

Secondary Examination, 2024

Mathematics

Time: $3\frac{1}{4}$ hours

M.M-70

Instructions:

- i) All questions are compulsory.
- ii) This question paper has two sections 'A' and 'B'.
- iii) Section 'A' contains 20 multiple choice type questions of 1 mark each that has to be answered on OMR Answer Sheet by darkening completely the correct circle with blue or black ballpoint pen.
- iv) After giving answer on OMR Answer Sheet do not cut or use eraser, whitener etc.
- v) Section 'B' contains descriptive type questions of 50 marks.
- vi) Total 5 questions are there in this section.
- vii) In the beginning of each question it has been mentioned how many parts of it are to be attempted.
- viii) Marks allotted to each question are mentioned against it.
- ix) Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
- x) If you need place for rough work, do it on left page of your answer book and cross (×) the page. Do not write the solution on that page.
- xi) Do not rub off the lines constructed in a question of construction. Do write the steps of construction in brief, if asked.
- xii) Draw neat and correct figure in solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A (Multiple Choice Type Questions)

1. The maximum number of tangents drawn from an external point to a circle will be
 (A) one
 (B) two
 (C) three
 (D) four

Solution:

From any external point, exactly two tangents can be drawn to a circle.

2. The distance of point (7,3) from y -axis will be
 (A) 3

- (B) $\frac{7}{2}$
 (C) 7
 (D) 8

Solution:

The distance of a point (x, y) from the y-axis is given by the absolute value of its x-coordinate.

For the point $(7, 3)$, the distance from the y-axis is $|7| = 7$.

3. If $p \sin \theta = q \cos \theta$, then the value of $\operatorname{cosec} \theta$ will be

- (A) $\frac{\sqrt{p^2+q^2}}{q}$
 (B) $\frac{\sqrt{p^2+q^2}}{p}$
 (C) $\frac{p}{\sqrt{p^2+q^2}}$
 (D) $\frac{q}{\sqrt{p^2+q^2}}$

Solution:

We are given the equation: $p \sin \theta = q \cos \theta$

Dividing both sides by $\cos \theta$:

$$p \tan \theta = q$$

$$\tan \theta = \frac{q}{p}$$

Using the identity:

$$\sin \theta = \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$$

Substituting $\tan \theta = \frac{q}{p}$:

$$\sin \theta = \frac{\frac{q}{p}}{\sqrt{1 + \frac{q^2}{p^2}}} = \frac{\frac{q}{p}}{\sqrt{\frac{p^2 + q^2}{p^2}}} = \frac{q}{\sqrt{p^2 + q^2}}$$

Now compute $\operatorname{cosec} \theta$:

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{\sqrt{p^2 + q^2}}{q}$$

Therefore, the correct answer is $\frac{\sqrt{p^2+q^2}}{q}$.

4. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 88^\circ \tan 89^\circ$ will be

- (A) 0
 (B) $\frac{1}{\sqrt{2}}$

- (C) $\frac{1}{2}$
 (D) 1

Solution:

Given:

$$\begin{aligned} & \tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \dots \tan 89^\circ \\ &= \tan (90^\circ - 89^\circ) \tan (90^\circ - 88^\circ) \tan (90^\circ - 87^\circ) \dots \dots \tan 89^\circ \\ &= \cot 89^\circ \cot 88^\circ \cot 87^\circ \dots \dots \tan 89^\circ [\because \tan (90^\circ - \theta) = \cot \theta] \\ &= \cot 89^\circ \tan 89^\circ \cot 88^\circ \tan 88^\circ \dots \dots \tan 45^\circ \text{ [Since, } \tan \theta \times \cot \theta = 1\text{]} \\ &= 1 \text{ [Since, } \tan 45^\circ = 1\text{]} \end{aligned}$$

5. If the roots of equation $3x^2 + 5x - q = 0$ are equal then the value of q will be

- (A) $-\frac{25}{12}$
 (B) $-\frac{25}{9}$
 (C) $\frac{9}{25}$
 (D) $-\frac{12}{25}$

Solution:

For the quadratic equation $3x^2 + 5x - q = 0$, the roots are equal if the discriminant is zero.

The discriminant Δ is given by: $\Delta = b^2 - 4ac$

Here, $a = 3$, $b = 5$, and $c = -q$.

