$.

(d) In the figure, if $BD \perp AC$ and $CE \perp AB$, prove that $\triangle AEC \sim \triangle ADB$.



Solution:

(i) In $\triangle$AEC and $\triangle$ADB , we have:

$\angle$AEC $= \angle$ADB $= 90°$[$\because$ CE $\perp$ AB and BD $\perp$ AC] and,

$\angle$EAC $= \angle$DAB [Each equal to $\angle$A]

Therefore, by AA-criterion of similarity, we have,

$\triangle$ AEC $\sim \triangle$ ADB

Hence proved.

(e) Find the arithmetic mean from following frequency table :

Class interval	$0-10$	$10-20$	$20-30$	$30-40$	$40-50$

Frequency	5	12	25	10	8

Solution:

To find the mean, use the formula:

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

Where:

x_i = Midpoint of each class = (Lower limit + Upper limit)/2

f_i = Frequency of the class

Class Interval	Frequency (f_i)	Midpoint (x_i)	$f_i x_i$
$0 - 10$	5	5	25
$10 - 20$	12	15	180
$20 - 30$	25	25	625
$30 - 40$	10	35	350
$40 - 50$	8	45	360

$$\sum f_i = 5 + 12 + 25 + 10 + 8 = 60$$

$$\sum f_i x_i = 25 + 180 + 625 + 350 + 360 = 1540$$

Therefore, mean $= \frac{1540}{60} = 25.67$

(f) Find the mode of the following data:

Class interval	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Frequency	7	8	2	2	1

Solution:

To find the mode:

Identify the modal class:

The class with the highest frequency is $3 - 5$ (frequency $= 8$).

Use the mode formula: Mode $= L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

where:

- $L = 3$ (lower boundary of modal class)
- $f_1 = 8$ (modal class frequency)
- $f_0 = 7$ (preceding class frequency)
- $f_2 = 2$ (succeeding class frequency)
- $h = 2$ (class width)

Substitute values:

$$\text{Mode} = 3 + \left(\frac{8-7}{2(8)-7-2}\right) \times 2$$
$$= 3 + \left(\frac{1}{16-9}\right) \times 2$$
$$= 3 + \left(\frac{1}{7}\right) \times 2$$
$$= 3 + \frac{2}{7}$$
$$= 3.29$$
Thus, the mode is approximately 3.29.

3. Solve the following pair of equations by converting these into linear pair of equations:
$\frac{1}{2x} - \frac{1}{3y} = 2; \frac{1}{3x} + \frac{1}{2y} = \frac{13}{16}$.

Solution:

Given: $\frac{1}{2x} - \frac{1}{3y} = 2; \frac{1}{3x} + \frac{1}{2y} = \frac{13}{16}$

Let: $u = \frac{1}{x}$, $v = \frac{1}{y}$

Thus, the equations transform into:

$\frac{u}{2} - \frac{v}{3} = 2$ ...(i)

$\frac{u}{3} + \frac{v}{2} = \frac{13}{16}$ ...(ii)

Multiply the first equation by 6 and the second equation by 6:

$3u - 2v = 12$ ...(iii)

$2u + 3v = \frac{39}{8}$ ...(iv)

Multiply the equation (iii) by 3 and equation (iv) by 2:

$$9u - 6v = 36$$
$$4u + 6v = \frac{78}{8}$$

Adding both equations:

$$13u = 36 + \frac{78}{8}$$
$$13u = \frac{288}{8} + \frac{78}{8} = \frac{366}{8}$$
$$u = \frac{366}{104} = \frac{61}{26}$$

Substituting $u = \frac{61}{26}$ in $3u - 2v = 12$:

$$3 \times \frac{61}{26} - 2v = 12$$
$$\frac{183}{26} - 2v = 12$$

$$-2v = 12 - \frac{183}{26} = \frac{312 - 183}{26} = \frac{129}{26}$$

$$v = -\frac{129}{52}$$

Now, find x and y.

Since $u = \frac{1}{x}$ and $v = \frac{1}{y}$:

$$x = \frac{26}{61},\ y = -\frac{52}{129}$$

Thus, the solution is: $x = \frac{26}{61},\ y = -\frac{52}{129}$

OR

Sum of areas of two squares is $117\ \text{m}^2$. If difference between their perimeters is 12 m, then find the sides of both the squares.

Solution:

Let the sides of the two squares be x m and y m, where $x > y$.

Given:

Sum of areas: $x^2 + y^2 = 117$

Difference of perimeters: $4x - 4y = 12$

Simplifying: $x - y = 3$

From $x - y = 3$, express x in terms of y:

$x = y + 3$

Substituting in the area equation:

$(y + 3)^2 + y^2 = 117$

$y^2 + 6y + 9 + y^2 = 117$

$2y^2 + 6y + 9 = 117$

$2y^2 + 6y - 108 = 0$

$y^2 + 3y - 54 = 0$

Factorizing the above equation, we get:

$(y + 9)(y - 6) = 0$

So, $y = -9$ (not possible) or $y = 6$.

Thus, $y = 6,\ x = 9$

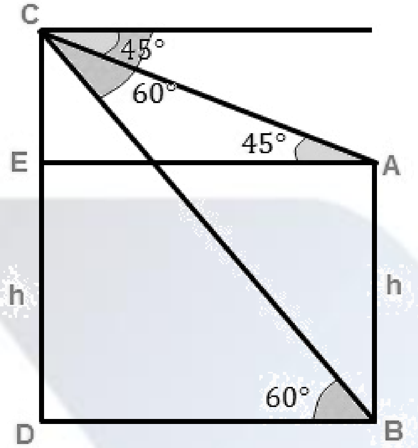
Therefore, the sides of the squares are 9 m and 6 m.