Set the discriminant to zero:

$$5^2 - 4(3)(-q) = 0$$

$$25 + 12q = 0$$

$$12q = -25$$

$$q = -\frac{25}{12}$$

6. The eleventh term of the A.P. $-62, -59, \dots, 7, 10$ will be

- (A) -34
 (B) -32
 (C) -30
 (D) -28

Solution:

The formula for the n th term of an arithmetic progression (A.P.) is:

$$a_n = a + (n - 1)d$$

Given:

First term, $a = -62$

Common difference, $d = -59 - (-62) = 3$

Eleventh term, $n = 11$

Substituting the values:

$$a_{11} = -62 + (11 - 1)3$$

$$= -62 + 30$$

$$= -32$$

So, the eleventh term is -32 .

7. If $P(E) = 0.05$, then the value of $P(\bar{E})$ will be

(A) 0.92

(B) 0.93

(C) 0.94

(D) 0.95

Solution:

We know that:

$$P(E) + P(\bar{E}) = 1$$

$$0.05 + P(\bar{E}) = 1$$

$$P(\bar{E}) = 1 - 0.05$$

$$P(\bar{E}) = 1 - 0.95$$

8. A bag contains 3 red and 5 black balls. One ball is drawn out at random. The probability of it being red ball will be

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{3}{5}$

(D) $\frac{1}{2}$

Solution:

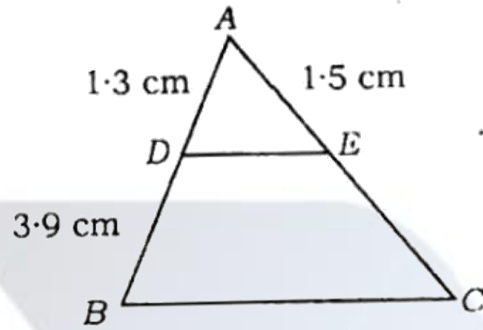
The total number of balls = $3 + 5 = 8$

Number of red balls = 3

Probability of drawing a red ball = $\frac{\text{Number of red balls}}{\text{Total number of balls}} = \frac{3}{8}$

So, the probability is $\frac{3}{8}$.

9. In the figure, $DE \parallel BC$, then the measure of CE will be



- (A) 5.5 cm
- (B) 5.0 cm
- (C) 4.8 cm
- (D) 4.5 cm

Solution:

In figure $DE \parallel BC$,

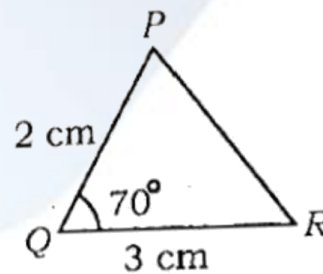
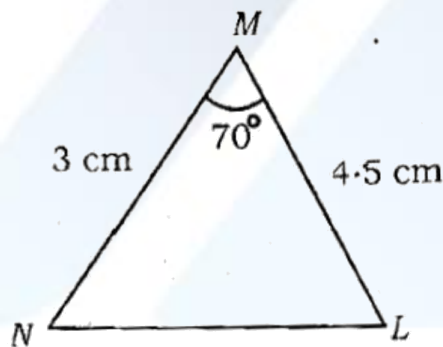
$$\frac{AD}{DB} = \frac{AE}{EC} \text{ (By basic proportionality theorem)}$$

$$\Rightarrow \frac{1.3}{3.9} = \frac{1.5}{EC}$$

$$\Rightarrow EC = 1.5 \times \frac{3.9}{1.3}$$

$$\Rightarrow EC = 4.5 \text{ cm}$$

10. In the figure, in $\triangle MNL$ and $\triangle PQR$, $\angle M = \angle Q = 70^\circ$, $MN = 3$, $ML = 4.5$ cm, $PQ = 2$ cm and $QR = 3$ cm. Then in the following correct will be



- (A) $\triangle NML \sim \triangle QPR$
- (B) $\triangle NML \sim \triangle QRP$
- (C) $\triangle NML \sim \triangle PQR$
- (D) None of these

Solution:

To check similarity between $\triangle MNL$ and $\triangle PQR$, we use the *AA* similarity criterion, which states that two triangles are similar if two corresponding angles are equal.

Given: $\angle M = \angle Q = 70^\circ$

We need to check another pair of corresponding sides for similarity.

Finding the ratio of corresponding sides:

$$\frac{MN}{QP} = \frac{3}{2} = 1.5$$

$$\frac{ML}{QR} = \frac{4.5}{3} = 1.5$$

Since the two corresponding sides have the same ratio and one angle is equal, $\triangle NML \sim \triangle PQR$ by the *SAS* similarity criterion.

So, the correct answer is: $\triangle NML \sim \triangle PQR$

11. The surface of a sphere of diameter $\frac{1}{2}$ cm will be

(A) $\frac{\pi}{2}$ cm²

(B) $\frac{\pi}{4}$ cm²

(C) $\frac{\pi}{3}$ cm²

(D) π cm²

Solution:

The surface area of a sphere is given by the formula: $A = 4\pi r^2$

Given the diameter = $\frac{1}{2}$ cm, the radius r is: $r = \frac{1}{4}$ cm

Now, calculating the surface area:

$$\begin{aligned} A &= 4\pi \left(\frac{1}{4}\right)^2 \\ &= 4\pi \times \frac{1}{16} \\ &= \frac{\pi}{4} \text{ cm}^2 \end{aligned}$$

Thus, the correct answer is: $\frac{\pi}{4}$ cm²

12. An arc of a circle of radius 6 cm subtends an angle of 30° at the centre. The measure of corresponding arc will be

(A) $\frac{\pi}{4}$ cm

(B) $\frac{\pi}{3}$ cm

(C) $\frac{\pi}{2}$ cm

(D) π cm

Solution:

The length of an arc is given by the formula: $L = \frac{\theta}{360^\circ} \times 2\pi r$

Given: $r = 6$ cm, $\theta = 30^\circ$

$$L = \frac{30^\circ}{360^\circ} \times 2\pi \times 6$$

$$L = \frac{1}{12} \times 12\pi$$

$$L = \pi \text{ cm}$$

Thus, the correct answer is π cm.

13. The tangent PQ of a circle of radius 5 cm meets at a point Q on the line passing through the centre O . If $OQ = 12$ cm, then the measure of PQ will be
- (A) 12 cm
 (B) 13 cm
 (C) 8.5 cm
 (D) $\sqrt{119}$ cm

Solution:

We use the tangent-secant theorem, which states that in a right-angled triangle formed by the radius, tangent, and line joining the center to the external point:

$$OQ^2 = OP^2 + PQ^2$$

Given: $OQ = 12$ cm, $OP = 5$ cm (radius)

Applying the Pythagorean theorem:

$$PQ^2 = OQ^2 - OP^2$$

$$PQ^2 = 12^2 - 5^2$$

$$PQ^2 = 144 - 25 = 119$$

$$PQ = \sqrt{119} \text{ cm}$$

Thus, the measure of PQ is $\sqrt{119}$ cm.

14. The HCF of the numbers 182 and 78 will be
- (A) 13
 (B) 26
 (C) 28
 (D) 39

Solution:

To find the HCF of 182 and 78:

Find the prime factorizations:

$$182 = 2 \times 7 \times 13$$

$$78 = 2 \times 3 \times 13$$

The common factors in both numbers are 2 and 13.

Multiply the common factors: $2 \times 13 = 26$

Thus, the correct answer is 26.

15. The radius of the base of a cylinder is 3.5 cm . If its height be 8.4 cm , then its curved surface area will be
- (A) $54.8\pi \text{ cm}^2$
 (B) $56.4\pi \text{ cm}^2$
 (C) $56.6\pi \text{ cm}^2$
 (D) $58.8\pi \text{ cm}^2$

Solution:

The formula for the curved surface area (CSA) of a cylinder is: $CSA = 2\pi rh$

Given:

$$r = 3.5 \text{ cm}$$

$$h = 8.4 \text{ cm}$$

$$CSA = 2\pi(3.5)(8.4)$$

$$CSA = 2\pi \times 29.4$$

$$CSA = 58.8\pi \text{ cm}^2$$

Thus, the correct answer is $58.8\pi \text{ cm}^2$.