4. From the top of a tower 60 metre high, the angles of depression of top and bottom of a building (house) are 45° and 60° respectively find the height of the building and its distance from the tower.

Solution:

Let the height of the tower (CD) be 60 m, and let the height of the building (AB) be h m.

Let the distance between the tower and the building be **d** m.



Step 1: Using the angle of depression for the top of the building (45°)
In right-angled triangle formed by the top of the tower, top of the building, and the ground:

$$\tan\ 45^\circ = \frac{(60 - h)}{d}$$

Since $\tan\ 45^\circ = 1$, we get:

$60 - h = d\ \Rightarrow\ h = 60 - d$ (Equation 1)

Step 2: Using the angle of depression for the bottom of the building (60°)
In right-angled triangle formed by the top of the tower, bottom of the building, and the ground:

$$\tan\ 60^\circ = \frac{60}{d}$$

Since $\tan\ 60^\circ = \sqrt{3}$, we get:

$$\sqrt{3} = \frac{60}{d}$$

$$\Rightarrow d = \frac{60}{\sqrt{3}}$$

$$\Rightarrow d = 20\sqrt{3}$$

Step 3: Finding the height of the building
Substituting $\mathrm{d} = 20\sqrt{3}$ into Equation 1: $h = 60 - 20\sqrt{3}$
Approximating $20\sqrt{3} \approx 34.64$:
$h \approx 60 - 34.64 = 25.36$ m
Therefore,
Height of the building ≈ 25.36 m
Distance from the tower ≈ 34.64 m

OR

Two poles of equal height are placed opposite to each other on either side of a road 80 metre wide. From a point on the road between these two poles, the angles of elevation of the top of the poles are $60°$ and $30°$ respoctively. Find the height of the poles and the distance of the point from the poles.

Solution:

Let the height of each pole be h metres and the width of the road be 80 metres. Let the point on the road be at distances x metres and $(80 - x)$ metres from the poles.

Using the tangent formula:

$\tan\ 60° = \frac{h}{x}$ and $\tan\ 30° = \frac{h}{80 - x}$

Since $\tan\ 60° = \sqrt{3}$ and $\tan\ 30° = \frac{1}{\sqrt{3}}$:

$\sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

$\frac{1}{\sqrt{3}} = \frac{h}{80 - x}$

$\Rightarrow h = \frac{(80 - x)}{\sqrt{3}}$

Equating both expressions for h:

$\sqrt{3}x = \frac{80 - x}{\sqrt{3}}$

$3x = 80 - x$

$4x = 80$

$x = 20$

Substituting $x = 20$ in $h = \sqrt{3}x$:

$h = 20\sqrt{3} \approx 34.64$ meters

Thus, height of each pole $= 20\sqrt{3}$ m

Distances from the point $= 20$ m and 60 m

5. A well of 3 metre in diameter is dug to a depth of 14 meter. An embankment is made by spreading the soil out of it, making a circular ring of 4 meters wide, evenly around the well. Find the height of the embankment.

Solution:

Given:

Diameter of well $= 3$ m $\rightarrow$ Radius $= 1.5$ m

Depth of well $= 14$ m

Width of embankment $= 4$ m

Step 1: Volume of soil dug out (cylindrical well)

$V_{\text{well}} = \pi r^2 h = \pi(1.5)^2(14) = \pi \times 2.25 \times 14 = 31.5\pi\ \text{m}^3$

Step 2: Volume of embankment (ring-shaped cylinder)

Outer radius $= (1.5 + 4) = 5.5$ m

Inner radius $= 1.5$ m

Let **h** be the height of the embankment.

$V_{\text{embankment}} = \pi(R^2 - r^2)h = \pi(5.5^2 - 1.5^2)h$
$= \pi(30.25 - 2.25)h = \pi(28)h$
Step 3: Equating Volumes
$31.5\pi = 28\pi h$
$$h = \frac{31.5}{28} = 1.125 \text{ m} = 1.13 \text{ m(approx)}$$
Therefore, the height of the embankment is 1.13 m (approx).

OR

A hollow sphere whose inner and outer diameters are 4 cm and 8 cm respectively is melted to form a cone whose base is 8 cm in diameter. Find the slant height and curved surface area of the cone.

Solution:

Let's solve the given problem step by step:
Step 1: Find the Volume of the Hollow Sphere
The volume of a sphere is given by: $V = \frac{4}{3}\pi r^3$
The volume of the hollow sphere is: $V = \frac{4}{3}\pi(R^3 - r^3)$
where $R = 4$ cm and $r = 2$ cm (since diameters are 8 cm and 4 cm).
$$V = \frac{4}{3}\pi(4^3 - 2^3)$$
$$= \frac{4}{3}\pi(64 - 8) = \frac{4}{3}\pi \times 56 = \frac{224}{3}\pi \text{ cm}^3$$

Step 2: Find the Height of the Cone
Since volume remains the same, the volume of the cone is:
$$V = \frac{1}{3}\pi r^2 h$$
Given $r = 4$ cm (since base diameter is 8 cm),
$$\frac{1}{3}\pi(4^2)h = \frac{224}{3}\pi$$
$$\frac{16}{3}\pi h = \frac{224}{3}\pi$$
$h = 14$ cm
Step 3: Find the Slant Height l
Using Pythagoras theorem:
$$l = \sqrt{r^2 + h^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212} \approx 14.56 \text{ cm}$$
Step 4: Find the Curved Surface Area
$CSA = \pi rl = \pi(4)(14.56) = 58.24\pi \approx 183.1 \text{ cm}^2$
Therefore, the final answer is:
Slant Height $= 14.56$ cm
Curved Surface Area $= 183.1 \text{ cm}^2$

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