16. The angle of a sector of a circle of radius 4 cm is 60° . Its area will be
- (A) $6\pi \text{ cm}^2$
 (B) $8\pi \text{ cm}^2$
 (C) $\frac{8}{3}\pi \text{ cm}^2$
 (D) $3\pi \text{ cm}^2$

Solution:

The area of a sector is given by the formula: $\text{Area} = \frac{\theta}{360^\circ} \times \pi r^2$

Given: $\theta = 60^\circ, r = 4 \text{ cm}$

$$\text{Area} = \frac{60^\circ}{360^\circ} \times \pi(4)^2$$

$$= \frac{1}{6} \times \pi \times 16$$

$$= \frac{16}{6} \pi$$

$$= \frac{8}{3} \pi \text{ cm}^2$$

Thus, the correct answer is: $\frac{8}{3} \pi \text{ cm}^2$

17. The discriminant of the quadratic equation $x^2 + x - 1 = 0$ will be
- (A) -4
 (B) 5

- (C) 4
(D) 2

Solution:

The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac$$

For the equation $x^2 + x - 1 = 0$:

$$a = 1,$$

$$b = 1,$$

$$c = -1.$$

$$\Delta = (1)^2 - 4(1)(-1) = 1 + 4 = 5$$

18. The sum of the roots of the quadratic equation $1 - 4x + 4x^2 = 0$ will be
(A) -2
(B) -1
(C) 1
(D) 2

Solution:

The given quadratic equation is: $1 - 4x + 4x^2 = 0$

Rewriting it in standard form: $4x^2 - 4x + 1 = 0$

For a quadratic equation of the form $ax^2 + bx + c = 0$, the sum of the roots is given by: $-\frac{b}{a}$

Here, $a = 4$ and $b = -4$,

$$\text{So, sum of the roots} = -\frac{-4}{4} = \frac{4}{4} = 1$$

19. The mean from the following table will be

Class-interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	4	7	5	8	6

- (A) 24.62
(B) 26.66
(C) 28.64
(D) 30.50

Solution:

To find the mean, use the formula: $\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$

Where:

x_i = Midpoint of each class interval

f_i = Frequency

Step 1: Calculate midpoints (x_i)

$$x_i = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Class Interval	Frequency (f_i)	Midpoint (x_i)	$f_i x_i$
0 – 10	4	5	20
10 – 20	7	15	105
20 – 30	5	25	125
30 – 40	8	35	280
40 – 50	6	45	270

Step 2: Compute the mean

$$\sum f_i = 4 + 7 + 5 + 8 + 6 = 30$$

$$\sum f_i x_i = 20 + 105 + 125 + 280 + 270 = 800$$

$$\text{Mean} = \frac{800}{30} = 26.66$$

20. The median class of the following table will be

Class-interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	6	11	18	6

(A) 10 – 20

(B) 20 – 30

(C) 30 – 40

(D) 40 – 50

Solution:

To find the median class, we follow these steps:

Step 1: Calculate the total frequency (N):

$$N = 8 + 6 + 11 + 18 + 6 = 49$$

Step 2: Find $\frac{N}{2}$:

$$\frac{49}{2} = 24.5$$

The median class is the class where the cumulative frequency just exceeds 24.5 .

Step 3: Calculate cumulative frequency (CF):

- 0 – 10 → 8

- $10 - 20 \rightarrow 8 + 6 = 14$
- $20 - 30 \rightarrow 14 + 11 = 25$ (crosses 24.5)
- $30 - 40 \rightarrow 25 + 18 = 43$
- $40 - 50 \rightarrow 43 + 6 = 49$

Since 25 is the first cumulative frequency exceeding 24.5, the median class is 20 – 30.

Section - B (Descriptive questions)

1. Do all the parts:

(a) If the distance between the points $(x, 5)$ and $(2, -3)$ is 17 units, then find the value of x .

Solution:

The distance formula is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given points $(x, 5)$ and $(2, -3)$, and distance $d = 17$:

$$17 = \sqrt{(2 - x)^2 + (-3 - 5)^2}$$

$$17 = \sqrt{(2 - x)^2 + (-8)^2}$$

$$17 = \sqrt{(2 - x)^2 + 64}$$

Squaring both sides:

$$289 = (2 - x)^2 + 64$$

$$225 = (2 - x)^2$$

Taking square root:

$$2 - x = \pm 15$$

Solving for x :

$$x = 2 \mp 15$$

$$x = 2 - 15 = -13 \text{ or } x = 2 + 15 = 17$$

So, the final answer is: $x = -13$ or $x = 17$

(b) If the points $(1,4)$, $(a, -2)$ and $(-3,16)$ are collinear, then find the value of a .

Solution:

The given points $(1,4)$, $(a, -2)$, and $(-3,16)$ are collinear, so their slopes must be equal.

Step 1: Find the slope between $(1,4)$ and $(a, -2)$

$$m_1 = \frac{-2 - 4}{a - 1} = \frac{-6}{a - 1}$$

Step 2: Find the slope between $(a, -2)$ and $(-3,16)$

$$m_2 = \frac{16 - (-2)}{-3 - a} = \frac{18}{-3 - a}$$

Step 3: Set slopes equal

$$\frac{-6}{a-1} = \frac{18}{-3-a}$$

Step 4: Cross multiply

$$-6(-3-a) = 18(a-1)$$

$$18 + 6a = 18a - 18$$

Step 5: Solve for a

$$18 + 18 = 18a - 6a$$

$$12a = 36$$

$$a = 3$$

(c) Prove that: $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$.

Solution:

$$\text{Given: } \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$$

L.H.S:

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

Since $\sin^2\theta + \cos^2\theta = 1$, we get:

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2}{\cos\theta}$$

$$= 2\sec\theta$$

$$= \text{R.H.S}$$

Hence proved.

(d) Find the median from the following frequency distribution:

Class-interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	9	20	15	9

Solution:

To find the median of the given frequency distribution, follow these steps:

Step 1: Compute the cumulative frequency (CF)

Class Interval	Frequency (f)	Cumulative Frequency (CF)
0 – 10	6	6

10 – 20	9	15
20 – 30	20	35
30 – 40	15	50
40 – 50	9	59

Step 2: Find the median class

Total frequency, $N = 59$

$$\text{Median position} = \frac{N}{2} = \frac{59}{2} = 29.5$$

The median class is 20 – 30 (since CF just before 29.5 is 15, and 35 includes 29.5).

Step 3: Apply the median formula

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h$$

Where:

- $L = 20$ (Lower boundary of median class)
- $\frac{N}{2} = 29.5$
- $CF = 15$ (Cumulative frequency before median class)
- $f = 20$ (Frequency of median class)
- $h = 10$ (Class width)

$$\begin{aligned} \text{Median} &= 20 + \left(\frac{29.5 - 15}{20} \times 10 \right) \\ &= 20 + \left(\frac{14.5}{20} \times 10 \right) \\ &= 20 + (0.725 \times 10) \\ &= 20 + 7.25 \\ &= 27.25 \end{aligned}$$

(e) Find the LCM of the numbers 92 and 510.

Solution:

To find the LCM of 92 and 510, find prime factorization:

$$92 = 2^2 \times 23$$

$$510 = 2 \times 3 \times 5 \times 17$$

Take the highest powers of all prime factors:

$$LCM = 2^2 \times 3 \times 5 \times 17 \times 23$$

Multiply the Factors:

$$4 \times 3 = 12$$

$$12 \times 5 = 60$$

$$60 \times 17 = 1020$$

$$1020 \times 23 = 23460$$

Thus, LCM (92, 510) = 23,460

(f) Prove that $\sqrt{3}$ is an irrational number.

Solution:

Let us assume on the contrary that $\sqrt{3}$ is a rational number.

Then, there exist positive integers a and b such that $\sqrt{3} = \frac{a}{b}$ where, a and b , are co-prime i.e. their HCF is 1.

Now,

$$\sqrt{3} = \frac{a}{b}$$

$$\Rightarrow 3 = \frac{a^2}{b^2}$$

$$\Rightarrow 3b^2 = a^2$$

$$\Rightarrow 3 \text{ divides } a^2 \text{ [}\because 3 \text{ divides } 3b^2\text{]}$$

$$\Rightarrow 3 \text{ divides } a \text{ ... (i)}$$

$$\Rightarrow a = 3c \text{ for some integer } c.$$

$$\Rightarrow a^2 = 9c^2$$

$$\Rightarrow 3b^2 = 9c^2 \text{ [}\because a^2 = 3b^2\text{]}$$

$$\Rightarrow b^2 = 3c^2$$

$$\Rightarrow 3 \text{ divides } b^2 \text{ [}\because 3 \text{ divides } 3c^2\text{]}$$

$$\Rightarrow 3 \text{ divides } b \text{ ... (ii)}$$

From (i) and (ii), we observe that a and b have at least 3 as a common factor. But, this contradicts the fact that a and b are co-prime. This means that our assumption is not correct.

Hence, $\sqrt{3}$ is an irrational number.

2. Do any five parts:

(a) Find the mode from the following table:

Class-interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	11	21	23	14

Solution:

To find the mode from the given frequency distribution table, we use the mode formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where:

- L = lower boundary of the modal class
- f_1 = frequency of the modal class
- f_0 = frequency of the class before the modal class
- f_2 = frequency of the class after the modal class
- h = class width

Step 1: Identify the Modal Class

The modal class is the class interval with the highest frequency.

From the given table:

Class Interval	Frequency
0 – 10	6
10 – 20	11
20 – 30	21
30 – 40	23
40 – 50	14

So, the modal class is 30 – 40.

Step 2: Identify the values

- $L = 30$ (lower boundary of the modal class)
- $f_1 = 23$ (frequency of the modal class)
- $f_0 = 21$ (frequency of the class before the modal class)
- $f_2 = 14$ (frequency of the class after the modal class)
- $h = 10$ (class width)

Step 3: Apply the Formula

$$\text{Mode} = 30 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10$$

$$= 30 + \left(\frac{2}{46 - 35} \right) \times 10$$

$$= 30 + \left(\frac{2}{11} \right) \times 10$$

$$= 30 + \left(\frac{20}{11} \right)$$

$$= 30 + 1.82$$

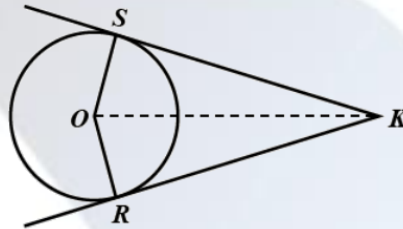
$$= 31.82$$

Thus, the mode is approximately 31.82.

(b) Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

The attached figure shows two tangents, SK and SR drawn to circle with centre O from an external point K.



To prove that: $SK = RK$

Proof:

Normal and tangent at a point on the circle are perpendicular to each other.

$$\angle OSK = \angle ORK = 90^\circ$$

Using Pythagoras Theorem,

$$OK^2 = OS^2 + SK^2 \dots(i)$$

$$OK^2 = OR^2 + RK^2 \dots(ii)$$

Subtracting (ii) from (i),

$$OK^2 - OK^2 = OS^2 + SK^2 - OR^2 - RK^2$$

$$\Rightarrow SK^2 = RK^2 \quad [\because OS = OR]$$

$$\Rightarrow SK = RK$$

Hence, proved.

(c) D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Prove that $CA^2 = CB \times CD$.

Solution:

Proof:

Step 1: Consider the given angles

Since $\angle ADC = \angle BAC$, we observe that triangles $\triangle ABC$ and $\triangle ADC$ are related by angle similarity.

Step 2: Establishing Similarity

In $\triangle ABC$ and $\triangle ADC$:

$$\angle ADC = \angle BAC \text{ (Given)}$$

$\angle ACB$ is common in both triangles.

Thus, by the AA similarity criterion, we conclude that: $\triangle ABC \sim \triangle ADC$

Step 3: Writing the Proportionality Relation

Since $\triangle ABC \sim \triangle ADC$, we get the proportionality: $\frac{CA}{CB} = \frac{CD}{CA}$

Step 4: Cross Multiplication

Rearranging the proportion: $CA \times CA = CB \times CD$

which simplifies to:

$$CA^2 = CB \times CD$$

Hence proved.

(d) The sum of the digits of a two-digit number is 9. Nine times this number is twice the number formed by reversing its digits. Find the number.

Solution:

Let the two-digit number be $10x + y$, where x and y are its digits.

Given conditions:

$$x + y = 9 \quad \dots(i)$$

$$9(10x + y) = 2(10y + x) \quad \dots(ii)$$

Expanding the second equation:

$$90x + 9y = 20y + 2x$$

$$90x - 2x = 20y - 9y$$

$$88x = 11y$$

$$8x = y$$

From $x + y = 9$, substitute $y = 8x$:

$$x + 8x = 9$$

$$9x = 9$$

$$x = 1$$

$$y = 8$$

Thus, the number is 18.

(e) Solve the quadratic equation $2x^2 - 5x + 3 = 0$.

Solution:

The quadratic equation is: $2x^2 - 5x + 3 = 0$

Using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Here, $a = 2$, $b = -5$, and $c = 3$

Calculate the discriminant:

$$b^2 - 4ac = (-5)^2 - 4(2)(3) = 25 - 24 = 1$$

Find the roots:

$$x = \frac{5 \pm \sqrt{1}}{2(2)}$$

$$x = \frac{5 \pm 1}{4}$$

$$x_1 = \frac{5 + 1}{4} = \frac{6}{4} = \frac{3}{2}, \quad x_2 = \frac{5 - 1}{4} = \frac{4}{4} = 1$$

So, the final answer is: $x = \frac{3}{2}, 1$

(f) If the perimeter of a garden of area 800 m^2 is 120 m and its length is twice the breadth, then find its length and breadth.

Solution:

Let the breadth of the garden be x metres.

Since the length is twice the breadth, the length = $2x$ metres.

Given conditions:

$$\text{Perimeter} = 120 \text{ m}$$

$$2(l + b) = 120$$

$$2(2x + x) = 120$$

$$6x = 120$$

$$x = 20$$

So, breadth = 20 m , and length = 40 m

3. Solve the following equations:

$$\frac{3}{2}x - \frac{5}{3}y = -2, \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

Solution:

We have the system of equations:

$$\frac{3}{2}x - \frac{5}{3}y = -2$$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

Step 1: Eliminate Fractions

Multiply the first equation by 6 (LCM of 2 and 3):

$$9x - 10y = -12$$

Multiply the second equation by 6:

$$2x + 3y = 13$$

Step 2: Solve by Elimination

Multiply the second equation by 3 to align coefficients of x :

$$6x + 9y = 39$$

Multiply the first equation by 2:

$$18x - 20y = -24$$

Now solve by elimination:

Multiply the second equation by 3:

$$6x + 9y = 39$$

Multiply by 3:

$$18x + 27y = 117$$

Subtract:

$$(18x + 27y) - (18x - 20y) = 117 + 24$$

$$47y = 141$$

$$y = 3$$

Step 3: Solve for x

Substituting $y = 3$ into $2x + 3y = 13$:

$$2x + 9 = 13$$

$$2x = 4$$

$$x = 2$$

Therefore, $x = 2$, $y = 3$

OR

Find the sum of 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Solution:

Given:

$$\text{Second term } a_2 = 14$$

$$\text{Third term } a_3 = 18$$

Using the formula for the n th term of an A.P.:

$$a_n = a + (n - 1)d$$

For the second term:

$$a + d = 14$$

For the third term:

$$a + 2d = 18$$

Step 1: Find a and d

Subtract the first equation from the second:

$$(a + 2d) - (a + d) = 18 - 14$$

$$d = 4$$

Substituting $d = 4$ in $a + d = 14$:

$$a + 4 = 14$$

$$a = 10$$

Step 2: Find Sum of 51 Terms

The sum of n terms of an A.P. is: $S_n = \frac{n}{2}[2a + (n - 1)d]$

For $n = 51$:

$$S_{51} = \frac{51}{2}[2(10) + (51 - 1)(4)]$$

$$= \frac{51}{2}[20 + 200]$$

$$= \frac{51}{2} \times 220$$

$$= \frac{11220}{2}$$

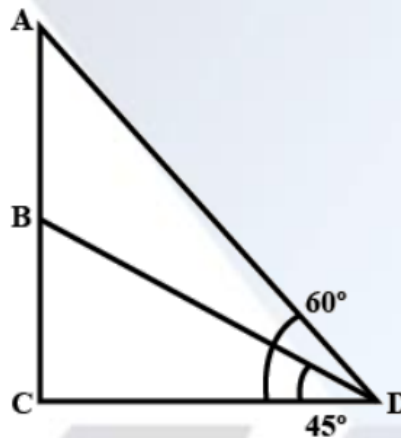
$$= 5610$$

So, the final answer is: $S_{51} = 5610$

4. A flagstaff stands on a tower. At a distance 10 m from the tower the angles of elevation of the top of the tower and flagstaff are 45° and 60° respectively. Find the length of the flagstaff.

Solution:

Let the height of the tower (BC) be h metres and the height of the flagstaff (AB) be H metres. The observer is 10 m away from the base of the tower.



Step 1: Find h

Using the angle of elevation of the top of the tower (45°):

$$\tan 45^\circ = \frac{h}{10}$$

Since $\tan 45^\circ = 1$,

$$h = 10 \text{ m}$$

Step 2: Find H

Using the angle of elevation of the top of the flagstaff (60°):

$$\tan 60^\circ = \frac{H}{10}$$

Since $\tan 60^\circ = \sqrt{3}$,

$$H = 10\sqrt{3} \text{ m}$$

Step 3: Find the Flagstaff's Height

The height of the flagstaff is:

$$H - h = 10\sqrt{3} - 10 = 10(\sqrt{3} - 1) \text{ m}$$

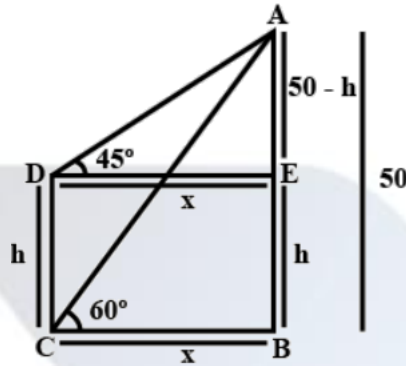
Therefore, the length of the flagstaff is $10(\sqrt{3} - 1)$ m.

OR

From the top of tower of height 50 m, the angle of depression of top and bottom of a pillar are 45° and 60° respectively. Find the height of the pillar.

Solution:

Let the height of the pole be h and the distance between the pole and tower be x .



It can be observed that,

$$\tan 60^\circ = \frac{50}{x}$$

$$\sqrt{3} = \frac{50}{x}$$

$$x = \frac{50}{\sqrt{3}}$$

And,

$$\tan 45^\circ = \frac{50 - h}{x}$$

$$1 = \frac{50 - h}{x}$$

$$50 - h = x$$

$$50 - h = \frac{50}{\sqrt{3}}$$

$$h = 50 - \frac{50}{\sqrt{3}}$$

$$h = \frac{50(\sqrt{3} - 1)}{\sqrt{3}}$$

$$h = 21.1 \text{ m}$$

Therefore, the height of the pole is 21.1 m.

5. A chord of a circle of radius 15 cm subtends an angle 60° at the centre. Find the area of minor and major sectors of the circle. ($\pi = 3.14, \sqrt{3} = 1.73$)

Solution:

We are given a circle with radius $r = 15$ cm, and a chord subtending an angle of 60° at the centre.

The area of a sector with central angle θ in degrees is given by:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

For the minor sector, $\theta = 60^\circ$:

$$\begin{aligned} \text{Area of minor sector} &= \frac{60^\circ}{360^\circ} \times 3.14 \times (15)^2 \\ &= \frac{1}{6} \times 3.14 \times 225 \\ &= \frac{706.5}{6} = 117.75 \text{ cm}^2 \end{aligned}$$

Since the total area of the circle is: $\pi r^2 = 3.14 \times 225 = 706.5 \text{ cm}^2$

The area of the major sector is:

$$\begin{aligned} \text{Area of major sector} &= \text{Total area} - \text{Area of minor sector} \\ &= 706.5 - 117.75 = 588.75 \text{ cm}^2 \end{aligned}$$

OR

By taking out a hemisphere from both the ends of a wooden solid cylinder, an item is formed. If the height of the cylinder is 10 cm and radius of base is 3.5 cm, then find the total surface area of the item.

Solution:

Given:

Height of the cylinder = 10 cm

Radius of the base = 3.5 cm

Step 1: Calculate the curved surface area (CSA) of the cylinder

$$\begin{aligned} \text{CSA of cylinder} &= 2\pi rh \\ &= 2 \times 3.14 \times 3.5 \times 10 \\ &= 219.8 \text{ cm}^2 \end{aligned}$$

Step 2: Calculate the surface area of two hemispheres

Each hemisphere has a curved surface area of:

$$\begin{aligned} \text{CSA of one hemisphere} &= 2\pi r^2 \\ &= 2 \times 3.14 \times (3.5)^2 \\ &= 76.93 \text{ cm}^2 \end{aligned}$$

For two hemispheres:

$$\text{Total CSA of hemispheres} = 2 \times 76.93 = 153.86 \text{ cm}^2$$

Step 3: Find total surface area

$$\begin{aligned} \text{Total Surface Area} &= \text{CSA of cylinder} + \text{CSA of two hemispheres} \\ &= 219.8 + 153.86 \\ &= 373.66 \text{ cm}^2 \end{aligned}$$

Therefore, the total surface area of the item is 373.66 cm^2 .